Fixed Complexity Soft-Output Detection Algorithm Through Exploration and Exploitation Processes

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***Abstract*—In this paper, we propose a soft-output Multiple-Input and Multiple-Output (MIMO) detector algorithm, which is based on two complementary techniques: *exploration* and *exploitation*. The proposed detector, called List Exploration and Exploitation (L2E), achieves near-optimal performance with low and fixed computational complexity. It has a high parallelism degree, which makes it suitable for efficient practical implementation. The soft-output values are calculated by means of squared Euclidean distances in a max-Log Likelihood Ratio (LLR) approximation. The average Bit Error Rate (BER) performances of the L2E are compared to the well-known List Sphere Decoding (LSD) algorithm and it is shown that our method considerably reduces the computation complexity while maintaining near-optimal performance in comparison to LSD algorithm.**

Keywords-MIMO detection; sphere-decoding; soft demodulation.

I. Introduction

MIMO systems can be used to increase data rates by adopting spatial multiplexing method or to improve the reception reliability by exploiting spatial diversity. In spatial multiplexing systems, the maximum likelihood (ML) detection leads to the optimal method for minimizing transmission errors. However, it involves high computation complexity and requires a brute-force search over all of the transmitted vectors. A promising and efficient alternative, with reduced computational complexity, is the original sphere decoding (SD) algorithm by Fincke and Phost and its variants [1]–[4].
Nevertheless, the computational complexity of the sphere decoding algorithm is still exponential in problem size [5]. Fast but suboptimal ML detection algorithms such zero forcing, minimum mean-squared error, semi-definite programming, and interference cancellation detectors have already been proposed in the literature [6]. This wide variety of detectors is mainly due to a lack of optimum performances and/or higher computational complexity.

It has already been shown that the soft-output detector improves the error performance compared with a hard-output detector [8]. A soft-output detector generates the LLR value of each bit, defined as the ratio of the probabilities that a zero or a one has been transmitted conditioned on the received vector. However, the computation complexity of exact a-posteriori probabilities (APP) is exponential in the MIMO system dimensions [9]–[11]. Recently, a number of soft-output MIMO detectors have been reported, which approximate the APP and provide soft outputs [7]. Several demodulation schemes use a list of candidate data vectors to obtain approximate LLRs. To the best of our knowledge there are three families of candidate list generation algorithms for the suboptimal soft-output maximum a-posteriori (MAP) detector. The first type of algorithms is a modified version of well-known hard tree search detector where the goal of is to find one tree leaf with the best metric. The goal of soft-output is to find and keep a list in which one seeks to efficiently identify all bit vectors that dominate the LLRs. The mostly well-known list type of soft-output MIMO decoding algorithms are LSD algorithm [8], sequential sphere decoding (LISS) algorithm [12], and the M-Algorithm [13]. The second type of algorithms is based on simple bit flips around the hard solution [14]. Therefore, a hard decoder is employed to find a maximum a posteriori symbol estimate, and a candidate list is generated by bit-flipping of the MAP estimate. This technique may produce a LLRs approximation with high probability. The third type of techniques is based on lattice-reduction (LR) aided detector following by bit-flipping technique, which generates a list from which it computes the APP of all bits comprising the symbol vector. The channel matrix properties can be improved using an efficient lattice reduction and by this way we can reduce the complexity of the hard demodulator.

Our contribution in this paper is multi-fold: Based on previous MIMO detection studies of the hard detector [15][16], we propose a new near-optimal soft-output demodulator based on list generation algorithm. The algorithm list's size is controlled using the number of considered candidates, N_c, during the exploitation phase, and the number of considered direction, N_d. The proposed soft-output MIMO detection algorithm L2E was closely approximate the max-log LLR functions based on the generated initial solutions list with a reduced and fixed computational complexity. In contrary, the List Sphere-Decoding algorithm has a variable computational complexity depending on the MIMO channel conditioning. Moreover, the growth of the LSD complexity is exponential in the low Signal-to-noise ratio (SNR) region.

