# A Microwave Triplexer Based on Coupled Resonators Approach

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*Abstract*—This paper presents the design and computer simulation results of a triplexer based on coupled resonators approach. It comprises 6 resonators yielding three passbands which are centered at 1.800 GHz, 2.077 GHz and 2.400 GHz covering the GSM, UMTS and IEEE 802.11b wireless bands. The specification of each band is Chebyshev with maximum reflection loss of -20 dB (ripple of 0.04321 dB). A Gradientbased local optimization technique is here utilized to determine the coupling coefficients by minimizing a proposed Cost Function. The triplexer can be built regardless of the type of resonator. It is only required that they be able to provide the optimized coupling coefficients.

#### Keywords-coupling coefficient; triplexer; coupling optimization

#### I. INTRODUCTION

Multiplexers are devices commonly used in microwave front-end systems for channel separation. The need to operate with multiple services in multiple bands by using a single communication device, demands the input signal to be separated in various bands. This implies that the circuits of microwave transceiver systems have to be larger. In view of this, it has become imperative to build devices even smaller. Conventional diplexers and triplexers are usually built with a junction that splits the input power into two and three parts, respectively, each one being directed to a filter tuned at the desired frequency band. Normally, matching circuits need to be designed to match the filters to the transmission lines coming from the junction. This makes the resulting system bulkier. Multiplexers based on coupling resonators have proved to meet the demand of size reduction, especially because they avoid the use of energy distribution network to split the incoming signal. For this case, matching the circuit is part of its design. Very few triplexers based on coupling resonators have been reported. This article presents computer simulation results of a triplexer based on coupled resonators, without energy distribution network. The theory presented here is an extension of the one developed in [1], which is applied for the design of power dividers and diplexers. The triplexer proposed here comprises 6 resonators operating with three passbands centered at 1.800 GHz, 2.077 GHz and 2.400 GHz. The specification is Chebyshev with maximum reflection loss of -20 dB. A Gradient-based local optimization technique is used to minimize a proposed Cost Function to determine the coupling coefficients. The procedure applies to any type of resonator, provided that they give the coupling coefficients calculated in the design. This paper deals with the state of the art for triplexers, gives the

description of the triplexer treated here, shows the theory involved for its design and presents the simulation results for the reflection and transmission losses.

#### II. THE STATE OF THE ART

Triplexers using power divider junction have been reported in [2,3], where matching circuits need to be designed. Also very few triplexers based on coupled resonators have been described, but no optimization was considered [4,5]. This paper presents the design of a triplexer for the wireless band based on the coupling matrix optimization and without power junction. As a result, no matching circuit is needed. Following the procedure, the coupling coefficients are the only control variables to be optimized, since the external quality factors are normally calculated using the filter theory developed in [6].

#### III. THE TRIPLEXER DESCRIPTION AND DESIGN

A triplexer is a four-port device which is described in Figure 1. It receives a signal at its input port and splits it in three parts, each one tuned at a specific frequency.



Figure 1. The triplexer diagram

The proposed triplexer is formed by 6 resonators with direct coupling between adjacent ones according to Figure 2. The lines linking the resonators represent the coupling between them.



Figure 2. The triplexer formed by 6 coupled resonators

These resonators can be in microstrip, waveguide or other technology. The coupling coefficients between adjacent resonators are designed such that the mid passband is collected from resonator 6 (corresponds to port 4) and the two side passbands are collected from resonators 4 (port 3) and 5 (port 3). A central resonator distributes power to the three branches of the triplexer. The design theory of coupled resonator filters is well developed in [6] and extended in [1] for power dividers and diplexers. Figure 3 shows a circuit of coupled resonators, which exhibits electric and magnetic coupling through their capacitors and inductors and can be analyzed from the Kirchhoff's circuit laws.



Figure 3. Multiport circuit of coupled resonators

In order to transform the circuit into a triplexer, the starting step is to consider suitable four ports, which are represented by resonators x, y and z whose resistors are  $R_x$ ,  $R_y$  and  $R_z$ . Ports 2, 3 and 4 are associated to resonators x, y and z, respectively. Figure 4 shows the triplexer from the resulting four-port network described above.



