

Uncertainty Quantification for Modeling and Simulation with Calibration

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Abstract—Calibration improves the consistency between simulation results and test data of a system, but it doesn't mean that the epistemic uncertainty of Modeling and Simulation (M&S) for subsystem is reduced, so propagation analysis with many uncertain inputs often leads to an overvaluation of uncertainty. As new system-level test is unavailable, it is unpractical to quantify M&S uncertainty with comparison between simulation results and test data. Taking advantage of the fact that calibration reduces the epistemic uncertainty of system-level simulation, we propose a method for Uncertainty Quantification (UQ), in which the uncertainty from comparison with existing system-level test data and the propagated uncertainty induced by additional cognitive defect for new system are used rationally. An example with virtual tests is displayed in which the method is demonstrated and validated.

Keywords—uncertainty quantification; modeling & simulation; calibration; verification & validation; reliability certification.

I. INTRODUCTION

M&S needs to experience verification, validation and accreditation (VV&A) procedure to assess its credibility for intended use [1][2]. However, it is still difficult to detect and eliminate all drawbacks even if the verification and validation are adequately implemented. Additionally, owing to inevitable discretization errors, simulation results of complicated physical processes often have systematic errors. Calibration is then used to rectify errors and improve consistency between simulation results and test data. It is a long-standing case to predict the performance of a new engineering system with calibrated codes. When system-level test could not be fulfilled for a new system, it would be a great challenge for engineering design or reliability certification to quantify the uncertainty of prediction offered by M&S [3].

In many cases, a new system is only a modified version of its prototype. Some system-level test data for the prototype generally exist and could be used for calibration and uncertainty quantification [4]. The differences between a new system and its prototype are mainly caused by redesign or by state shift arising from long period stockpile, which may bring on recertification or assessment in engineering. In the case that system-level test is forbidden, numerical simulations for the modified design parameters or the additional engineering factors, and uncertainty quantification

of the simulation results are consequently the main approaches to supply information for the recertification and assessment.

According to the concept of Verification and Validation (V&V), the parameter space of an engineering system and its environment may be divided into application domain and validation domain which corresponding to the new system and its prototype respectively. A complex system may be divided into an arbitrary number of progressively simpler hierarchy tiers [1]. Without system-level test of the new system, the uncertainty information of M&S in application domain may have two sources, one is obtained by extrapolation from the uncertainty in validation domain which is quantified by comparison between the simulation results and the existent system-level test data [4], the other is obtained by propagation of the M&S uncertainty from lower level tiers to system level tier [5].

The uncertainty quantification with single information source has been widely studied, such as the UQ method based on comparison and propagation. In a probability frame, Oberkampf and Roy offered a quantification method of M&S errors according to comparison between simulation results and the statistics of test data such as sample average and standard deviation [1]. Helton gave a discussion about sensitivity analysis and Monte Carlo sampling used for uncertainty propagation [6]. Liu et al. used non-intrusive polynomial chaos to quantify the propagation of parameter uncertainties in Jones-Wilkins-Lee equation-of-state (JWL-EOS) for explosive in a detonation system [7]. With the assumption that new system-level test can not be implemented, Ma et al. put forward a method to extrapolate the uncertainties from validation domain to application domain [4][8]. Up to now, it is still a choke point for UQ of M&S that how to fuse two kinds of information that comes from comparison and propagation, respectively.

Techniques of information fusion have become more and more important for reliability analysis as the data lack is just about a ubiquitous problem. Information from comparison and propagation are obtained from different cognitive approaches. It is necessary in engineering and rational in science to fuse them.

The uncertainties of M&S are mainly epistemic and are suitably represented or fused with interval theory. The UQ method should obey two basic principles, the unknown true value should be covered by the estimated uncertainty interval and the estimation of the uncertainty should be minimized

based on the available information [4]. If the estimation is only based on the extrapolated uncertainty originated by comparison, the additional cognitive defect of M&S for a new system may be neglected, and the true-value-covered principle may be violated on account of underestimation of the uncertainty. As a result, the risk to accept an unreliable system may be augmented. If the estimation is based on direct summation of the uncertainties from comparison and propagation, the estimated uncertainty may be irrationally magnified which may lead to violating the uncertainty-minimized principle and consequently increase the risk to reject a reliable system.