The computation complexity of the proposed L2E algorithm is independent from the SNR and thus has a constant value over all SNR regions. In our algorithm, the complexity depends only on the number of transmitter/receiver antennas, the number of considered candidates N_c, and the number of considered directions N_d. Monte Carlo simulations show that the proposed soft-output list detector has better complexity and performance trade-offs than the well-known LSD detector. Moreover, the L2E detector has an inherent parallel structure, thus it is very suitable for massive parallel architectures.

The remainder of this paper is organized as follows. Section II introduces the mathematical model of the studied MIMO bit-interleaved coded modulation (BICM-MIMO) system and the associated maximum likelihood detection (MLD) problem. The hard-output version of the exploration and exploitation detector algorithm (H2E) will be introduced and it is extended to compute soft-outputs. The proposed algorithm will be called as list exploration and exploitation detector (L2E). Computational complexity issues are given in Section IV. Section V provides Monte Carlo simulation results of the proposed algorithm and gives same discussions. Finally, Section VI is devoted to concluding remarks;

II. Mathematical Model

In this section, we introduce the BICM-MIMO model and the LLR generation with perfect channel state information (CSI) at the receiver is described.

A. MIMO-BICM system Model

Herein, we consider a MIMO system with N transmit antennas and M receive antennas associated to a BICM schema where a block of information bits is mapped to transmit symbols through a channel encoder and a symbol mapper separated by a code-bit interleaver [17] [18]. Let us consider a MIMO-BICM system with $M \times N$ channel matrix $\mathbf{H}_c$. At the receiver, a detector calculates the log-likelihood ratios for the coded bits, which are deinterleaved and passed to the subsequent channel decoder. The coded bit stream is mapped to N-dimensional transmit vector symbols $\mathbf{x}_c \in \phi^N$, where ϕ is a 2^Q-QAM modulation. The individual coded bits are denoted by b_{ij}, where the indexes i and j refer to the i^{th} bit in the binary label of the j^{th} entry of the transmitted symbol vector $\mathbf{x}_c = [x_1^c, x_2^c, ..., x_N^c]^T$. In considered MIMO-BICM system, the transmitted signal and the received signal are related through a complex baseband input-output relation as:

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{x}_c + \mathbf{w}_c \tag{1}$$

where $\mathbf{w}_c$ is an independent and identically distributed complex zero-mean Gaussian noise with variance $\sigma^2/\sqrt{2}$ per complex entry, $\mathbf{y}_c$ is the received symbol vector, and $\mathbf{x}_c$ is the transmitted symbol vector with the average transmit power of each antenna normalized to one, $i.e.$ $E[\mathbf{x}_c \mathbf{x}_c^H] = \mathbf{I}_N$. The $M \times N$ channel matrix $\mathbf{H}_c$ contains uncorrelated complex Gaussian fading gains where the element h_{ij}^c represents the complex transfer function from the j^{th} transmit antenna to the i^{th} receive antenna. Thus, the channel matrix $\mathbf{H}_c$, which is assumed to be known by the receiver, is modelled as an independent and identically distributed complex Gaussian variable with zero mean and variance $1/2$ per complex entry. Treating real and imaginary part of (1) separately, and with the real-valued channel matrix and the real-valued vectors, the system model can be rewritten as

$$\begin{bmatrix} \Re(\mathbf{y}_c) \\ \Im(\mathbf{y}_c) \end{bmatrix} = \begin{bmatrix} \Re(\mathbf{H}_c) & -\Im(\mathbf{H}_c) \\ \Im(\mathbf{H}_c) & \Re(\mathbf{H}_c) \end{bmatrix} \begin{bmatrix} \Re(\mathbf{x}_c) \\ \Im(\mathbf{x}_c) \end{bmatrix} + \begin{bmatrix} \Re(\mathbf{w}_c) \\ \Im(\mathbf{w}_c) \end{bmatrix}$$