Figure 4. The triplexer resulting from the N- resonator network.

Referring to Figure 3, port 1 is the feeding port; port 2 is associated to resonator 4; port 3 associated to resonator 5 and port 4 to resonator 6. Manipulating the equations of Kirchhoff's circuit laws, the following expressions can be obtained for the scattering parameters:

$$S_{11}(z) = 1 - \frac{2cof_{11}([B(z)])}{q_{e1}\Delta_B(z)}$$
(1)

$$S_{21}(z) = \frac{2cof_{14}([B(z)])}{\sqrt{q_{e1}q_{e4}}} \Delta_B(z)$$
(2)

$$S_{31}(z) = \frac{2cof_{15}([B(z)])}{\sqrt{q_{e1}q_{e5}} \,\Delta_B(z)}$$
(3)

$$S_{41}(z) = \frac{2cof_{16}([B(z)])}{\sqrt{q_{e1}q_{e6}} \,\Delta_B(z)} \tag{4}$$

where  $q_{e1}$ ,  $q_{e4}$ ,  $q_{e5}$  and  $q_{e6}$  are the external quality factors at ports 1, 4, 5 and 6, and  $cof_{ij}([B(z)])$  is the element *ij* of the cofactor of matrix [B] calculated at  $z = j2\pi f$ . Also,  $\Delta_B(z)$  is the determinant of matrix [B] calculated at the same complex frequency. Matrix [B] of the triplexer depicted in Figure 3 can be expressed by

$$\begin{bmatrix} \frac{1}{q_{e1}} + P & -jm_{12} & 0 & 0 & 0 & 0 \\ -jm_{12} & P & -jm_{23} & 0 & 0 & 0 \\ 0 & -jm_{23} & P & -jm_{34} & -jm_{35} & -jm_{36} \\ 0 & 0 & -jm_{34} & \frac{1}{q_{e4}} + P + jm_{44} & -jm_{45} & 0 \\ 0 & 0 & -jm_{35} & -jm_{45} & \frac{1}{q_{e5}} + P - jm_{44} & 0 \\ 0 & 0 & -jm_{36} & 0 & 0 & \frac{1}{q_{e6}} + P \end{bmatrix}$$

where  $m_{12}$ ,  $m_{23}$ ,  $m_{34}$ ,  $m_{35}$ ,  $m_{36}$ ,  $m_{45}$  and  $m_{44}$  are the normalized coupling coefficients to be optimized. Since the four-port network is asynchronously tuned, normalized coupling coefficients  $m_{44}$  and  $m_{55}$  have to be introduced in the coupling matrix. In order to have symmetrical passbands in relation to  $f_0$ , one has to set  $m_{55} = -m_{44}$ . Also, for the passbands to exhibit the same Chebyshev specifications, the relation  $m_{34} = m_{35} = m_{36}$  must be established. Variable *P* is the prototype normalized frequency, which is related to the denormalized frequency by the expression

$$P = \frac{j}{FBW} \left( \frac{f}{f_0} - \frac{f_0}{f} \right)$$
(5)

where *f* is expressed in GHz and  $FBW = (f_2 - f_1)/f_0$  is the fractional bandwidth. For the triplexer proposed here,  $f_1 = 1.795 GHz$ ,  $f_2 = 2.405 GHz$ , and  $f_0 = 2.077 GHz$ .

The Cost Function to be optimized is given by

$$CF = \sum_{i=1}^{N} \left| \Delta_{B}(z_{i}) - \frac{2cof_{11}[B(z_{i})]}{q_{e1}} \right|^{2} + \sum_{j=1}^{N-3} \left\| 1 - \frac{2cof_{11}[B(z_{j})]}{q_{e1}\Delta_{B}(z_{j})} \right\| - 10^{\frac{L_{B}}{20}} \right\|^{2}$$
(6)

where N is the number of resonators,  $Z_{ri}$  is the *i*-th reflection zero and  $z_{pj}$  is the complex frequency where the *i*-th peak of the reflection loss is observed. Since the number of reflection zeros for each passband is 2, only one peak in the reflection loss response is observed. This means that 3 peaks are observed in the reflection loss response of the triplexer (N-3). The coupling coefficients and external quality factors form a set of parameters known as control variables. In this work, only the coupling coefficients are to be optimized, since the calculated external quality factors have proved to be very effective for good simulation results of the coupling coefficients. The first term in the Cost Function is derived from (1) and is minimum when calculated at  $Z_{ri}$ . Likewise, the second one is related to the points where the peaks of  $|s_{11}|$  occur. Theory shows that they occur for transmission loss given by  $10^{\frac{L_R}{20}}$ .

