On the Proportional-Integral-Derivative Based Trading Algorithm under the Condition of the log-Normal Distribution of Stock Market Data

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***Abstract*—Our paper deals with a novel trading algorithm based on the conventional feedback control methodology. A profitable trading algorithm design for stock markets constitutes a very challenging problem of the modern financial engineering. We apply a model-free version of the classic Proportional-Integral-Derivative (PID) control to the modern Algorithmic Trading (AT). The proposed control theoretical application of the classic PID methodology is combined with a specific statistical information on the available historical stock market data. We consider a generic condition of the log-normal distribution of the available stock data. The log-normal property mentioned above implies a new efficient calibration rule for the gain coefficients (gains tuning) for the resulting PID type trading algorithm. We finally apply the developed PID based optimal AT strategy to a specific real-world example of the Binance Bitcoin / USD market. This application illustrates the effectiveness of the proposed trading algorithm.**

***Index Terms*—algorithmic trading, financial engineering, model-free PID control, statistical decision making.**

I. INTRODUCTION

Consider an idealized stock market model in discrete time. This trading abstraction includes some ideal assumptions, among others, the ”no transaction costs” and ”one stock portfolio” conditions. Moreover, one also assumes ”zero interest” ”continuous trading” and some further simplifying hypothesises. We refer to [4][14][16] for the necessary technical details. The discrete time model consideration is mainly motivated by the real stock market dynamics as well as by the decision making mechanism. We next introduce the trading ticks $t = 1,...,$ and the corresponding time-intervals of the trading buckets $[t, t+1)$. Note that the development of the efficient and robust trading algorithms for the financial markets constitutes a sophisticated problem. Recall that one deals with a stochastic dynamic behaviour in that case. The highly frequent non-regular stochastic nature of the modern markets makes it impossible any suitable forecasting of the prices of financial instruments traded on the stock markets.

A systematic, control theory based approach to the AT was initially developed in [5]-[8] [20]. It studies the model-free PID trading strategy and proposes to react to the stock price variations instead of modeling them. An interesting, frequency domain involved extension of the above approach can be found in [14]. Let us also refer to [4] for a novel PID related trading algorithm with a switched structure.

Following [20], we introduce the current gain $\Delta g(t)$ and the current investment level $\Delta I(t)$ for a time instant t. Moreover, by $g(t)$ and $I(t)$ we next denote the cumulative profit and the cumulative investment, respectively. The initial investment level I_1 is assumed to be given. Consider the nonlinear discrete-time PID type feedback with a saturation rule:

$$\begin{aligned} &\delta I(t+1) = K_P(t)\Delta g(t) + K_D(t)\dot{\Delta} g(t) + \\ &K_I(t)\int_{t-T}^{t} h(\tau)\Delta g(\tau)d\tau, \\ &\Delta I(t+1) = \chi(\delta I(t+1)), \quad \text{for} \quad t = 1,..., \\ &I(1) = I_1. \end{aligned} \tag{1}$$

Here $K_P(\cdot)$, $K_D(\cdot)$ and $K_I(\cdot)$ are dynamic gains associated with the proportional, integral, derivative, and second order derivative terms of regulator (1). We put

$$K(\cdot) := \{K_P(\cdot),\ K_D(\cdot),\ K_I(\cdot)\}.$$

Similar to the classic PID control (see e.g., [21]), the integral term in (1) is defined on a given time interval $[t-T, t]$. The time instant T belongs to the given discrete time grid. Moreover, $h(\cdot)$ in (1) is a suitable ”memory loss” function. One can consider the generic exponentially weighted ”memory loss" function $h(\cdot)$ with $h(t) = 1$. The saturation function $\chi(\cdot)$ in 1 can be defined as follows

$$\chi(\delta I) := \begin{cases} \delta I, & \text{if } \delta I^{\min} \leq |\delta I| \leq \delta I^{\max}; \\ \pm\delta I^{\max}, & \text{if } |\delta I| > \delta I^{\max}; \\ 0, & \text{if } |\delta I| < \delta I^{\min}. \end{cases} \quad (2)$$

where $\delta I^{\max}$ and $\delta I^{\min}$ are prescribed maximal and minimal current investment levels, respectively.

