Investigating the Robustness of Detection vis-à-vis the Detector Threshold in WSN with Fading MAC and Differing Sensor SNRs – Optimal Sensor Gains vs. Uniform Sensor Gains

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Abstract—In Wireless Sensor Networks, if the Multiple Access Channel between distributed sensors and multiple antennas is fading and the envelope of the channel gain distribution is unknown and time-varying, fusion at the antennas is usually incoherent. Often, the overall sensor power is upper bounded by a constraint on the onboard battery power. Then, the optimal sensor power allocation scheme which minimizes the probability of missed detection is kno-wn to outperform uniform sensor power allocation scheme. Further, if the observation signal-to-noise ratios at the sensors are non-identical, optimizing the probability of detection must take into account the combined effect of the differing sensor signal-to-noise ratios and the fading nature of the channel *as seen by the sensors*. Neyman-Pearson formulation of this problem sets out by setting an upper bound on the permissible probability of false alarm. Consequently, the detector threshold is governed by the power allocation scheme – uniform or optimal. We examine here the inter-dependencies between the probability of false alarm, the probability of detection and the detector threshold. We demonstrate that for robust detection vis-à-vis variations in detector threshold, there is an additional compelling case for optimal power allocation over uniform power allocation.

Keywords-Wireless Sensor Networks; Multiple Access Fading Channel; Optimal Power Allocation; Detector Threshold; Robustness.

I. INTRODUCTION

We address the problem of distributed detection over a resource constrained Wireless Sensor Network (WSN). The schematic of the system taken from [1] is shown in Fig 1. On-board batteries with limited power drive the sensors. The sensed signal received by the sensors is corrupted by additive noise, amplified by the sensor gain and transmitted over a fading channel to the Fusion Centre (FC). To detect the sensed parameter/event at the FC, we employ the Neyman-Pearson (NP) formulation.

A. State of the Art

Uniform Power Allocation (UPA) to the sensors is shown to be sub-optimal when the Multiple Access Channel (MAC) is fading [1]. The authors there show that Optimal Power Allocation (OPA) is superior to UPA under the following conditions: (i) the channel is fading; (ii) the sensor observation noise is i.i.d.; (iii) the sensor observation Signal-to-Noise Ratio (SNR) is time-invariant; (iv) there is an overall sensor power constraint; and (v) the False Alarm (FA) rate has a fixed acceptable upper bound. Thus, there is a saving in onboard power even with i.i.d. sensor observation noise and time-invariant sensor observation SNR. However, with non-identical sensor observation SNRs, the OPA of [1] may lead to wastage of system resources. OPA for the case of sensor noise with different SNRs is addressed in [2].

B. Motivation for this Work

It was seen in Section I-A that UPA is sub-optimal when the MAC is fading and that OPA is superior to UPA under some conditions [1]. Suppose the sensor observation SNRs are non-identical. Then, the OPA of [1] which does not take into account the combined effect on the overall performance of (i) the differing sensor SNRs; and (ii) the particular fading characteristic of the channel path seen by the individual sensors, has been shown to result in wastage of system resources [2].

Within a fixed permissible probability of FA, P_{FA}, the NP scheme admits a choice of the detector threshold. Of particular concern that we focus upon here is from a designer's perspective. It lies in examining the interval allowable to choose the detector threshold, τ, within the constraints placed by the tolerable P_{FA} concurrent with the desired probability of detection, P_D, albeit for a given total sensor power constraint, P_T.

Moreover, as τ varies within the admissible interval of τ, it is desired to study the nature of degradation of performance. In relation to P_{FA}, the detector threshold, τ, impacts P_D. The natural question that arises here is therefore the 'goodness' of the choice of τ for 'enhanced performance'. To answer this question, we study the nature of inter-relationships between P_{FA}, τ and P_D. The comparison here is across OPA and UPA. Thus, equipped with a priori knowledge of the nature of impact of τ on system performance, the user can choose τ to balance conflicting criteria, dictated by the demands of the specific application.