The paper is organized as follows. In Section 2, properties of M&S experienced calibration are analyzed. Section 3 offers a quantification method of total uncertainty of M&S based on information fusion, in which the basic component is an extrapolated uncertainty from comparison on system level, and an incremental component is the propagated uncertainty related to new system. Section 4 gives a comparison-based UQ methods needed in Section 3. An example to show these methods is displayed in Section 5 with a shock problem. Finally, we give a conclusion in Section 6.

II. PROPERTIES OF CALIBRATED M&S

There are two approaches to improve consistency between simulation results and test data. One is to enhance the cognitive ability by which the epistemic uncertainties in M&S are reduced. This is also an ideal approach for M&S development. The other is based on existent cognitive ability, to make artificially errors produced in M&S compensated with each other. Calibration of M&S depends mostly on the mechanism of error compensations. However, when M&S is used as prediction, the calibration only works well as the modification of the new system is not very large compared to the systems on which the calibration is made.

In this paper, the mathematical model is divided into an entity model and a physics model. The former represents the specific engineering system depicted by design parameters, such as material type, shape, size, mass, and initial or boundary conditions when the system works. The latter represents the abstract laws of hylic world, such as equations of state and constitution, turbulence model, detonation model, the universal conservation equations of mass, momentum, and energy etc. The uncertainties of physics models are usually greater than that of entity models, as dynamic measuring and converse reckon are generally involved to determine the parameters and forms of physics models.

Calibration is achieved by comparison with test data, in which the forms and parameters of models, methods and parameters of computation, knobs, and computer code are adjusted and then fixed. Knobs here are referred to the ad hoc parameters added to a model to simply obtain agreement with test data but lack definite physics significances or lack actual evaluating information [2]. Via sufficient verification and validation, knobs could be reduced but it is difficult to eliminate absolutely due to the existence of discretization errors and the deficiency in modeling and simulation for complex systems [9].

Generally, calibration is executed based on a range of entity models, to which we call calibration domain in the model space. Validation activities executed after calibration also have their validation domains, in which the M&S uncertainties may be quantified according to test data. After calibration is finished, the computer code and the parameters that need adjustment should be fixed for intended use. The fixation is usually relative and periodical, as the evaluation on parameters in physics model probably depend on methods and parameters of numerical computation under the expectation of good agreement with test data. The version of code and the parameters of physics models may vary with the development of M&S.

Comparison and propagation are two basic approaches to gain information of uncertainties and, from the point of methodology on cognition, they are pertaining to induction and deduction, respectively. As the former is based on practice and observation to apperceive the realities, information obtained from comparison is generally with an inherent credibility than that from propagation and it may dominate in information fusions when conflict occurs between them.

The characters in numerical simulations with calibration are summarized as follows:

- Uncertainties on system level can be effectively reduced by calibration. However, as the entity model departing from calibration domain, the error compensation may be gradually fading away and the simulation results for a new system may have lager deviations from the true values;
- Uncertainties obtained by comparison in validation domain could be extrapolated into application domain, while the extrapolated uncertainties do not include the uncertainties that introduced by additional cognitive defect of the M&S for a new system in application domain;
- Calibration can not reduce certainly the M&S uncertainties under system level, so the traditional propagation gives usually an overestimation of the M&S uncertainties on system level;
- The epistemic M&S uncertainties that come from comparison in validation domain should have a dominate weight than that come from propagation when information fusion is implemented;
- Without system-level test of new system, there are two independent information sources of M&S uncertainties for system level. One is that from the comparison in validation domain and the other is the additional uncertainty propagated from under system levels which induced by extra cognitive defects that M&S encounters. They can be fused based on interval theory and their additive property.

