A Proposal of New Autonomous Decentralized Structure Formation Based on Huygens' Principle and Renormalization

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***Abstract*—This paper proposes an autonomous distributed algorithm that can construct spatial structures for clustering in MANETs. Since the topology of a MANET changes frequently, a fast, light-weight, and autonomous clustering mechanism is required. However, existing autonomous clustering mechanisms are based on differential equations and thus demand a lot of calculations for generating the spatial structures that yield clustering. This paper proposes an autonomous clustering algorithm that is based on Huygens' principle. The most remarkable characteristics of our proposal are calculation simplicity and fast convergence on the cluster structures. We verify the basic characteristics of the proposed algorithm.**

Keywords-autonomous decentralized control; structure formation; asymptotic stability; Huygens' principle; renormalization transformation.

I. INTRODUCTION

In large-scale communication networks, hierarchical architectures are effective for scalable network control. Let us consider how to introduce a hierarchical structure into a network. In a network having fixed topology, we can consider the desired hierarchical structure when designing the network. Unfortunately, this is not possible in a network with a dynamic topology. A typical example of such a network is the mobile ad hoc network (MANET) [1]. A MANET consists of mobile terminals that offer routing functions and data forwarding. Two terminals can directly communicate if their coverage areas overlap. If the areas do not overlap, the terminals do not directly communicate, but by relaying data through terminals between the two terminals, they can establish multihop communication. To achieve multihop communication, routing is one of most important issues in MANETs.

The primitive approach to route finding is called flooding [2]. In flooding, the sender terminal sends route finding packets to all adjacent terminals, which resends them to all their adjacent terminals until at least one copy of the packet reaches the destination terminal. The total amount of route search packets sent in MANETs increases exponentially with network size (the number of terminals). One of the challenges in MANETs, realizing scalable routing control [3], [4], is best addressed by setting a hierarchical structure through clustering [5]–[7].

Hereafter, we call a MANET terminal a node. An autonomous clustering mechanism for generating a hierarchical structure requires several characteristics, as follows:

- Each node acts autonomously based on local information about its neighboring nodes.
- The generated cluster structure should reflect the state information of the network (e.g., battery power of nodes).
- The generated cluster structure should be flexible so that it can adapt to the dynamic environment.
- The convergence time of clustering should be sufficiently shorter than the timescale of topology change enforced by node movement. This is because clustering should dynamically adapt to the network topology.
- Action rules of each node should be as simple as possible in order to reduce the battery power consumed by computation or processing at the node.

Since ad hoc networks are expected as an effective communication tool after serious disaster, the above requirements are essential for realizing clustering in ad hoc networks.

Takano *et al.* has proposed a clustering mechanism based on the Fokker-Planck equation and includes the drift motion given by back-diffusion [8], [9]. Let us call it the back-diffusion based approach. This mechanism satisfies the first two requirements listed above. Hamamoto et al. [10] recently proposes a guaranteeing mechanism of the asymptotic stability, and clarifies that, by using the guaranteeing mechanism, the back-diffusion based approach can satisfy the third requirements listed above. In addition, it also implies that we might be able to make the clustering algorithm that satisfies all the requirements listed above by replacing back-diffusion algorithm with other simple and fast-converging rule. This is because the guaranteeing mechanism of the asymptotic stability does not depend on details of the clustering mechanism. In this paper, we use Huygens' principle [11] as the simple and fast-converging rule, and propose a new clustering mechanism that satisfies all the requirements listed above.

The paper is organized as follows: Section II explains the guaranteeing mechanism of the asymptotic stability, which is the foundation of this research. Section III proposes an autonomous clustering mechanism based on Huygens' principle. Section IV discusses the initial condition of the proposed mechanism with the goal of ensuring robust controllability of cluster size. In addition, it also shows cluster structures generated by the proposed mechanism using numerical examples and verifies that they reflect the network condition. The

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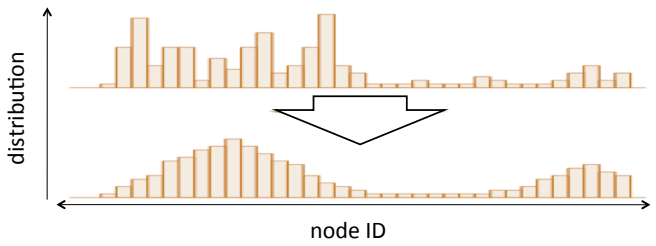

Fig. 1. Concept of cluster forming.

conclusion is discussed in Section V.