The rest of the paper is organized as follows. Section II describes the system model with the power constraint. The detection algorithm and its formulation comprise Section III. Power allocation schemes are discussed in Section IV. Simulation set up and results are presented and analyzed in Section V. Concluding remarks form Section VI.

II. System Description and Model

To facilitate readability, we briefly describe the system setup on the lines of the formulation in [2].

A. The Sensor

The sensor network comprises L sensors transmitting to N antennas over NL channels as in Fig. 1 [3]. The sensed parameter/event is $\Theta \in \{0, \theta\}$. Let H_0 and H_1 be the hypotheses corresponding respectively to $\Theta = 0$ and $\Theta = \theta$. Further, in terms of the priors p_0 and p_1, let

$$\Theta \triangleq \left\{ \begin{array}{ll} H_0 : 0 & w.p.\ p_0; \\ H_1 : \theta & w.p.\ p_1. \end{array} \right.$$

At the ℓth sensor, the additive noise, η_ℓ, is characterized as $\eta_\ell \sim \mathcal{CN}(0, \sigma_\ell{}^2)$. Thus the sensor SNRs are not identical. The gain of the ℓth sensor is $\alpha_\ell \in \mathbb{C}$. Then, the sensor output is

$$\alpha_\ell(\Theta + \eta_\ell);\ \ell = 1, 2, \cdots, L.$$

The constraint on the overall sensor power, P_T, is given by

$$
\begin{aligned}
P_T &= E\left[\sum_{\ell=1}^{L} | \alpha_\ell(\Theta + n_\ell) |^2 \right] & (1) \\
&= \sum_{\ell=1}^{L} | \alpha_\ell |^2 \, (p_1\theta^2 + \sigma_\ell^2) \\
&= \alpha^H \left[p_1\theta^2 \mathbf{I}_L + \mathbf{D}(\sigma) \right] \alpha, & (2)
\end{aligned}
$$

in matrix notations. Here, $E[\cdot]$ is the expectation operator; $\alpha,\ \sigma \in \mathbb{C}_{L\times 1}$; x^H is the Hermitian of x; $\mathbf{I}_L$ is the identity matrix of size L; and $\mathbf{D}(u) \in \mathbb{C}_{m\times m}$ is the diagonal matrix with the entries of the vector $u \in \mathbb{C}_{m\times 1}$ on the diagonal.

B. Multiple Access Channel (MAC)

The sensors feed into a multiple access fading channel. The random gain from the ℓth sensor to the nth antenna is $h(n, \ell)$.
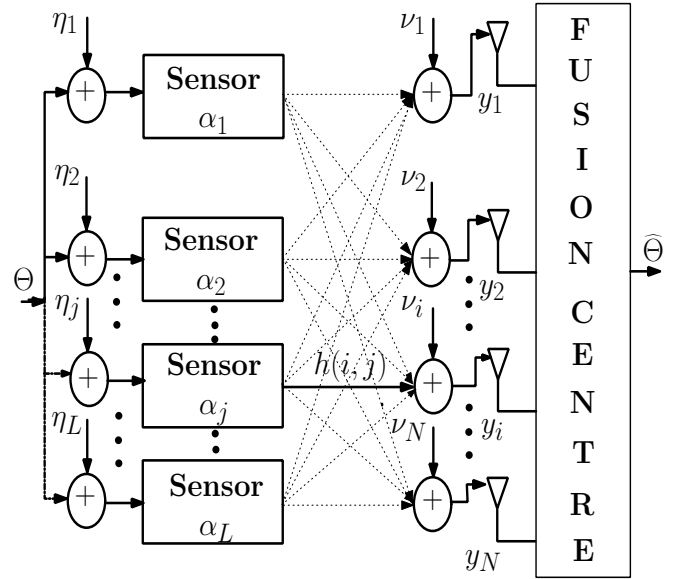

Fig. 1. Schematic of the Setup

The channel gain matrix is thus $\mathbf{H} = [h(n, \ell)] \in \mathbb{C}_{N\times L}$.