where $\Re(z)$ and $\Im(z)$ denote the respective real and complex parts of a complex number z. Let $m = 2M$ and $n = 2N$, then the dimension of the real channel matrix is given by $m \times n$. Likewise, the dimension of the vectors are given by $\mathbf{y} \in \mathbb{R}^m$, $\mathbf{w} \in \mathbb{R}^m$, and $\mathbf{x} \in \xi^n$ where $\xi \equiv \Re(\mathcal{O})$. In this paper, we restrict our study to the case where $\xi^n \equiv \{\pm 1\}^n$.

B. Optimum Soft-output Demodulation

Given the channel matrix $\mathbf{H}$, the received vector $\mathbf{y}$, and assuming an ideal interleaver, the optimum soft-output maximum a posteriori decoder minimizes the BER by evaluating the LLRs of the a posteriori probability of each bit b_{ij}.

$$\mathcal{L}(b_{ij}) = \log \frac{P(b_{ij} = 1|\mathbf{y})}{P(b_{ij} = 0|\mathbf{y})} \tag{2}$$

where $P(b_{ij}|\mathbf{y})$ is the probability mass function of the code bits conditioned on $\mathbf{y}$. The exhaustive evaluation of (2) has a high computational complexity. Thus, using Bayes' theorem and the max-log approximation as shown in [7][9], the equation (2) can be further rewritten as:

$$\mathcal{L}(b_{ij}) \approx \min_{\mathbf{x} \in \mathcal{X}_{ij}^0} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 - \min_{\mathbf{x} \in \mathcal{X}_{ij}^1} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \tag{3}$$

where $\mathcal{X}_{ij}^1$ and $\mathcal{X}_{ij}^0$ are the sets of symbols vectors having b_{ij} equal to 1 and 0, respectively. The set ξ^n can be seen as the union of the previous two subsets $\xi^n = \mathcal{X}_{ij}^1 \cup \mathcal{X}_{ij}^0$. The computation complexity of (3) is exponential in the number of transmit antennas. Thus, we propose a novel soft-output detector, called L2E, which keeps a limited number of candidates in order to evaluate the equation (3). Hence, the LLR of the i^{th} bit b_{ij} in the j^{th} symbol x_j can be approximated as

$$\mathcal{L}(b_{ij}) \approx \min_{\mathbf{x} \in \Gamma \cap \mathcal{X}_{ij}^0} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 - \min_{\mathbf{x} \in \Gamma \cap \mathcal{X}_{ij}^1} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 \tag{4}$$

where Γ denotes the candidates list, which is the subset of the feasible set ξ^n. The computational complexity the soft-output detector is affected by the list size and it increases approximately linearly (see Section IV).

III. Proposed soft-output detector

In this section, we propose an soft-output MIMO detector based on the exploration and exploitation strategies. The proposed L2E detector allows a sub-optimal solution for the optimum soft-output demodulation problem that limits the complexity of the receiver design.

A. Exploitation technique

The exploitation step can be defined as a simple and naive local search technique. This section gives a mathematical basis to understand how the exploitation (*intensification*) is applied on the subset $\xi_{st}^n \subset \xi^n$. The subset ξ_{st}^n is generated by the first phase, which is the exploration (*diversification*) step. In the remainder of this section, we show that a simple greedy policy of position switching between neighbouring feasible solutions to locally minimize the objective function $f(\mathbf{x}) = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$ over the subset ξ_{st}^n.

1) Definition 1: A neighbourhood operator is a function $\mathcal{Z} : \xi^n \longmapsto \mathcal{Z}(\xi^n)$ that assigns to every solution $\mathbf{u} \in \xi^n$ a set of neighbours $\mathcal{Z}(\mathbf{u}) \subseteq \xi^n$. The subset $\mathcal{Z}(\mathbf{u})$ is called neighbourhood of $\mathbf{u}$ and it is equivalent to a neighbourhood graph, which has ξ^n as vertex set, and which contains directed edge $\mathbf{v} \longmapsto \mathbf{u} \Leftrightarrow \mathbf{v} \in \mathcal{Z}(\mathbf{u})$.