II. PRELIMINARY

A. Concept of Cluster Formation

In our clustering model, each node has certain value and cluster formation is conducted by the distribution of the values of nodes. The initial value is determined by considering the network condition (e.g., battery power of each node). The clustering algorithm is to extract a coarse grained spatial structure from the initial distribution of the values and this procedure corresponds to clustering. Fig. 1 shows an example cluster formation in a simple 1-dimensional network. The horizontal axis represents node ID, and the vertical axis represents the value of the distribution for each node. The top part of Fig. 1 represents the initial initial distribution, which is reflecting network state (e.g., battery power of each node). The bottom part of Fig. 1 represents the generated coarse grained spatial structure. The peaks of the coarse grained distribution correspond to clusters and they are reflecting the initial condition.

The back-diffusion based approach is an example of this mechanism, and has a relatively faster convergence rate than conventional bio-inspired approach [12]. However, this clustering mechanism does not consider the change of the initial condition, and therefore it cannot adapt to the dynamic environment. That is why this mechanism does not satisfy the third requirement listed in the previous section.

B. Guaranteeing Mechanism of Asymptotic Stability

To adapt the spatial structure to the dynamic environment, the guaranteeing mechanism of the asymptotic stability of cluster structures has been proposed by Hamamoto *et al.* [10]. In this mechanism, generated cluster structure can adapt dynamic environment, and it also can generate stable spacial structure under the fixed initial condition.

Let us consider a one-dimensional network model for simplicity, and let $q(i,t)$ be the value of distribution at node ID i at time t. The distribution $q(i,t)$ determines the cluster structure. Examples of the initial condition $q(i,0)$ and cluster structure $q(i,t)$ obtained at time t are shown in Fig. 1. The conventional back-diffusion based approach described in Takano *et al.* [9] presents a rule governing the temporal evolution of the distribution $q(i,t)$. However, as shown in the previous section, it is difficult to guarantee the stability of $q(i,t)$ for large t. In other words, the cluster structure is not stable in a dynamic environment.

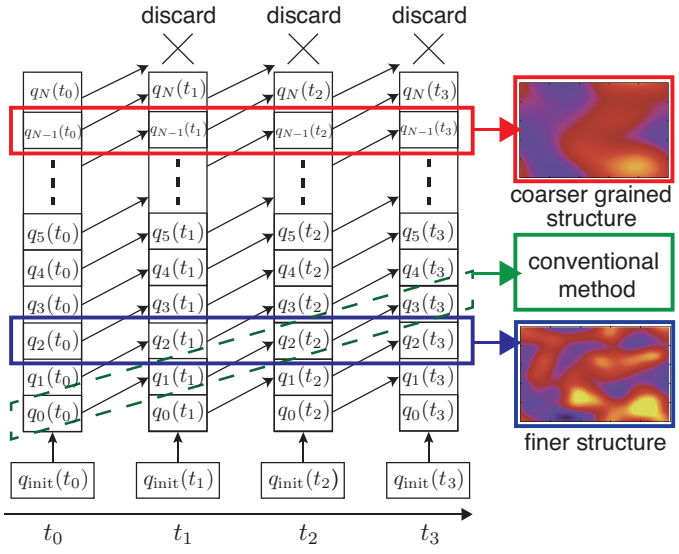

Fig. 2. Outline of the guaranteeing mechanism of asymptotic stability.