Using the decision about the current investment $\Delta I(t+1)$ and the corresponding stock price $p(\omega, t+1)$, we next calculate the current profit obtained at the time instant $(t+1)$:

$$\Delta g(t+1) = \frac{(p(\cdot, t+1) - p(\cdot, t))}{p(\cdot, t)} \Delta I(t+1) \quad (3)$$

Note that the investment level $\Delta I(t+1)$ in (1) constitutes a "control input". We next call it "investment decision". Note that it is deployed at a current time instant t under the natural unknownness of the market price $p(\omega, t+1)$. Here, $\omega \in \Omega$ and Ω is a probability state space with a specific probability measure. The stock price $p : \Omega \times \mathbb{Z}_+ \to \mathbb{R}$ is assumed to be a measurable (stochastic) function. Note that the current profit $g(t+1)$ is an a posteriori value such that $p(\cdot, t+1)$ in (3) denotes a concrete realization of a stochastic price $p(\omega, t+1)$. The block diagram of the proposed model-free PID trading strategy is illustrated in Figure 1.

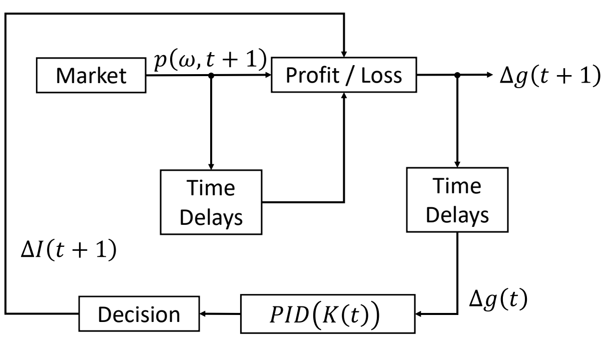

Fig. 1. Model-free PID based trading algorithm

The general formula for $\delta I(t+1)$ in (1) can easily be specified in the discrete-time case:

$$\begin{aligned} &\delta I(t+1) = (K_P(t) + K_I(t) + K_D(t))\Delta g(t) + \\ &(K_I(t)h(t-1) - K_D(t))\Delta g(t-1) + \\ &K_I(t) \sum_{\tau=t-T}^{t-2} h(\tau)\Delta g(\tau). \end{aligned} \quad (4)$$

The main problem in the conventional PID control theory as well as in the model-free version under consideration constitutes in searching of adequate gains $K(\cdot)$ tuning rules (see e.g., [19] and references therein). In the conventional application areas of the classic model-based PID controllers these tuning techniques are usually well established [19][21]. In the sophisticated model-free stochastic case, the design of a suitable PID tuning scheme represents a challenging problem. It constitutes in fact a key problem of an intelligent investment decision.

The remainder of this paper is organized as follows: Section 2 contains a general statistical analysis of a generic stock data. In Section 3, we apply the obtained log-normal distribution of the available data to backtesting driven tuning (calibration) of the PID gains. In this section, we also consider an application of the proposed PID based trading algorithm to a specific stock market, namely, to the Binance BTC (bitcoin) / USD futures. Section 4 summarizes our paper.

II. Statistical Analysis of the Stock Data

The conceptually important PID gains tuning problem mentioned in Section I will be considered here from the statistical point of view. For these aims let us examine the log-normal hypothesis for the probability distribution of the following "price/volume" ratio:

$$\theta(\omega, t+1) := \frac{p(\omega, t+1)}{v(t+1)}.$$

The (a posteriori) statistical analysis of a wide spectrum of stock markets has demonstrated that the distribution of the closing prices normalized by investment volume $v(t+1)$, namely the value $\theta(\omega, t)$ fits well a specific log-normal law (see [1] and references therein). Recall that $v(t+1)$ can be calculated as follows:

$$v(t+1) := \frac{\Delta I(t+1)}{p(\cdot, t)}.$$

Note that in the trader praxis the investment volume is usually restricted by a maximal investment volume. In the case of PID based trading algorithm under consideration, the maximal investment volume is a direct consequence of the bounded structure of investment $\Delta I(t+1)$ in (1).