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2) Definition 2: A local minimum solution with respect to a neighbourhood operator $\mathcal{Z}$ is a solution $\mathbf{u}^*$, such that for all $\mathbf{u} \in \mathcal{Z}(\mathbf{u}^*) \Rightarrow f(\mathbf{u}^*) \leq f(\mathbf{u})$. The exploitation technique starts at a given solution $\mathbf{u} \in \xi^n$ and makes it as the current solution $\mathbf{x}_0 = \mathbf{u}$. At each iteration, it examines all the neighbours of the current solution and seeks to best one having a better objective function value than $\mathbf{x}_0$, *i.e.*, $\mathbf{v} \in \mathcal{Z}(\mathbf{x}_0)$ such that $f(\mathbf{v}) \leq f(\mathbf{x}_0)$. If such a solution is found, it becomes the current solution, *i.e.*, $\mathbf{x}_0 = \mathbf{v}$. These iterations are repeated until there is no better solution in the neighbourhood $\mathcal{Z}(\mathbf{x}_0)$ of the current solution. For the exploitation step, we perform a simple 1-flip local search algorithm where only one variable is flipped, per step, to reach the nearest neighbour. The local neighbourhood is the Hamming ball with distance one. More expensive, p-flip methods can be adopted where at most p variables are simultaneously flipped. In general, for a total number of n variables, the cardinality of the neighbourhood set is equal to $\binom{n}{p}$.

B. Exploration technique

Given a channel matrix $\mathbf{H}$, the singular value decomposition of this matrix is defined as $\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^T$, where the diagonal matrix $\mathbf{D}$ contains the singular values $[\lambda_k]_{k=1}^n$, supposed to be indexed in increasing order, i.e., $\{\lambda_i \leq \lambda_j, 1 \leq i < j \leq n\}$. The unitary matrices $\mathbf{U}$ and $\mathbf{V}$ contain, respectively, the left $[\mathbf{u}_k]_{k=1}^m$ and right $[\mathbf{v}_k]_{k=1}^n$ singular vectors of the matrix $\mathbf{H}$.

Let's now $\mathbf{x}_{zf} = \mathbf{H}^+\mathbf{y}$, where $\mathbf{H}^+$ is the pseudo-inverse of channel matrix $\mathbf{H}$, be a linear solution given by the zero-forcing (ZF) detector. For all $\mathbf{x} \in \xi^n$, the vector $\mathbf{z} = \mathbf{x} - \mathbf{x}_{zf}$ can be expressed as a linear combination of the basis vectors formed by the columns of the matrix $\mathbf{V}$. Moreover, the value of the objective function $f(\mathbf{x})$ can be expressed as

$$\begin{aligned} f(\mathbf{x}) &= \|\mathbf{H}(\mathbf{x} - \mathbf{x}_{zf})\|^2 \\ &= \Sigma_{k=1}^n a_k^2 \lambda_k^2 \end{aligned} \tag{5}$$

where the coefficients $[a_k]_{k=1}^n$ are real numbers. Since the singular values of the channel matrix are ordered increasingly, we can note that the increase in the objective function is much slower along the first N_d singular values than the last $(n - N_d)$ values. Let us consider an n-dimensional line Δ_k passing through the point $\mathbf{x}_{zf}$ with the directed vector $\mathbf{v}_k$, i.e $\Delta_k = \{\mathbf{z} \in \mathbb{R}^n; \mathbf{z} = \mathbf{x}_{zf} + \gamma\mathbf{v}_k, \gamma \in \mathbb{R}\}$. It is obvious that if we choose $\mathbf{x} \in \xi^n$ that are close to the first N_d lines $\{\Delta_k\}_{k=1}^{N_d}$ associated to the N_d smallest singular values, we can create a subset ξ_{st}^n that contains N_c possible solutions in the vicinity of each line. The purpose of the exploration step is to create a subset $\xi_{st}^n \subset \xi^n$, which contains $N_c N_d$ feasible solutions.

