Wireless Relay System with Two Sections in κ-μ Short-Term Fading Channel

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***Abstract*—The wireless relay communication mobile radio system will be considered in this article. This system has two sections. The desired signal in both sections is subjected to κ-μ short- term fading. The outage probability is calculated for two cases. For the first case, the outage probability is defined as probability that the signal envelope in any sections falls below the specified threshold. For the second case, the outage probability is defined as probability that output signal envelope falls below the determined threshold. The influence of the Rician κ factor and multipath fading severity parameter on the outage probability is analyzed for both cases.**

Keywords-κ-μ distribution; cumulative distribution function; probability density function; outage probability

I. INTRODUCTION

In this article, the wireless communication relay system operating over κ-μ multipath fading environment is considered. The relay system has two sections. The κ-μ distribution describes signal envelopes in channels with dominant components [1] [2].

The κ-μ distribution is characterized by two parameters [3]. The parameter κ is Rician factor, which is defined as the ratio of dominant component power and scattering components power. The parameter μ is related to the number of clusters in propagation channel. The κ-μ small scale fading is more severe for less values of parameter μ. Also, the κ-μ multipath fading is more severe for lesser value of dominant component power and higher values of scattered components power. The κ-μ distribution is general distribution from which Rayleigh, Rician and Nakagami-m distribution can be derived [4]. The κ-μ distribution reduces to Nakagami-m distribution for κ=0, to Rician distribution for μ=1, and to Rayleigh distribution for κ=0 and μ=1. When Rician factor goes to infinity, κ-μ multipath fading channel becomes no fading channel. Also, when parameter μ goes to infinity, there is no fading in the channel [5].

There are two ways to define the outage probability in wireless relay systems. One way to define the outage probability is as a probability that signal level at any section falls below of the determined threshold. In this case, the outage probability can be calculated from cumulative distribution function of the minimum of the signal envelope at sections of wireless communication system. In the second definition, the outage probability can be calculated as probability that signal envelope at output of wireless relay communication system falls below the determined threshold. For this case, the outage probability can be calculated as cumulative distribution function of product of two κ-μ distributions [6].

There are some works in technical literature dealing with performance analysis of wireless relay communication systems in the presence of multipath fading [7]-[14]. Beside, several papers treated minimum, product and ratio of random variables and how they could be applied in performance analysis of relay communication systems [13] [14]. Since the distribution of random variables is of interest in wireless communications, the probability density functions of minimum of ratios of Rayleigh, Rician, Nakagami-m, Weibull and α-μ random variables are derived in [13]. These formulas are used for studying the outage probability in multi-hop systems working over fading channels in the presence of cochannel interference. The general, closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the ratio of the products of two independent α-μ variables are presented in [14]. The obtained solutions are applicable in analysis of multihop wireless communication systems in the presence of different fading.

According to the authors' knowledge, relay radio system in the presence of κ-μ short-term fading in the first and the second sections is not processed in publicly available literature. Consequently, such system is analyzed in this work. The Rician factor and fading severity parameter are the same for both sections. In this paper, probability density function and cumulative distribution function of minimum of two κ-μ random variables are determined. The cumulative distribution function is the outage probability of the proposed wireless relay system. Furthermore, PDF and CDF of product of two κ-μ random variables are evaluated and also the outage probability of proposed system can be calculated from cumulative distribution function. The signal envelope at output of considered relay system can be evaluated as product of two κ-μ random variables. The results obtained here can be applied in performance analysis and design of relay radio systems working over κ-μ multipath fading channels.

This article consists of five sections. After an introduction to the topic and description of the works in the field, in the second section, the probability density function and cumulative distribution function of minimum of two κ-μ random variables are performed. Then, in third section, PDF and CDF of product of two κ-μ random variables are carried out. Numerical results are given and discussed in the forth section, and some conclusions, with described possibilities for further work, are given in the last section.

II. PROBABILITY DENSITY FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION OF MINIMUM OF TWO κ-μ RANDOM VARIABLES

The probability density function of κ-μ random variable x_1 is:

$$p_{x_1}(x_1)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot x_1^{2i_1+2\mu-1}e^{-\frac{\mu(k+1)}{\Omega_1}x_1^2},x_1\geq 0, \quad (1)$$

where κ is Rician factor, μ is severity parameter and Ω_1 is signal envelope average power.