III. UQ OF M&S WITH CALIBRATION

The most important problem is to fuse information from comparison and propagation and to keep the UQ method observe the true-value-covered and uncertainty-minimized principles [4].

It is known that both aleatory and epistemic uncertainties can be quantified by test, but only the epistemic uncertainties can be reduced by test data. For nondeterministic M&S, the aleatory uncertainties that come from comparison and propagation respectively could be depicted by probability and be fused by Bayesian theory with the weight relative to their information quantities [10][11]. As the deterministic M&S only produces epistemic uncertainties, uncertainties from propagation should only be distributed little or zero weight in fusions when uncertainties from comparison exist.

In the case that system-level test of a new system is unavailable and no direct information from comparison could be used, we consider the following schemes for uncertainty quantification:

- Only use the extrapolated uncertainty from comparison in validation domain;
- Only use the uncertainty from propagation;
- Use both of above information.

The first scheme may leave out the additional uncertainties induced by the extra cognitive defects for new systems. The second scheme does not make use of the existing comparison to reduce the epistemic uncertainty and the uncertainty induced by propagation method and numerical computations are difficult to be quantified into a total M&S-uncertainty on system level. The third scheme has the most reasonable idea, but needs a proper design to avoid the disadvantages appearing in the former two schemes.

Based on the third scheme, we disassemble the uncertainty from propagation ($U_{\Delta}^{propagation}$) to two parts. One refers to the propagated uncertainty for the systems in validation domain ($U_{validation}^{propagation}$). The other refers to an additional propagated uncertainty induced by an extra cognitive defects of new systems in application domain ($U_{\Delta}^{propagation}$). They have an approximate relationship

$$U_{application}^{propagation} \approx U_{validation}^{propagation} + U_{\Delta}^{propagation}. \quad (1)$$

According to the three sources of M&S uncertainty, we have

$$U_{application}^{propagation} \approx \sum_{i=1}^3 U_{application}^{propagation} \approx \sum_{i=1}^3 (U_{validation}^{propagation} + U_{\Delta}^{propagation}), \quad (2)$$

where $1U_{propagation}$, $2U_{propagation}$, $3U_{propagation}$ represent the propagated uncertainties on system level that born from entity modeling, physics modeling and numerical computation, respectively.

In validation domain, as the information from comparison has an overwhelming weight than that from propagation, we neglect the contribution of $U_{validation}^{propagation}$ for information fusion. In application domain, the additional propagated uncertainty $U_{\Delta}^{propagation}$ has no corresponding uncertainty from comparison, so $U_{\Delta}^{propagation}$ must be counted wholly in the total M&S uncertainty.

Thus, the M&S uncertainty for a new system is composed of two components and they have an additive property, which is represented as

$$U_{application}^{M\&S} \approx U_{application}^{extrapolation} + U_{\Delta}^{propagation} \quad (3)$$

So, the steps of UQ for M&S may be arranged as follows:

- To calibrate the M&S with available test data and finish the fixation of concerned parameters and the computer code;
- To quantify M&S uncertainty based on comparison with test data in validation domain;
- To extrapolate uncertainties in validation domain to obtain M&S uncertainty ($U_{application}^{extrapolation}$) for the new system based on the relationship between uncertainties and parameters of the entity model;
- To quantify the additional propagated M&S uncertainty ($U_{\Delta}^{propagation}$) for the new system;
- To obtain the total M&S uncertainties of the new system by (3).

IV. UQ BASED ON COMPARISON

Test data may have aleatory uncertainties owing to the randomness in manufacture of physical models and in test measure. As these uncertainties are essentially not induced by deterministic M&S, it is necessary for a comparison-based UQ to build properly statistics to get rid of the impacts of the aleatory uncertainties on the quantification of epistemic uncertainties.

There are two cases when comparison is carried out:

- One simulation to one test (one-to-one);
- One simulation to many tests (one-to-many).