The log-normal probability distributions in the stock market data have comprehensively been studied for the stock price differences and for option prices. Let us refer to the celebrated Black and Scholes model [12]. A full review of this subject can be found in [13]. Additionally, the log-normal properties of the volatility and related market values in the stock prices and indices are also considered in [15]. In this section, we follow [1] and analyze the (stationary) statistical law for $\theta(\omega, t+1)$:

$$\rho(\theta) = \frac{a}{\sqrt{2\pi}\sigma(\theta - s)} \exp -(0.5\sigma^2)(\ln(\theta - s) - \mu)^2, \quad (5)$$

where $\mu \in \mathbb{R}$ is a mean, $\sigma \in \mathbb{R}_+$ is a dispersion and $s \in \mathbb{R}$ denotes a shifting parameter. Note that (5) is a so called "three-parameters" $\{\mu, \sigma, s\}$ log-normal distribution (see e.g., [22]). The necessary parameters of the proposed log-normal distribution $\rho(\theta)$ can be determined using the historical stock market data. This way, we incorporate the generic backtesting into the resulting PID based trading algorithm we develop.

As mentioned above, (5) constitutes an adequate distribution hypothesis for the price/volume ratio $\theta(\omega, t+1)$. The quality

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of this statistic hypothesis can be established by the standard Chi-Quadrat-Test for distributions (see e.g., [22]). In this paper, we consider the normalized value χ^2/q for this purpose. Here q is the number of degrees of freedom.

Consider now the daily market prices and investment volumes data

$$\{p(1),...,p(T)\},\ \{v(1),...,v(T)\}.$$

Following [24], we now assume the log-normal distribution for the value

$$\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big),$$

where $t=1,...,T-1$. Note that this assumption or the equivalent assumption on the normal distribution of the logarithmic return

$$\ln\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big)$$

is well motivated. We refer to [1][24] for the necessary statistical consideration. We now consider the following value:

$$\ln\big(\theta(\omega,t+1)/\theta(\cdot,t)\big)=\ln\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big)- \ln\big(\frac{v(t+1)}{v(t)}\big). \tag{6}$$

From (6) we next obtain

$$\ln\big(\frac{v(t+1)}{v(t)}\big)=\ln\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big)+\ln\theta(\cdot,t)- \ln\theta(\omega,t+1). \tag{7}$$

Expression (7) makes it possible to make a decision related to the investment volume $v(t+1)$. We evidently have

$$v(t+1)=\exp\big[\ln\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big)+\ln\theta(\cdot,t)- \ln\theta(\omega,t+1)+\ln v(t)\big]. \tag{8}$$

Considering (8), we observe that $\theta(\cdot,t)$ and $v(t)$ and the corresponding logarithms are known values. Moreover, the price/volume ration $\theta(\omega,t+1)$ can be simulated using the above log-normal distribution (5). As mentioned above, we can also forecast the value $\ln\big(p(\omega,t+1)/p(\cdot,t)\big)$ using a suitable log-normal distribution (see [24]). Note that the necessary parameters of the log-normal distributions for values $\theta(\omega,t+1)$ and $p(\omega,t+1)/p(\cdot,t)$ needed to be determined using the available historical data.

III. Application of the log-Normal Distribution to the PID Gains Calibration

The main problem of interest for many researchers working in the financial engineering is to anticipate the behavior of stock markets giving a certain amount of historical data. For this purpose we study the PID type trading algorithm (1) in combination with the statistical log-normal characteristics discussed in Section II. We now apply formula (8) and obtain $M\in\mathbb{N}$ simulations $v^j(t+1),\ \ j=1,...,M.$ of the expected investment value at $(t+1)$. Using the given algebraic structure of the PID algorithm (1), we next calculate the necessary (three-dimensional) gain coefficient $K(\cdot)$ as a solution of the following optimization problem

$$\begin{aligned}&\sum_{j=1}^{M}\big(\chi(\delta I(t+1))-v^j(t+1)p(\cdot,t)\big)^2\rightarrow\min\\ &\delta I(t+1)=K_P(t)\Delta g(t)+K_D(t)\dot{\Delta}g(t)+\\ &K_I(t)\int_{t-T}^{t}h(\tau)\Delta g(\tau)d\tau,\end{aligned} \tag{9}$$

where the nonlinear function $\chi(\cdot)$ is given by (2). Evidently, (9) constitutes a specific nonlinear regression. This nonlinear optimization problem finally leads to some optimal gains

$$K^{opt}(\cdot):=\{K_P^{opt}(\cdot),\ K_D^{opt}(\cdot),\ K_I^{opt}(\cdot)\}.$$

for the PID type trading algorithm (1). Finally, we use $K^{opt}(\cdot)$ and define the deployed (optimal) investment level $\Delta I^{opt}(t+1)$ by expressions (1)-(2). The really deployed investment volume $v^{opt}(t+1)$ can be calculated as follows:

$$v^{opt}(t+1):=\frac{\Delta I^{opt}(t+1)}{p(\cdot,t)}.$$

Note that the trading decision at the future time instant $(t+1)$ can be expressed by the current optimal investment level $\Delta I^{opt}(t+1)$ calculated above. It also can be determined by the pair $\{\mathrm{sign}[\Delta I^{opt}(t+1)],\ \ |v^{opt}(t+1)|\}$, namely, by the current trade direction $\mathrm{sign}[\Delta I^{opt}(t+1)]$ and by the associated absolute value $|v^{opt}(t+1)|$ of the investment volume. The resulting PID based strategy (1) combined with the optimal gain selection (9) is next called Optimal PID (OPID) trading algorithm.

We now present an application of the developed PID based AT technique to a real-world stock market data. Consider the Binance Bitcoin / USD futures and apply the proposed AT technique. The BTC futures price dynamics is presented in Figure 2.

Fig. 2. Binance BTC / USD one day price index

We have applied the novel OPID trading algorithm to the above example. The corresponding profit dynamics is presented in Figure 3.

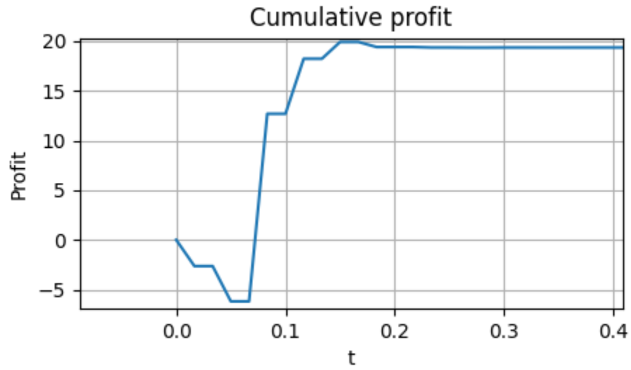

Fig. 3. Binance BTC / USD one day price index

Note that the operation time of the OPID in that example is timely restricted. The stock market under consideration has an obvious high-frequency behaviour. This fact naturally implies some (expected) difficulties of the common trading algorithms. In particular, this concerns the widely used moving average based trading strategies.

As one can see, the developed OPID constitutes a High-Frequency Trading (HFT) strategy. The time ticks in the above trading example under consideration are very dense. The length of the corresponding intervals of trading buckets is equal to 1.728 sec. Let us also note that the typical ”hyperregulation and stabilization” dynamics of the proposed PID based strategy is visible (see Figure 3). The presented practical example illustrates the implementability of the developed PID involved AT scheme.

IV. Conclusion and Future Work

In this paper, we developed a combined trading algorithm that involves the PID control methodology and some a priori given statistical characteristics of the stock market data. We studied an idealized market under the assumption of only one asset. However, the proposed analytic approach and the resulting trading methodology can easily be extended to the real-time multi-asset trading. Moreover, it can incorporate some additional efficient data driven optimization techniques.

The given historical stock market data and the statistical properties mentioned above are constructively used for an adequate calibration (tuning) of the PID controller gains. This calibration is based on the backtesting procedure. The resulting optimal trading strategy generates adequate (profitable) decisions of the ”buy/sell/hold” market orders at every subsequent time instant. The proposed approach to the AT involves a combination of two mathematically rigorous tools, namely, the classic PID control methodology and applied statistics. It also contains a specific data driven optimization procedure. This combination finally leads to a novel and very promisingly trading strategy. The resulting algorithm and the corresponding real-world scenario based simulation technique conceptually extend the family of the feedback based trading algorithms.

Note that the developed PID based trading approach can be used as an additional tool in the several theoretic frameworks of the modern financial engineering. For example, it can be applied in combination with the well established financial time series analytics. The proposed PID based trading algorithm is also compatible with the generic price prediction techniques (see e.g., [17]).

Finally note that the algorithmic trading strategy proposed in our paper constitutes an initial (conceptual) development. We are mostly concentrated on the formal algorithmic aspects of the proposed technique. The financial solutions for the stock market considered in our contribution need additional comprehensive simulations and prototyping, further backtesting, adequate data driven optimization and applications to the real markets. We also expect a profitable application of the proposed trading methodology specifically in the High-Frequency Trading.

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