C. L2E algorithm

By construction, the exploration step generates a subset ξ_{st}^n of feasible solutions where each solution will be processed independently by the exploitation step. The latter step will iterate once over each element of the subset ξ_{st}^n. In this exploration and exploitation optimization process, the objective function will be evaluated nN_dN_c times. The L2E will create the list Γ of $2nN_dN_c$ elements, which yielded the lowest objective function values, and then the Γ list will be used to evaluate the output soft metrics. A summary of the proposed soft-output MIMO detector is shown in Figure 1.

Data: Channel matrix $\mathbf{H}$, received vector $\mathbf{y}$, N_d and N_c.
Result: LLR-values of the sub-optimal solution's entries.
begin
  Extract the N_d right singular vectors of the channel matrix $\{\mathbf{v}_k\}_{k=1}^{N_d}$;
  Compute $\mathbf{H}^+$;
  Compute $\mathbf{x}_{zf} = \mathbf{H}^+\mathbf{y}$;
  for $k = 1..N_d$ **do**
    Generate the line Δ_k defined by $\mathbf{x}_{zf}$ and directed vector $\mathbf{v}_k$;
    Find all intersection points between the line Δ_k and all hyperplanes $\mathbf{x}(i) = 0$ and project them on ξ^n;
    Evaluate the objective function for all feasible solution and keep the best N_c solution in order to update ξ_{st}^n;
  Create Γ using ξ_{st}^n;
  Perform the exploitation step over Γ;
  Compute LLR-values;

Figure 1. L2E Algorithm

IV. L2E Complexity

The complexity is measured in terms of the number of real multiplications required to decode one block of transmitted information bits. The assumption of a block-constant channel is almost universal in the analysis of MIMO systems. Thus, computing the pseudo-inverse matrix $\mathbf{H}^+$ is not needed for each received vector. We assume that each block contains L transmitted vector. To find the linear solution $\mathbf{x}_{zf}$, the received signal vector will be multiplied by the pseudo-inverse of channel matrix. The resulting complexity for L transmitted vectors is hence equal to Ln^2 multiplications. For the N_d studied directions, the exploration step produces an initial list of feasible solutions using the intersection points between lines $\{\Delta_k\}_{k=1}^{N_d}$ and hyperplanes $\{\mathbf{x}(i) = 0\}_{i=1}^n$. Thus, the computation complexity of the exploration step would be LN_dn^2 multiplications. The exploitation step will be performed on subset ξ_{st}^n of feasible solutions (the ξ_{st}^n subset's cardinality is N_dN_c). The exploitation step needs $2nN_dN_c$ multiplications. Finally, the LLR-values computation needs $4nN_dN_c$.

For a given N_d and N_c, the computational complexity of the proposed soft-output L2E detector is constant, over the entire SNR range, compared to that of the list sphere-decoding.

V. Simulation Results

In this section, we carry out some simulation results to evaluate the bit error rate performances of the soft-output L2E MIMO detector. We consider a MIMO-BICM system as proposed in [8], with N transmit antennas and M receive antennas. This system use a parallel concatenated turbo error control coding with rate $R = 1/2$. Each constituent convolutional code has memory 2, feedback polynomial $G_r(D) = 1 + D + D^2$, and feedforward polynomial $G(D) = 1 + D^2$. The interleaver size of the turbo code is 512 information bits. We choose the number of inner iterations for the turbo decoding module to

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be 10. As in [8], we generate independent Rayleigh flat fading channels between transmit/receive antennas and we assume a perfect channel estimation at the MIMO receiver side.