In the case of one-to-one, each physical model to undergo test must be measured and the results are used for M&S. The simulation results and the test data have one-to-one relationship. The difference between them after test error is recouped reflect directly the error of M&S for this physical model. If the number of test or simulation is n , we have M&S uncertainty as:

$$U^{m\&s} = \text{MAX} |y_i^{m\&s} - y_i^{test}| + \delta U^{test}, \quad i = 1, 2, \dots, n, \quad (4)$$

where $y_i^{m\&s}$ and y_i^{test} are results of M&S and test data of physical model i , respectively. U^{test} is uncertainty of the test data. As n is small δ is advised to be evaluated as $+1$ to evade the risk of underestimation for $U^{m\&s}$. But as n is great enough, δ could be evaluated as 0 or -1 to evade the risk of overestimation.

The case of one-to-many corresponds to the repeated tests, in which the mathematical modeling is based just on one set of parameters that evaluated generally from the average values of design for the physical models, and only one set of M&S result is output. Although all physical models are manufactured according to a same design, their test results may be stochastic owing to the random of manufacture and test measurement.

In order to screen the interference of aleatory uncertainty induced by these random factors, we suggest to dig out the M&S uncertainty from the difference between the M&S result and the average of the test data,

$$\bar{y}^{test} = \frac{1}{n} \sum_{i=1}^n y_i^{test}, \quad (5)$$

$$s = \left(\frac{1}{n-1} \sum_{i=1}^n (y_i^{test} - \bar{y}^{test})^2 \right)^{1/2}, \quad (6)$$

$$U^{m\&s} = |y^{m\&s} - \bar{y}^{test}| + t_{(1-\beta)/2, \nu} \cdot \frac{s}{\sqrt{n}}. \quad (7)$$

In the following example, we assign the confidence $\beta=0.95$, and $t_{(1-\beta)/2, \nu}$ is the quantile of $(1-\beta)/2$ for t -distribution with the freedom of $\nu = n - 1$.

When n is great enough and U^{test} is exactly estimated, the result from (4) and that from (5)-(7) should be accordant with each other.

V. AN EXAMPLE

A. Description of System

The system is simply composed of a 1-dimensional shock problem as shown in Figure 1, in which the left part is a high-pressure gas and the right part is a high-density metal with initial parameters of pressure, density, inner energy and velocity p_1, ρ_1, e_1, v_1 and p_2, ρ_2, e_2, v_2 , respectively, where v_1, v_2, e_2, p_2 are zero. A left-marching expanding wave and a right-marching shock wave occur from the material interface. The metal is pushed by the high pressure of gas to move rightward. The response of interest for this system is the interface displacement D toward right when time is at $10 \mu s$.

B. Design of Tests

Tests are needed in the procedures of calibration and validation to prove the UQ method is valid or not. As the exact solution of system response D^* exists, we use virtual tests to replace the real tests. It is needed to predefine a set of entity and physics models that express the real aging of the products with different stockpile time. In this paper, we call them as true aging models, which are only used to give virtual test data and are not necessary to be displayed in the context, as if in real tests the true parameters of the aging models are unknown.

The way to do the virtual test (called briefly test in the following text) is as follows. At first, we evaluate design parameters of the physical models according to the true aging models. Then change the design parameters to be random by adding virtual random variables (to simulate the

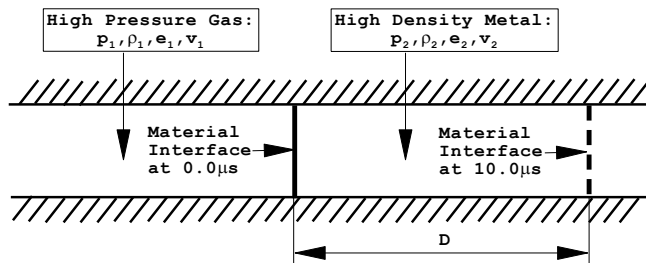


Figure 1. Entity model of the system

random process of manufacture). The exact solution of each physical model is obtained based on the sampling value of the randomized model parameters. Finally, we get results that regarded as real test data from the exact solution plus a sampling value of another random variable that added to the exact solution (to simulate the random process of real test).