C. Antennas and the Fusion Center

There are N antennas at the receiving end. The additive noise, $v_n \sim \mathcal{CN}(0, \sigma_v{}^2)$, at the nth antenna, $n = 1, 2, \cdots, N$, is taken to be i.i.d. For simplicity, we assume the antenna noise to be of unit variance. The output of the N antennas received by the FC is $y \in \mathbb{C}_{N\times 1}$. Thus,

$$y = \mathbf{H}\alpha\Theta + \mathbf{H}\mathbf{D}(\alpha)\eta + v, \tag{3}$$

where $\eta \in \mathbb{C}_{L\times 1}$ and $v \in \mathbb{C}_{N\times 1}$. From the observed output, $\widehat{\Theta} \in \{0, \hat{\theta}\}$, at the FC, the problem is to detect the parameter, $\Theta \in \{0, \theta\}$, emitted by the source and to analyze the system performance.

III. Detection Algorithm

For detection, we assume y to be Gaussian. Thus,

$$
\begin{aligned}
H_0 &: \quad y \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{R}); \\
H_1 &: \quad y \sim \mathcal{CN}(\theta\mathbf{H}\alpha, \mathbf{R}). \tag{4}
\end{aligned}
$$

Here, $\mathbf{0}_N$ is the $N \times 1$ zero vector and $\mathbf{R}$ is the $N \times N$ covariance matrix of the received signal given by

$$\mathbf{R} = \mathbf{H}\mathbf{D}(\alpha)\mathbf{D}(\sigma)\mathbf{D}(\alpha)^H \mathbf{H}^H + \mathbf{I}_N. \tag{5}$$

Define

$$\delta \triangleq \alpha^H \mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}\alpha \tag{6}$$

and

$$Q(x) \triangleq (1/\sqrt{2\pi}) \int_x^\infty \mathrm{e}^{-t^2/2} \mathrm{d}t.$$

Then, the probability of false-alarm, P_{FA}, becomes

$$P_{FA} \leq Q\left(\frac{\theta\sqrt{\delta}}{2} + \frac{\tau}{\theta\sqrt{\delta}}\right). \qquad (7)$$

In (7), τ is the detector threshold which is a consequence of the likelihood ratio,

$$\frac{\Pr\{y \mid H_1\}}{\Pr\{y \mid H_0\}}.$$

Specifically, τ influences detection in accordance with

$$\left(\theta y^H \mathbf{R}^{-1} \mathbf{H}\alpha\right) \begin{array}{c} H_1 \\ \gtrless \\ H_0 \end{array} \left(\frac{1}{2}\theta^2 \alpha^H \mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}\alpha + \tau\right) = \frac{\theta^2 \delta}{2} + \tau. \qquad (8)$$

Finally, the probability of missed detection, P_{MD}, and hence, the probability of detection, P_D, are given by

$$P_{MD} = 1 - P_D \leq \left[1 - Q\left(Q^{-1}(P_{FA}) - \theta\sqrt{\delta}\right)\right]. \qquad (9)$$

IV. POWER ALLOCATION ALGORITHMS

We discuss here two schemes, viz., Uniform PA and Optimal PA. The relations of Section II and Section III are valid for both these schemes, although they represent different quantities in the two schemes.

A. Uniform Power Allocation (UPA)

The total sensor power with UPA is equally distributed among L sensors as P_T/L, thus giving the sensor gains as

$$\alpha_{uni,\ell} = \sqrt{P_T/L}, \ \ell \in \{1, 2, \cdots, L\}.$$

Setting $\alpha = \alpha_{uni}$ in (5) and (6), we obtain R and δ. We stipulate the maximum allowable false alarm P_{FA}. Then, taking the equality in (7), we solve for the corresponding limiting detector threshold, $\tau = \tau_{uni}$. Similarly, taking the equality in (9), we solve for the corresponding limiting probability of detection, $P_D = P_{Duni}$.