The random variable x_2 follows also κ-μ distribution:

$$p_{x_2}(x_2)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}\cdot$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1}\cdot\frac{1}{i_2!\Gamma(i_2+\mu)}\cdot$$

$$x_2^{2i_2+2\mu-1}e^{-\frac{\mu(k+1)}{\Omega_2}x_2^2},\ x_2\geq 0 \quad (2)$$

Cumulative distribution function of x_1 is [5, eq. 2]:

$$F_{x_1}(x_1)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)}\right)^{i_1+\mu}\cdot\gamma\left(i_1+\mu,\frac{\mu(k+1)}{\Omega_1}x_1^2\right),x_1\geq 0, \quad (3)$$

where $\gamma(n, x)$ is incomplete Gamma function of argument x and order n [15].

Cumulative distribution function of x_2 is:

$$F_{x_2}(x_2)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}\cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1}\frac{1}{i_2!\Gamma(i_2+\mu)}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\Omega_2}{\mu(k+1)}\right)^{i_2+\mu}\gamma\left(i_2+\mu,\frac{\mu(k+1)}{\Omega_2}x_2^2\right),\ x_2\geq 0 \quad (4)$$

In wireless relay system, the outage probability can be defined in two different ways, as it is described above. When the signal level at any section falls under the defined threshold, the outage probability can be calculated from cumulative distribution function of the minimum of the signal envelopes at sections of wireless communication system.

Let analyze the minimum x of two random variables x_1 and x_2. It is define as:

$$x=\min(x_1,x_2) \quad (5)$$

PDF of x is [8]:

$$p_x(x)=p_{x_1}(x)F_{x_2}(x)+p_{x_2}(x)F_{x_1}(x) \quad (6)$$

Cumulative distribution function of minimum x of two κ-μ random variables is [13, eq. (26)]:

$$F_x(x)=1-\left(1-F_{x_1}(x)\right)\left(1-F_{x2}(x)\right)=$$

$$=1-(1-\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)}\right)^{i_1+\mu}\gamma\left(i_1+\mu,\frac{\mu(k+1)}{\Omega_1}x^2\right))\cdot$$

$$\left(1-\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}\cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1}\cdot\frac{1}{i_2!\Gamma(i_2+\mu)}\cdot\right.$$

$$\left.\cdot\frac{1}{2}\left(\frac{\Omega_2}{\mu(k+1)}\right)^{i_2+\mu}\gamma\left(i_2+\mu,\frac{\mu(k+1)}{\Omega_2}x^2\right)\right) \quad (7)$$

Cumulative distribution function of minimum of two κ-μ random variables is the outage probability of wireless relay communication system with two sections over κ-μ multipath fading channel.

III. PROBABILITY DENSITY FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION OF PRODUCT OF TWO κ-μ RANDOM VARIABLES

By the second definition, the outage probability of wireless relay system can be calculated as probability that signal envelope at the output of wireless relay communication system lessens below the predetermined threshold. For this case, the outage probability can be calculated as cumulative distribution function of product of two κ-μ distributed signals.

Product of two κ-μ random variables x_1 and x_2 is:

$$x=x_1\cdot x_2,\ x_1=\frac{x}{x_2} \quad (8)$$

Conditional PDF of x is [10]:

$$p_x(x/x_2)=\left|\frac{dx_1}{dx}\right|p_{x_1}\left(\frac{x}{x_2}\right)=\frac{1}{x_2}p_{x_1}\left(\frac{x}{x_2}\right) \quad (9)$$

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After integration, the expression for $p_x(x)$ becomes:

$$p_x(x) = \int_0^\infty dx_2 \frac{1}{x_2} p_{x_1}\left(\frac{x}{x_2}\right) p_{x_2}(x_2) =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \cdot \frac{1}{i_1!\Gamma(i\ + \mu)} \cdot$$

$$\cdot \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2 + \mu)}$$

$$x^{2i_1+2\mu-1}\int_0^\infty dx_2\, x_2^{-1-2i_1-2\mu+1+2i_2+2\mu-1} e^{-\frac{\mu(k+1)}{\Omega_1}\frac{x^2}{x_2^2}-\frac{\mu(k+1)}{\Omega_2}x_2^2} =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \cdot \frac{1}{i_1!\Gamma(i\ + \mu)} \cdot$$

$$\cdot \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2 + \mu)}$$

$$x^{2i_1+2\mu-1}\left(\frac{\Omega_2 x^2}{\Omega_1}\right)^{i_2-i_1} K_{2i_2-2i_1}\left(2\sqrt{\frac{\mu^2(k+1)^2x^2}{\Omega_1\Omega_2}}\right) \tag{10}$$