The parameters that need to add a virtual random variable are the design values $\rho_1^*, \rho_2^*, e_1^*, \gamma_1^*, \gamma_2^*$ and the exact solution of system response D^* . Their virtual random variables $\tilde{\rho}_1, \tilde{\rho}_2, \tilde{e}_1, \tilde{\gamma}_1, \tilde{\gamma}_2$ and \tilde{D} are supposed to follow normal distributions with zero-means and deviations of $4.5\text{kg/m}^3, 36.0\text{kg/m}^3, 0.2\text{MJ/kg}, 0.01, 0.2$ and 0.1mm , respectively. The true parameters (unknowns in real tests) of physical models are formed as $\rho_1^{test} = \rho_1^* + \tilde{\rho}_1$, $\rho_2^{test} = \rho_2^* + \tilde{\rho}_2$, $e_1^{test} = e_1^* + \tilde{e}_1$, $\gamma_1^{test} = \gamma_1^* + \tilde{\gamma}_1$, $\gamma_2^{test} = \gamma_2^* + \tilde{\gamma}_2$. And the true test data are formed as $D^{test} = D^* + \tilde{D}$.

C. Calibration

In this system, two types of parameters are calibrated, namely numerical parameters and physics parameters. The numerical methods do not need to be calibrated as the physical process is not complicated. Like the sequence of V&V, numerical parameters should be calibrated before physics parameters. Calibration on numerical parameters is just based on the test data of fresh products considering the adequacy of test data and the least disturbance of aging models. And calibration on physics parameters is based on the test data of aged products.

The numerical parameters to be calibrated are artificial viscosity coefficients corresponding to the selected steps of space and time. The physics parameters to be calibrated are from aging models for e_1 and γ_2 of aged materials. All the calibrated numerical and physics parameters form a fixed association in M&S for intended use.

Calibration could be executed through following steps:

- Based on the demand analysis of M&S, determine numerical methods, numerical and physics parameters or knobs need to be calibrated, and the approach to get the reference solution for M&S (Here the reference solutions are test data);
- Choose fresh products to be tested and give their design values of physics parameters as $\rho_1^* = 2500.0\text{kg/m}^3$, $e_1^* = 6.0\text{MJ/kg}$, $\gamma_1^* = 3.0$, $\rho_2^* = 20000.0\text{kg/m}^3$, $\gamma_2^* = 5.0$. Obtain ρ_1^{test} , e_1^{test} , γ_1^{test} , ρ_2^{test} , γ_2^{test} and the corresponding test data D^{test} for five physical models by plus the sampling values of their virtual random variables and the design values or exact solutions;
- Obtain the optimally calibrated numerical parameters through comparison between one numerical result $D^{m\&s}$ and five test data $D^{test} = 3.856, 3.852, 3.841, 4.010, 3.757$ mm, such as

the artificial viscosity coefficients $\alpha = 1.5$ and $\beta = 0.06$ corresponding to the initial grid width $\Delta x = 0.1$ mm and time step $\Delta t = 0.0016 \mu s$. The artificial viscosity model used is

$$q = \begin{cases} 0, & \dot{\epsilon}_{kk} \geq 0 \\ \rho l |\nabla \cdot \bar{v}| (\alpha l |\nabla \cdot \bar{v}| + \beta c), & \dot{\epsilon}_{kk} < 0 \end{cases}$$

where q is the viscous pressure, l is the grid size, c is the sound speed and $\nabla \cdot \bar{v}$ is the divergence of velocity. The numerical result corresponding to these optimal values is $D^{m\&s} = 3.891$ mm;