It is shown in [1] that this UPA results in wastage of system resources if the channel is fading and/or if the sensor observation SNRs are not identical [2], albeit time-invariant. This brings us to the optimal PA scheme.

B. Optimal Power Allocation (OPA)

In the context of a fading MAC, if the sensor observation SNRs are time-varying and/or non-identical, the optimal PA scheme proposed in the setting of [1] is indeed non-optimal. Even though the channel considered there is fading, the sensor noise is i.i.d. Hence, if the sensor observation noise is not identical, due to very poor SNR of a certain sensor, amplified noise transmitted by it over even a noise-free channel may lead to (i) missed detection, (ii) false alarm and (iii) wastage of resources. A comprehensive optimization algorithm must therefore consider the combined effect on detection of the
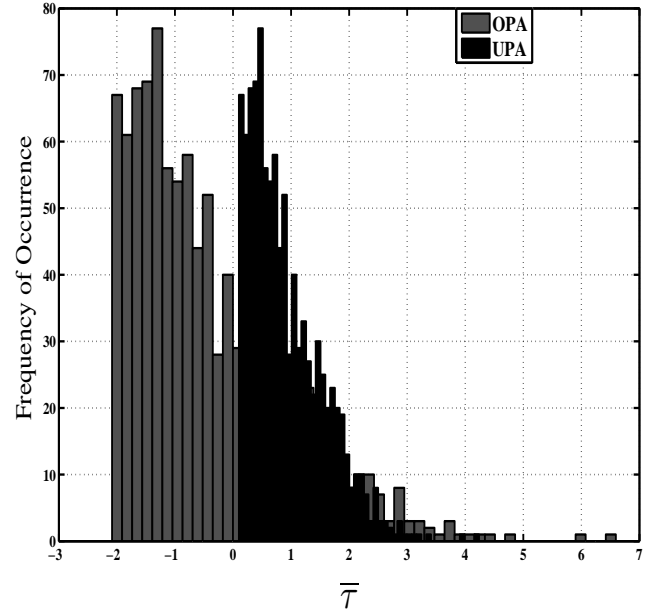

Fig. 2. Distribution of mean detector threshold for UPA and OPA
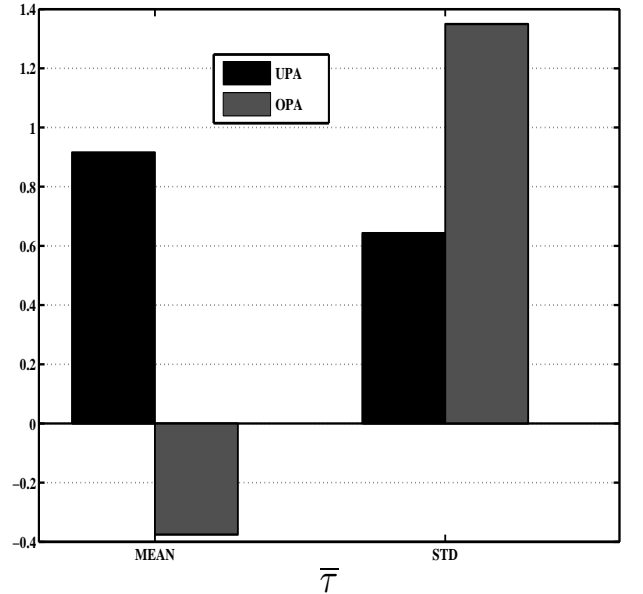

Fig. 3. Statistics of mean detector threshold

differing sensor SNRs and the fading channel, subject to a total power constraint [2].