Let us consider discrete time t_k $(k = 1, 2, \dots)$, and distribution $q(i,t_k)$. Since we need an autonomous decentralized algorithm, the temporal evolution of distribution $q(i,t_k)$ is determined by its local information. By introducing the temporal evaluation operator of $\mathcal{T}$, the temporal evolution is formally described as

$$q(i,t_{k+1}) = \mathcal{T}(q(i-1,t_k),\, q(i,t_k),\, q(i+1,t_k)) \qquad (1)$$

This rule states that the distribution of node i at the next time is completely determined by the values of the present distribution at node i and its adjacent nodes.

To guarantee the mechanism of asymptotic stability, we consider a vector of the distribution. Each node i has the following $N+1$ dimensional vector

$$\boldsymbol{q}(i,t_k) = \{q_0(i,t_k),\, q_1(i,t_k),\, \dots,\, q_N(i,t_k)\} \qquad (2)$$

Here, we define the rule for the temporal evolution of the vector $\boldsymbol{q}(i,t_k)$. Let $q_{\mathrm{init}}(i,t_k)$ be the distribution describing the network state (e.g., battery power of a node) at time t_k. Then we set

$$q_0(i,t_{k+1}) = q_{\mathrm{init}}(i,t_{k+1}) \qquad (3)$$

If $q_{\mathrm{init}}(i,t_k)$ is independent of time, $q_0(i,t_{k+1}) = q(i,0)$, that is, the initial condition of the conventional mechanism. Note that, in general, we allow the time-dependence of $q_{\mathrm{init}}(i,t_k)$. Next, for $q_{n+1}(i,t_{k+1})$ $(n = 0,\, 1,\, \dots,\, N-1)$, we set

$$q_{n+1}(i,t_{k+1}) = \mathcal{T}(q_n(i-1,t_k),\, q_n(i,t_k),\, q_n(i+1,t_k)) \qquad (4)$$

Although the above rule may look complicated, we can easily understand it through graphical representation. Fig. 2 explains the temporal evolution of vector (2) at node i. The horizontal axis represents discrete time as t_0, t_1, $\dots$, and $q_{\mathrm{init}}(t_k)$ expresses a certain metric of network state of a node at time t_k. Each component of the vector is a distribution value and (4) is the temporal evolution rule for the nth component $(n = 1, 2, \dots, N-1)$. The temporal evolution of each component will be updated to the upper-right component in

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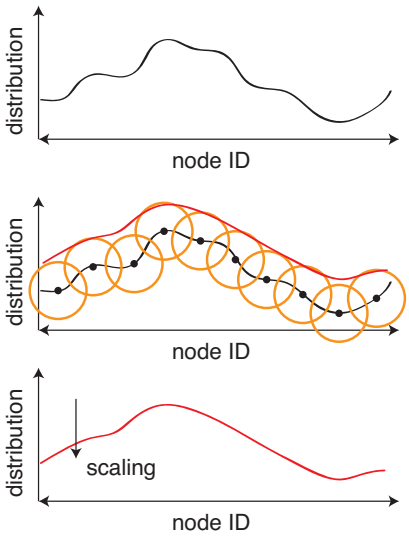

Fig. 3. Renormalization transformation as per Huygens' principle.

Fig. 2. The component at the bottom, $q_0(i, t_{k+1})$, is overwritten by the network condition $q_{\rm init}(i, t_{k+1})$ at the present time. The component at the top will be discarded.

The temporal evolution of the conventional mechanism corresponds to the sequence indicated by the green broken line in Fig. 2. In the guaranteeing mechanism of asymptotic stability, we focus on the sequence of the same vector components. If we choose small n for the nth component, we obtain a finer-grained spatial structure as indicated by the blue line. A large n yields a coarse grained spatial structure as indicated by the red line.

III. Design of the Autonomous Structure Formation Technology based on Huygens' Principle

A. Huygens' principle and Renormalization

Huygens' principle [11] describes the temporal evolution of the wavefront and can explain the laws of reflection and refraction. Let us consider spherical waves originating at each point on a wavefront. The envelope of these spherical waves gives the temporal evolution of the wavefront. This is called Huygens' principle or the Huygens-Fresnel principle. Renormalization is a way to extract simple and important macroscopic characteristics from a large-scale and complex system, and its procedure is defined as the renormalization transformation. This procedure is suitable for generating a simple cluster structure extracted from the spatial structure of the network state. The renormalization transformation is defined as the combination of coarse-grained transformation and scaling. In this paper, we adopt the renormalization transformation based on Huygens' principle as temporal evolution operation $\mathcal{T}$. Concrete procedures of the renormalization transformation are shown below.