- Obtain the optimally calibrated physics parameters based on the aging model and the comparison between numerical results and test data about stockpile time of 10 years, 30 years and 50 years. The aging models describe the changing of physics parameters are

$$e_1(t) = e_1(0)(1.0 + a_1 t + b_1 t^2), \quad (8)$$

$$\gamma_2(t) = \gamma_2(0)(1.0 + a_2 t + b_2 t^2), \quad (9)$$

where the time t is in "year" and the calibrated parameters are

$$\begin{cases} a_1 = -0.5 \times 10^{-4}, \\ b_1 = -1.5 \times 10^{-5}, \\ a_2 = 1.5 \times 10^{-4}, \\ b_2 = 3.0 \times 10^{-5}. \end{cases} \quad (10)$$

- Finish calibration by fixing the calibrated numerical methods and parameters.

D. Validation

In model space, the validation domain is defined as the stockpile time from 0 years to 50 years, in which the validation tests are for the stockpile time of 0 year, 10 years, 20 years, 30 years, 40 years, 50 years. For each stockpile time there are five repeated tests but only one numerical result.

By (7), in which $\beta = 0.95$, and $t_{(1-\beta)/2, \nu} = t_{0.025, 4} = 2.7764$, uncertainty in validation domain are quantified as $U(t) = 0.142, 0.128, 0.283, 0.152, 0.202, 0.257$ mm for stockpile time $t = 0, 10, 20, 30, 40, 50$ years.

E. Uncertainty Quantification in Application Domain

In model space, the application domain is defined as the stockpile time great than 50 years. There is no system-level test in this domain.

In order to quantify the first item in the right hand of (3), we have to determine the function showing uncertainty varies with the stockpile time. Here, a second order polynomial is used

$$U(t) = a_0 + a_1 t + a_2 t^2. \quad (11)$$

Based on uncertainties in validation domain, we have

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 10 & 100 \\ 1 & 20 & 400 \\ 1 & 30 & 900 \\ 1 & 40 & 1600 \\ 1 & 50 & 2500 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0.142 \\ 0.128 \\ 0.283 \\ 0.152 \\ 0.202 \\ 0.257 \end{pmatrix}. \quad (12)$$

The minimum-norm solution of this over-determined equation is

$$(a_0, a_1, a_2) = (0.14, 0.0026, -1.38 \times 10^{-5})$$

With this solution, we get the extrapolated M&S uncertainty $U_{80\text{ year}}^{extrapolation} = 0.261$ mm for 80 years stockpile by (11). It is just the value of $U_{application}^{extrapolation}$ in (3).

Uncertainties of physics parameters for different stockpile time are listed in Table 1, in which the aleatory uncertainties are induced from manufacture and the epistemic uncertainties are induced from the cognitive defect of the statistical population average of physics parameters. The additional epistemic uncertainty of 80 years stockpile comparing to 0~50 years stockpile is 0.2 MJ/kg for ${}^e U$ and 0.2 for ${}^{\gamma_2} U$.

The sensitivities of system response D to each physics parameter are in Table 2.

For system of 80 years stockpile, the uncertainty propagated from the additional uncertainties of physics parameters is

$$\begin{aligned} U_{\Delta}^{propagation} &\approx |\partial D / \partial e_1| \times \Delta({}^e U) + |\partial D / \partial \gamma_2| \times \Delta({}^{\gamma_2} U) \\ &= 0.499 \times (0.8 - 0.6) + 0.103 \times (0.7 - 0.5) \\ &= 0.120(\text{mm}). \end{aligned}$$

The uncertainty of 80 years stockpile quantified by (3) is:

$$U_{80\text{ year}}^{M\&S} \approx U_{80\text{ year}}^{extrapolation} + U_{\Delta}^{propagation} = 0.381(\text{mm}). \quad (13)$$

If all the epistemic uncertainties are propagated, we get the uncertainty that comes from propagation as

$$\sum_{i=1}^5 |\partial D / \partial x_i| \cdot ({}^x U) \approx 0.496(\text{mm}).$$

Moreover, if the aleatory uncertainties are also propagated, the uncertainty from propagation will reach 0.928 mm. However, it couldn't be regarded as total M&S uncertainty. From here, we see that the method depicted by (3)-(7) can reduce epistemic uncertainties through calibration and filter the aleatory uncertainties by properly defined statistics.