In this backdrop, (9) shows that for a fixed P_{FA}, maximizing δ is equivalent to minimizing P_{MD}. It is clear from (6) that this requires choosing α that maximizes δ. Thus, the problem of maximizing P_D reduces to finding that optimal sensor gain, α_{opt}, such that

$$\alpha_{opt} = \underset{\alpha}{\operatorname{argmax}}\left[\alpha^H \mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}\alpha\right], \qquad (10)$$
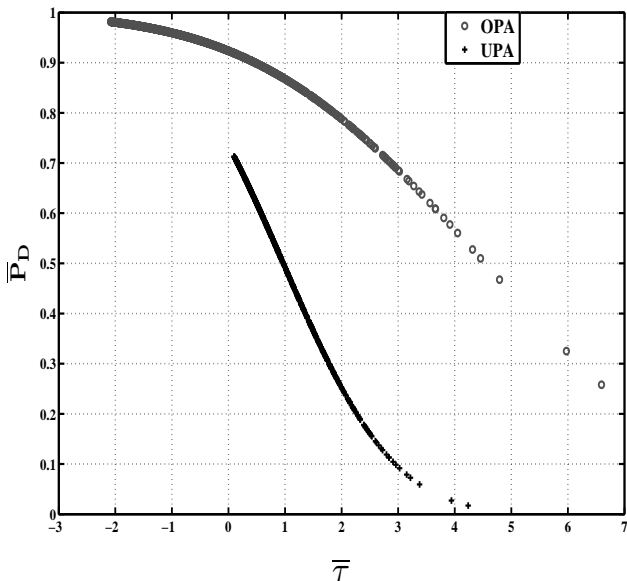
Fig. 4.   Mean probability of detection vs. Mean detector threshold

subject to the power constraint of (2) rewritten as

$$P_T = \sum_{\ell=1}^{L} \left[|\alpha_\ell|^2 \left(p_1\theta^2 + \sigma_\ell{}^2 \right) \right]$$
$$\triangleq \sum_{\ell=1}^{L} |\alpha_\ell \beta_\ell|^2 \triangleq \sum_{\ell=1}^{L} |\gamma_\ell|^2. \qquad (11)$$

Here, $\beta_\ell = \sqrt{(p_1\theta^2 + \sigma_\ell^2)}$. By sampling on a grid on the surface of the sphere of radius $\sqrt{P_T}$ and centered at the origin of the L dimensional complex space, we obtain candidates for γ. Since p_1, θ and σ_ℓ are known apriori, β_ℓ can be calculated $\forall \ell$ and hence, α_ℓ also using $\alpha_\ell = \gamma_\ell/\beta_\ell$ from (11). The candidates of α thus derived are used in (10) yielding α_{opt}. Now, we may write

$$\alpha_{opt,\ell} = \frac{\gamma_{opt,\ell}}{\beta_\ell} = \frac{\gamma_{opt,\ell}}{\sqrt{(p_1\theta^2 + \sigma_\ell^2)}} \ . \qquad (12)$$

Clearly, $\alpha_{opt,\ell}$ from (12) depends on σ_ℓ. Hence, for sensors with different observation SNRs and for different realizations of the random channel matrix, H, we run the optimization algorithm again to find a new α_{opt}. After substituting $\alpha = \alpha_{opt}$ in (5) and (6), we obtain the corresponding R and δ respectively. Hereafter, similar to the procedure in Section IV-A, we specify the desired maximum allowable false alarm P_{FA}, the same as was stipulated in Section IV-A. Considering the equality in (7), we solve for the corresponding limiting detector threshold, $\tau = \tau_{opt}$. Likewise, taking the equality in (9), we solve for corresponding probability of detection, $P_D = P_{Dopt}$, the desired optimum.

It was seen in Section I-B that the question we seek to answer concerns the 'goodness' of the choice of τ for 'enhanced performance'. Towards this end, we study the nature of inter-relationships between P_{FA}, τ and P_D, across both OPA and UPA. The relative computational burden with OPA over UPA must indeed be justified by commensurate profit in performance.