Let us consider a one-dimensional network and a distribution on the network. The panel at the top of Fig. 3 shows an example of the distribution at the present time. We consider the shape of the distribution as the wavefront. The panel at the middle of Fig. 3 shows the temporal evolution of the wavefront as given by Huygens' principle. This procedure has smoothing effect such that the fine-grained structure that takes the shape of the distribution becomes smooth. The temporal evolution of the distribution causes an increase in the value of the distribution, that is, the wavefront proceeds upward. In order to compensate for this increase, we introduce scaling as shown in the panel at the bottom of Fig. 3. We define the renormalization transformation as the combination of such temporal evolution and scaling.

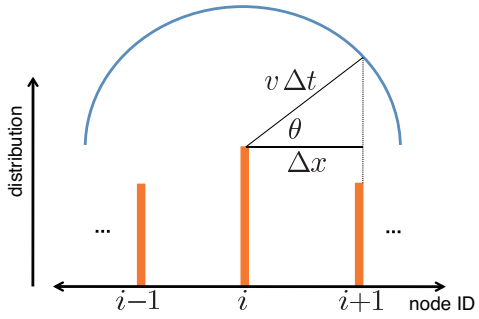

Fig. 4. The wavefront of the spherical wave reaches an adjacent node.

Let the value of distribution at node i at time t_k be $q(i, t_k)$, and let the set of nodes that are adjacent to node i at time t_k be $M(i, t_k)$. In addition, $\tilde{q}(i, j, t_{k+1})$ is the wavefront of sphere wave at node i at time t_{k+1} that originated from node j at time t_k. Our renormalization transformation is expressed as

$$q(i, t_{k+1}) = \frac{1}{b} \max_{j \in M(i,t_k)} \tilde{q}(i, j, t_{k+1}), \tag{5}$$

where, the maximizing operation in (5) means Huygens' principle; it determines the most advanced wavefront of spherical waves that originated from the node itself and its neighborhood, and $b > 1$ is the scaling parameter.

Next, we consider the concrete form of $\tilde{q}(i, j, t_{k+1})$. Let the propagation speed of spherical wave be v, the distance between two adjacent nodes be Δx, and the interval of the temporal evolution (renormalization transformation (5)) be Δt (i.e., $t_{k+1} - t_k = \Delta t$). Here, Δx is not physical distance but is a kind of hop count, so we can choose $\Delta x = 1$. We consider the situation that the temporal evolution (5) is determined only by adjacent nodes, v is chosen as $1 \le v\,\Delta t < 2$. As shown in Fig. 4, the wave front of spherical wave originated from node i influences both node i and its adjacent nodes. It is expressed as

$$\tilde{q}(i \pm 1, i, t_{k+1}) = q(i, t_k) + v\,\Delta t\,\sin\theta, \tag{6}$$

$$\tilde{q}(i, i, t_{k+1}) = q(i, t_k) + v\,\Delta t, \tag{7}$$

where θ is a constant and, from $v\,\Delta t\,\cos\theta = \Delta x$,

$$\theta = \arccos\left(\frac{\Delta x}{v\,\Delta t}\right) \tag{8}$$

Since v, Δt, and $\sin\theta$ are constants and we can know them in advance, the temporal evolution (5) is a simple operation. In addition, with iterations of the temporal evolution (5), the maximum value of distribution $q(i, t_k)$ converges to $p^* := v\,\Delta t/(b-1)$, regardless of initial condition $q(i, 0)$.