TABLE I. UNCERTAINTIES OF PHYSICS PARAMETERS

| Uncertainty type | Aleatory | | Epistemic | |
|---------------------------------|----------|----|-----------|------|
| | 0~50 | 80 | 0~50 | 80 |
| Stockpile (Year) | 0~50 | 80 | 0~50 | 80 |
| $\rho_1 U$ (kg/m ³) | 13.50 | | 5.00 | |
| $\rho_2 U$ (kg/m ³) | 108.00 | | 40.00 | |
| ${}^e U$ (MJ/kg) | 0.60 | | 0.60 | 0.80 |
| ${}^{\gamma_1} U$ (1) | 0.03 | | 0.01 | |
| ${}^{\gamma_2} U$ (1) | 0.60 | | 0.50 | 0.70 |

TABLE II. SENSITIVITY OF D TO PHYSICS PARAMETERS

| $\frac{\partial D}{\partial \rho_1}$ $\left(\frac{mm}{kg/m^3}\right)$ | $\frac{\partial D}{\partial \rho_2}$ $\left(\frac{mm}{kg/m^3}\right)$ | $\frac{\partial D}{\partial e_1}$ $\left(\frac{mm}{MJ/kg}\right)$ | $\frac{\partial D}{\partial \gamma_1}$ (mm) | $\frac{\partial D}{\partial \gamma_2}$ (mm) |
|--|--|--|--|--|
| 1.085×10^{-3} | -1.356×10^{-4} | 0.499 | 1.400 | -0.103 |

F. Validity of the UQ Method

In practice, the system-level test is not available in application domain. We have specially arranged five new tests of 80 years stockpile here aims to prove the UQ method is valid. This procedure starts with uncertainty assessment by comparison the numerical results and test data as specified in (7). Then compare the assessment result with the prediction result in (13). If the latter is greater than the former just in a little amount, it may offer a positive evidence of validity for the UQ method in the sight of obeying the true-value-covered and uncertainty-minimized principles.

Implementation of the tests for 80 years stockpile is similar to the tests as narrated above, i.e., the values of physics parameters in physical models are obtained by the true aging models and sampling, the test data are obtained by exact solutions and sampling, the physics parameters for M&S are from (10) and their deviations from the true aging models agree with the uncertainty in Table 2.

The five test data $D^{test} = 3.455, 3.338, 3.216, 3.395, 3.174$ mm and the numerical result is $D^{m\&s} = 3.499$ mm, in which the parameters for M&S are $\rho_1 = 25000kg/m^3, e_1 = 5.4MJ/kg, \gamma_1 = 3.0, \rho_2 = 20000kg/m^3, \gamma_2 = 6.02$. The assessed M&S uncertainty $U_{80year}^{M\&S} = 0.331$ mm.

The result in (13) shows that the uncertainty (0.381mm) obtained by prediction with (3) is slightly greater than the uncertainty (0.331mm) obtained by assessment, from which the success of the UQ methods is exhibited.

VI. CONCLUSION

When system-level test is unavailable, the prediction by M&S and its uncertainty are the most important information for reliability certification or assessment, and propagation is the most imaginable UQ method. As the system becoming complicated and its hierarchy having more multiple tiers, the numerical errors and the great number of uncertain input factors will make it impractical to quantify the M&S uncertainty just by propagation. Based on the reality that the prototype of a new system generally has some test data and the awareness that the epistemic uncertainty of M&S could be reduced by calibration with existed test data, an UQ

method is put forward to synthesize the uncertainties in validation domain and the propagated additional uncertainties. With an example the method is shown to observe the true-value-covered and uncertainty-minimized principles of UQ for M&S that is used as prediction.

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