V. Simulation Details, Results and Discussions

The dependence that we seek to study in Section I-B is through simulations over a slew of P_{FA}. Thus, for each of the several different values of fixed P_{FA}, the goal here is to compare and analyze the behavior of τ_{uni} and P_{Duni} from UPA of Section IV-A vis-à-vis the behavior of τ_{opt} and P_{Dopt} from OPA of Section IV-B, for a given power constraint, P_T. We first present the simulation settings.

A. Simulation Details

We simulate with the parameter being sensed, $\theta = 1$, no. of sensors, $L = 5$, no. of antennas, $N = 3$, probability of the null hypothesis, $p_0 = 0.4$ and the observation SNRs, θ^2/σ_ℓ^2 being 5 dB, 10 dB, 0 dB, 15 dB and 20 dB respectively for the sensors $\ell = 1, \cdots, L$. The power constraint is $P_T = 1$. We run Monte Carlo simulations with 1000 realizations of the random channel envelope, H, corresponding to a fixed P_{FA}. For each of these 1000 realizations, we implement the following.

1. By the UPA scheme of Section IV-A, obtain τ_{uni} and P_{Duni} through α_{uni}.

2. By the OPA scheme of Section IV-B, obtain τ_{opt} and P_{Dopt} through α_{opt}.

We run 1000 such epochs after taking 1000 samples of P_{FA} drawn from a uniform distribution supported on $(0, 0.2)$. The choice of this upper bound of 0.2 for P_{FA} is such that it is 50% of p_0 which has been set to 0.4.

B. Results and Discussions

As stated in Section I, we seek to study of the nature of inter-relationships between P_{FA}, τ and P_D. Specifically, we report here the initial results of an ongoing investigation into the influence of the detector threshold on P_{FA} and P_D. Towards this end, for each of the 1000 instances of $P_{FA} \in (0, 0.2)$, we average τ and P_D over 1000 realizations of the random channel matrix, H. Let the averaged values for the UPA scheme be $\overline{\tau}_{uni}$ and $\overline{P}_{Duni}$, and the corresponding quantities for the OPA scheme be $\overline{\tau}_{opt}$ and $\overline{P}_{Dopt}$. Thus, we get 1000 each of the above four averaged quantities.

Fig. 2 shows the histograms of $\overline{\tau}_{uni}$ and $\overline{\tau}_{opt}$. The standard deviation of the mean of the detector threshold, $\overline{\tau}_{uni}$, is 0.643 with the UPA scheme, whereas the corresponding standard deviation for the OPA scheme is 1.35. This comparison along with the mean values of $\overline{\tau}_{uni}$ and $\overline{\tau}_{opt}$ is depicted in Fig. 3. The implication is that in comparison to the UPA scheme, the OPA scheme admits a greater leeway in the choice of the
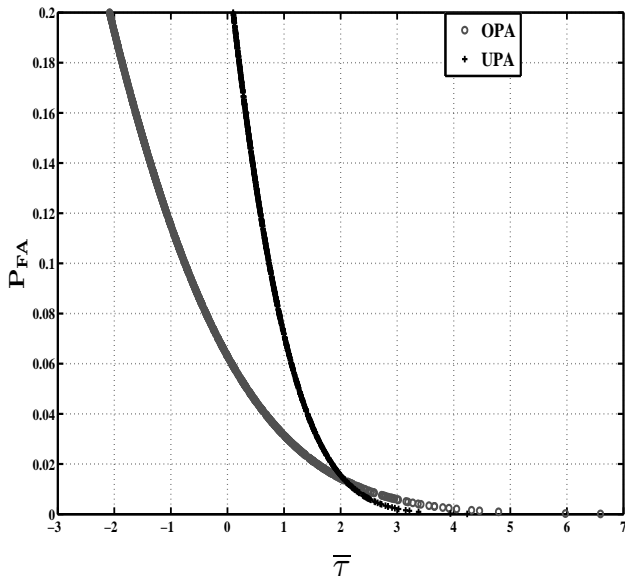
Fig. 5.   Probability of false alarm vs. Mean detector threshold

detector threshold. This is empirical evidence in support of the claim that in relation to the detector threshold, detection with the OPA scheme is more robust than that in the UPA scheme.