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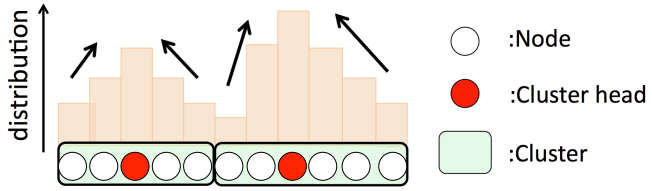

Fig. 5. Determination of the cluster and cluster head.

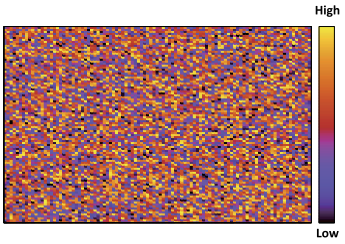

Fig. 6. An example of randomized initial condition

B. Amplification of Amplitude of the Distribution

Our renormalization transformation (5) makes the distribution flat and we can obtain coarse-grained spatial structure. However, differ from physical phenomena, there are situations that the distribution does not change when the difference of distribution values are small. This is because the positions of nodes in the network are discrete. If the value of the distribution at a node can affect that of the adjacent node, the following relation is required,

$$|q(i \pm 1, t_k) - q(i, t_k)| > v\,\Delta t\,(1 - \sin\theta) \qquad (9)$$

When the smoothing proceeds and the condition (9) is no longer met, two adjacent nodes do not interact and the distribution is unchanged. To avoid this phenomenon, we introduce amplification of the amplitude of the distribution in addition to the renormalization transformation (5). The additional operation is

$$q(i, t_{k+1}) \leftarrow p^* + a\,\left(q(i, t_{k+1}) - p^*\right), \qquad (10)$$

after the renormalization transformation (5). This operation means that the difference between the value of distribution and p^* is amplified by a factor of a times. Here, aforementioned $p^* = v\,\Delta t/(b-1)$ is the fixed point of the renormalization transformation, and also is the convergence point. The value of the parameter a should be chosen as $a > b$.

Finally, we explain how to determine clusters and cluster heads from the generated spatial structure (Fig. 5). By following the direction of the steepest gradient of the distribution, we can find a node with local maximum value. We define it as a cluster head, and the nodes belonging to the same cluster head belong to the same cluster.

IV. Performance Evaluation

This section investigates the convergence speed with respect to the range of the initial distributions, and proposes a way to guarantee the controllability of our proposed mechanism. In addition, we verify the initial distribution dependency of our clustering.

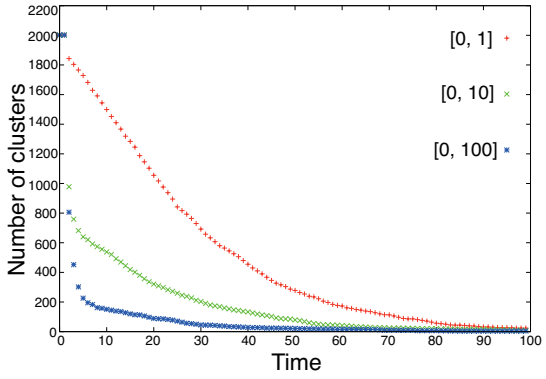

Fig. 7. The number of generated clusters for randomized initial conditions w.r.t. time.

A. Dependence Characteristics on the Range of Initial Distribution

First, we investigate the convergence speed with respect to the range of the initial distributions. The network model in this evaluation is a two-dimensional lattice network with torus boundary and it has 100×100 nodes. The reason of torus boundary condition is to eliminate the effect of the network edge, and to concentrate our attention on the characteristics of clustering mechanism itself. The initial distribution of $q(i, 0)$ for all the node position, i, are given by a uniform distribution; three kinds of uniform distributions are examined: their ranges are $[0,\ 1]$, $[0,\ 10]$, and $[0,\ 100]$. An example of an initial condition is shown in Fig. 6.

We calculate the temporal evolution of the distribution by using (5) and (10), every discrete time. The interval of discrete time is set to be $\Delta t = 1$. Here, we investigate the change in the number of generated clusters. Fig. 7 shows the temporal evolutions of cluster number, from three different initial conditions. The parameters were set as $v = 1.5$, $a = 1.2$, $b = 1.1$. From this result, we can recognize that the number of clusters strongly depends on the range of the initial condition. Huygens' principle or the maximizing operation in (5) has strong impact when the adjacent nodes have very different values.