### IV. PERFORMANCE OF THE TRIPLEXER

The design was initially done in the normalized frequency from the desired specifications in the high frequency range. A triplexer with Chebyshev characteristics [6] was simulated with maximum reflection loss of -20 dB (ripple 0.04321 dB) for each passband. The center frequency is 2.077 GHz in the denormalized frequency domain. A good guess for the external quality factors can be obtained if one considers the triplexer as a filter formed by 6 resonators. The filter normalized frequency prototype elements are then used to calculate the initial guess for the external quality factors at ports 1, 2, 3 and 4 in the computer simulations. A Gradient-based local optimization technique [7] was utilized to obtain the coupling coefficients  $m_{12}, m_{23}, m_{34}$  and  $m_{44}$ . The normalized external quality factors are calculated by employing expressions [6]

$$q_{e1} = g_0 g_1 \tag{7}$$

$$q_{e6} = g_6 g_7 \tag{8}$$

where  $g_0, g_1, g_6$  and  $g_7$  are the prototype normalized frequency elements of a Chebyshev filter of 6 elements. For the triplexer here simulated, the bandwidth of each passband was taken to be 0.125 Hz in the normalized frequency range. The calculated external quality factors were found to be  $q_{e1} = 5.30$  and  $q_{e4} = q_{e5} = q_{e6} = 15.9$ . Having been calculated, rather than optimized, reduction in the number of parameters to be optimized and in the computer time is achieved. Furthermore, very good results for the optimized coupling coefficients can also be attained. As for the coupling coefficients, the initial guess for the computer simulation was  $m_{12} = m_{23} = m_{44} = 0.6$  and  $m_{34} = 0.5$ . Since the algorithm searches for a local minimum, the results are sensitive to the initial guess. After 90 iterations and a precision of  $10^{-9}$ , the final optimized coupling coefficients values were  $m_{12} = 0.6775$ ,  $m_{23} = 0.6595$ ,  $m_{34} = 0.1166$  and  $m_{44} = 0.9213$ . Figure 5 shows the response of the triplexer against frequency in the normalized frequency. The selectivity of the channels can be improved as the number of resonators increases, which makes the reflection loss greater in the rejection band.



Figure 5. The triplexer response in the normalized frequency.

The denormalized frequency response of the triplexer is graphed in Figure 6. As can be seen, it is slightly compressed in the passband centered at 1.800 GHz when compared with the other side passband (centered at 2.447 GHz) due to the corresponding frequency transformation in (5). Even so, very similar responses were attained for the three output passbands. It was observed that the number of resonators is equal to the number of reflection zeros. Furthermore, in



Figure 6. The triplexer response in the denormalized frequency.

order to have three outputs with similar Chebyshev specifications, the triplexer must have a number of resonators multiple of three, so that each one of the three passbands presents the same number of reflection zeros.

## V. CONCLUSIONS AND FUTURE WORK

Design and simulation results of a triplexer based on coupled resonators approach was presented here. Six resonators were employed without using any energy distribution network. A Chebyshev response for the three passbands was achieved, with maximum reflection loss -20 dB (ripple 0.04321) and fractional bandwidths of 2.5%, 3.3% and 2.3% for the left, central and right passbands, respectively. A Gradient-based local optimization technique was used, which consisted in minimizing a proposed Cost Function whose control variables were the coupling coefficients. The external quality factors were excluded from the optimization procedure, since the calculated values yielded satisfactory optimized coupling coefficients. Very good simulation results were obtained for the reflection loss and transmission loss. As a future work, the triplexer will be implemented in microstrip with triangular resonators aiming to achieve size reduction.

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