The foregoing result has credibility only if the P_{Dopt} compares favorably with P_{Duni}. That it is indeed true is borne out by Fig. 4. It shows that $\overline{P}_{Duni}$ and $\overline{P}_{Dopt}$ vs. their respective $\overline{\tau}$. We observe four significant features here.

1) The lowest value of $\overline{P}_D$ for the UPA and OPA schemes are respectively 0.0172 and 0.2581. That is, the OPA scheme outperforms the UPA scheme in respect of the empirical worst case detection. This is after taking even the outliers into consideration, without which, the disparity is further pronounced.

2) The highest value of $\overline{P}_D$ for the UPA and OPA schemes are respectively 0.7127 and 0.9816. That is, the OPA scheme improves upon the UPA scheme in respect of the empirical best case detection.

3) The mean of $\overline{P}_D$ for the UPA scheme which is 0.5166 is considerably lower than that for the OPA scheme which is 0.9216.

4) Finally, on any interval of common support of the detector threshold, the rate of fall of $\overline{P}_D$ with the OPA scheme is smaller than that with the UPA scheme.

Often, the price for a higher P_D is a higher P_{FA}. Hence, in laying claims to a higher P_D, one must bring into perspective the associated P_{FA}. Fig. 5 shows P_{FA} vs. $\overline{\tau}_{uni}$ and $\overline{\tau}_{opt}$. Here too, we may note these characteristics.

1) For the same distribution of P_{FA}, the range of $\overline{\tau}$ over which P_D may be optimized is larger for OPA than it is for UPA.

2) The probability of false alarm falls more rapidly with the threshold in the case of UPA than in the case of OPA. This must be expected in view of the fact that

for optimal detection, the OPA offers a larger interval for the detector threshold in comparison to the UPA as borne out by Fig. 4.

3) For the same value of $\overline{\tau}$, compared with the UPA scheme, the OPA scheme yields not merely a higher probability of detection (vide Fig. 4), but concurrently operates at a lower probability of false alarm, except for $\overline{\tau} \in (2, 4.5)$. Moreover, even for this interval of $\overline{\tau}$, it is noteworthty that $P_{Dopt} - P_{Duni} > 0.5$.

In essence, the investigation here makes a strong prima facie case for the OPA scheme over the UPA scheme in terms of robustness of detection w.r.t. the detector threshold when operating under an overall sensor power constraint. In fact, the enhancement in performance is concurrently over conflicting requirements.

VI. Conclusion

In Wireless Sensor Networks, we relaxed the AWGN condition on the Multiple Access Channel and considered the envelope of the channel gain distribution to be unknown and time-varying. Moreover, the observation SNRs at the multiple sensors were take to be non-identical. The detector threshold in the Neyman-Pearson formulation holds a key to the detection probability in relation to the probability of false alarm. With an overall sensor power constraint, we used an optimal detection scheme which takes into consideration the combined effect of the sensor noise and the fading MAC on detection. We examined the impact of the detector threshold on the probability of detection and the probability of false alarm under the uniform and the optimal PA schemes. Optimizing the probability of detection independently in each of the power allocation schemes was the common basis. For the case of a single power constraint, we showed through simulations that the optimal PA scheme outperforms the uniform PA scheme concurrently on three counts: (a) Relative robustness of the probability of detection vis-à-vis the detector threshold; (b) Comparatively high probability of detection; and notwithstanding this, (c) relatively low probability of false alarm.

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