This characteristic triggers a loss of control over cluster size (or the number of clusters). The number of clusters in the initial state (the number of local maximums in Fig. 6) is about $2,000$ as shown in Fig. 7. The horizontal axis denotes time but also corresponds to iteration number of temporal evolution or the component of the vector (2). If we need $1,000$ clusters, we can choose about 20 iterations or the 20th component of vector $\boldsymbol{q}(i, t_k)$, for the initial condition of $[0,\ 1]$. However, we cannot choose the appropriate value for $[0,\ 100]$. Since we cannot know the network condition in advance, we cannot control the number of clusters.

Let us consider how this situation corresponds to difficulty in controllability. If we describe the initial condition of nodes by their battery power, we should express the battery power in numerical value. There are many ways to express the battery power in numerical value: ampere-hour [Ah], mili-ampere hour [mAh], coulomb [C], etc. Incidentally, 1 Ah = 1,000 mAh = 3,600 C. The above difficulty in controllability implies that

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the initial distribution having different range gives different clustering structure even if the distributions come from the same physical situation (only difference is in way to express it).

B. Robust Controllability of Cluster Size

The cause of the above problem is the excessive sensitivity of cluster forming with respect to the range of the distribution. The mechanism of the excessive sensitivity with respect to the range of the distribution can be recognized by Fig. 8. Fig. 8 shows the behaviors of the proposed clustering mechanism for the distributions having wide and narrow ranges. First, each node performs temporal evolution obeying Huygens' principle, and next, scaling. Small-valued node, which is next to the large-valued node, is greatly influenced by the large-valued node, and the difference of the values between them is rapidly decreasing. So, if the range of the initial distribution is wide, the distribution is rapidly uniformized.

In order to avoid the above problem, we redefine the initial condition. The details are as follows. We do not use the network condition directly as the initial condition, but we use

$$q(i,0) = \log(1 + q_{\text{init}}(i,0)) \tag{11}$$

In the vector formulation, we replace (3) with

$$q_0(i,t_{k+1}) = \log(1 + q_{\text{init}}(i,t_{k+1})) \tag{12}$$

The reasons for introducing a logarithmic function are as follows:

- It is possible to maintain the magnitude relation of the value of the original initial distribution.
- As a value of the original initial distribution is large, new value is smaller in the sense of the ratio.

Fig. 9 shows similar evaluations by using the redefined initial condition (11). We recognize that the impact of the initial condition is sufficiently weakened. In particular, the initial conditions of $[0, 10]$ and $[0, 100]$ yield almost the same result. This means we have a robust clustering mechanism that can control the number of clusters by appropriately choosing the number of iterations or the component of the vector (2).

C. State Dependent Characteristics of Clusters

Since the initial condition reflects the network state (e.g., battery power of each node), the generated cluster must be influenced by the initial condition. This subsection introduces a numerical example that shows that the cluster structures generated by the proposed mechanism do reflect the network condition.

We use the same network model and parameter setting as the previous subsection. To verify the dependence on initial conditions, we use two types of initial conditions. One is a randomized condition, that was used in the previous subsection. This initial condition is determined by logarithm of random values, which obey a uniform distribution with range of $[0, 10]$, as shown in Fig. 6. The other initial condition has spatial patterns as shown in Fig. 11. Three areas have relatively high values, and the value of these areas is determined by random values which obey a uniform distribution with range