In the $N \times M$ MIMO system (i.e., $n = m = 8$) case, the Figure 2 compares the performance of proposed soft output L2E algorithm versus the list sphere decoding (LSD) with candidate list of maximal length $N_{cand} = 1024$ as show in [8] and the shifted spherical list APP detector [19]. It can be seen that for this MIMO systems scenario, the shifted spherical list APP detector, with $N_{cand} = 1024$, has a slightly better performance than the L2E decoder with parameters ($N_d = 3$ and $N_c = 4$). However, the soft-output L2E use not more than nN_dN_c feasible solutions to generate the log-likelihood ratios of different outputs.

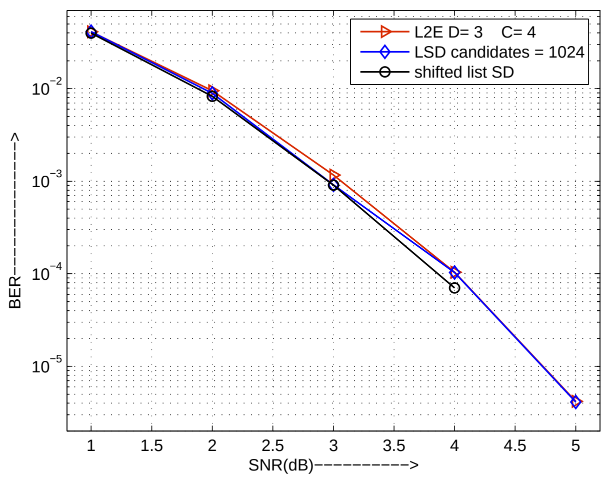

Figure 2. BER performances versus the SNR of L2E, LSD and shifted list sphere decoder, $N = M = 4$.

In Figure 3, the bit error rate performances between following soft-output MIMO detectors has been compared: list sphere-decoding, list exploration and exploitation detector, and soft-output semi-definite programming (SDP). Numerical results are presented for a MIMO system with $N = 8$ and $M = 8$ in a Rayleigh fading channel. The bit error rate difference is not even noticeable between the L2E with parameters $N_d = 3$ and $N_c = 4$ and the soft output SDP. At the bit error rate of 10^{-4}, L2E performance is only less than 0.2 dB from the LSD with candidates list length $N_{cand} = 1024$.

VI. Conclusion

In this paper, we proposed a soft-output MIMO detector algorithm called the list exploration and exploitation. We have shown that the proposed algorithm achieves near-optimal performance with low and fixed computational complexity. Furthermore, it is suitable for efficient practical implementation because of its parallelism. We have compared the bit error rate performances of the proposed detector are compared to the well-known list sphere decoding algorithm and it is shown that our method maintains near-optimal performances in comparison with LSD while considerably reducing the computation complexity.

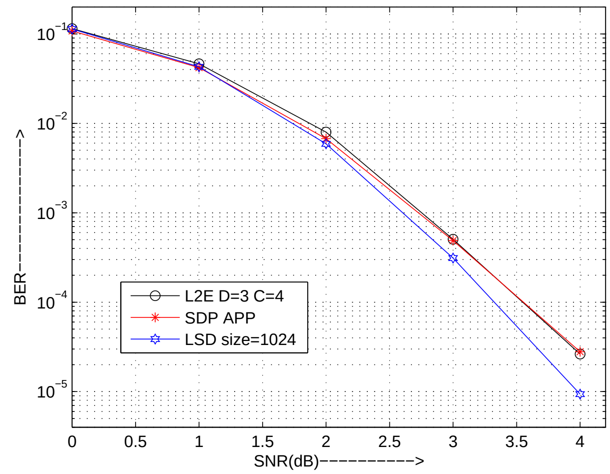

Figure 3. BER performances versus SNR of L2E, LSD and soft-output SDP detector, $N = M = 8$.

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