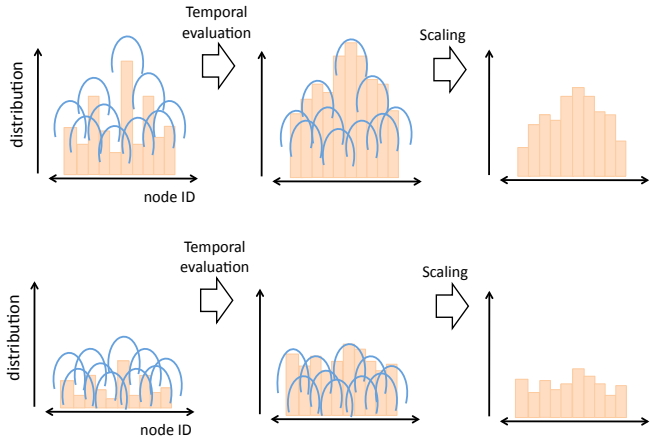

Fig. 8. An example of behavior of the our proposed mechanism on two different distribution ranges

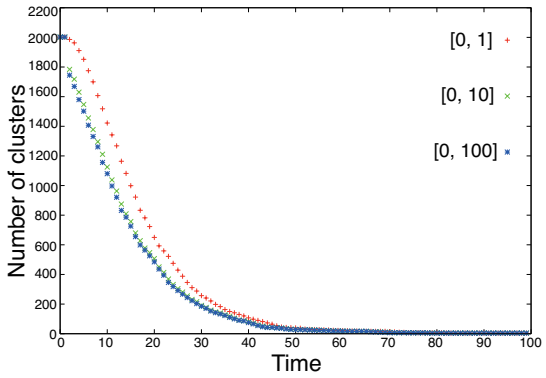

Fig. 9. The number of generated clusters for logarithmic initial conditions, w.r.t. time.

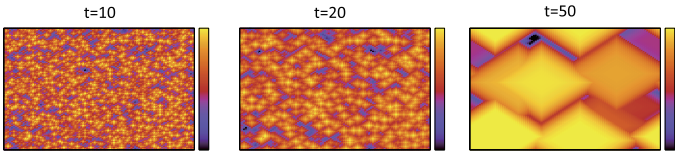

Fig. 10. The cluster structures generated from randomized initial condition.

$[0, 10]$. The value of other area is determined by random values which obeys the uniform distribution with range $[0, 1]$. The logarithm of these values is used as the initial condition.

Figs. 10 and 12 show the cluster structures generated from the initial conditions of Figs. 6 and 11, respectively. The three panels of each figure show the number of iteration or, equivalently, the vector component. If we choose few iterations, we obtain a finer-grained cluster structure, and if we choose more iterations, we get a coarse grained cluster structure. We can also recognize that the cluster structures reflect the spatial structures of the initial conditions.

V. Conclusion and Future Work

In this paper, we proposed an autonomous clustering mechanism based on Huygens' principle and renormalization. In

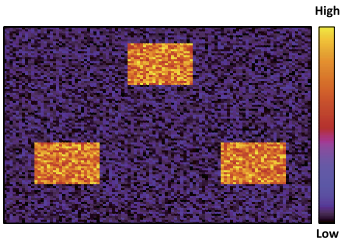

Fig. 11. An example of initial conditions with the spatial structure

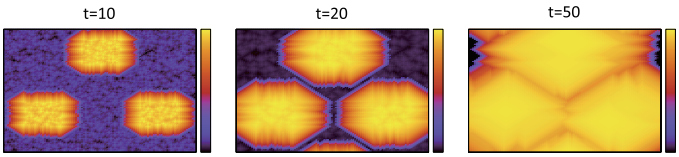

Fig. 12. The cluster structures generated from the initial conditions with spatial structure.

verification, we used a two-dimensional lattice network to evaluate the characteristics of the proposed algorithm. The pros of the proposed algorithm are in its simplicity and in the ability to keep the spatial structure of the initial condition in the configuration of clusters. However, unfortunately, the convergence speed of cluster configuration strongly depends on the value of the initial distribution. Since we can not know the value of distribution for each node in advance, the difference of convergence speed causes the situation that we cannot control the number of clusters. To avoid this problem, we introduced new distribution defined by the logarithm of the original distribution. Consequently, the difference of the convergence speed is significantly reduced, and the number of clusters becomes controllable. The above obtained characteristics are suitable for clustering in MANET. As future work, we will consider the adaptability of our mechanism in dynamic environment.

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