Cumulative probability density of x is:

$$F_x(x) = \int_0^x dt\, p_x(t) =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \cdot \frac{1}{i_1!\Gamma(i\ + \mu)} \cdot$$

$$\cdot \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2 + \mu)} \cdot$$

$$\cdot \int_0^\infty dx_2\, x_2^{2i_2-2i_1-1} e^{-\frac{\mu(k+1)}{\Omega_2}x_2^2} \cdot \int_0^x dt\, t^{2i_1+2\mu-1} e^{-\frac{\mu(k+1)}{\Omega_1}\frac{x^2}{x_2^2}} =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \cdot \frac{1}{i_1!\Gamma(i\ + \mu)} \cdot$$

$$\cdot \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2 + \mu)} \cdot$$

$$\cdot \int_0^\infty dx_2\, x_2^{2i_2-2i_1-1} e^{-\frac{\mu(k+1)}{\Omega_2}x_2^2}$$

$$\cdot \frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)x_2^2}\right)^{i_1+\mu} \gamma\left(i_1+\mu, \frac{\mu(k+1)}{\Omega_1}\frac{x^2}{x_2^2}\right), \tag{11}$$

where $\gamma(n, x)$ is incomplete Gamma function.

After substituting (10) into (11), the previous expression for CDF becomes:

$$F_x(x) = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \cdot \frac{1}{i_1!\Gamma(i_1 + \mu)} \cdot$$

$$\cdot \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2 + \mu)} \cdot$$

$$\cdot \frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)}\right)^{i_1+\mu} \frac{1}{i_1+\mu}\left(\frac{\mu(k+1)}{\Omega_1}x^2\right)^{i_1+\mu} \cdot$$

$$\cdot \sum_{j_1=0}^{\infty} \frac{1}{(i_1+\mu+1)(j_1)} \cdot \left(\frac{\mu(k+1)x^2}{\Omega_1}\right)^{j_1} \cdot$$

$$\cdot \int_0^\infty dx_2\, x_2^{2i_2-2i_1-1+2i_1+2\mu-2i_1-2\mu-2j_1} e^{-\frac{\mu(k+1)}{\Omega_1}\frac{x^2}{x_2^2}-\frac{\mu(k+1)}{\Omega_2}x_2^2} =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \cdot \frac{1}{i_1!\Gamma(i\ + \mu)} \cdot$$

$$\cdot \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2 + \mu)} \cdot$$

$$\cdot \frac{1}{2}\frac{1}{i_1+\mu} x^{2i_1+2\mu} \cdot \sum_{j_1=0}^{\infty} \frac{1}{(i_1+\mu+1)(j_1)} \cdot \left(\frac{\mu(k+1)x^2}{\Omega_1}\right)^{j_1} \cdot$$

$$\left(\frac{\Omega_2 x^2}{\Omega_1}\right)^{\frac{i_2}{2}-\frac{i_1}{2}-\frac{j_1}{2}} K_{i_2-i_1-j_1}\left(2\sqrt{\frac{\mu^2(k+1)^2x^2}{\Omega_1\Omega_2}}\right) \tag{12}$$

Here, $K_n(x)$ is the modified Bessel function of the second kind [16, eq. (3.471.9)].

IV. Numerical Results

The cumulative distribution function of minimum of two κ-μ random variables versus signal envelope is presented in Fig. 1. The CDF is plotted for $\mu_1=\mu_2=2$ and variable parameters κ_1 and κ_2. It is visible that CDF increases with increasing of the signal envelope. The cumulative distribution function decreases for larger values of Rician factor κ_1. Also, one can see from Fig. 1 that CDF is bigger for higher values of Rician factor κ_2.

The cumulative distribution function of product of two κ-μ random variables depending on the signal envelope is presented in Fig. 2. The parameters μ_1 and μ_2 are equal to each other and have a value of 2. It is possible to see from Fig. 2 that CDF becomes bigger with increasing of the signal envelope. Fig. 2 shows that CDF is less for higher values of Rician factors κ_1 and κ_2.

The system performance is better for smaller values of the outage probability, i.e., cumulative distribution function. This can be achieved by increasing the Ricean factors κ_1 and κ_2.

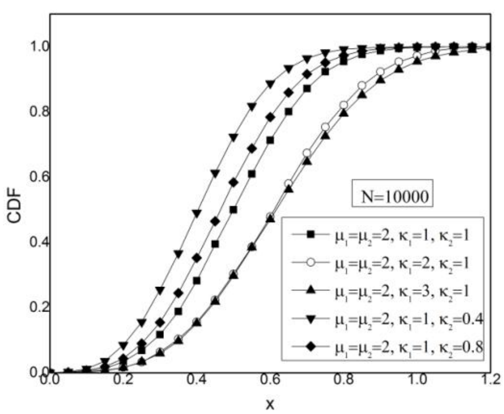

Figure 1. The cumulative distribution function of minimum of two κ-μ random variables.

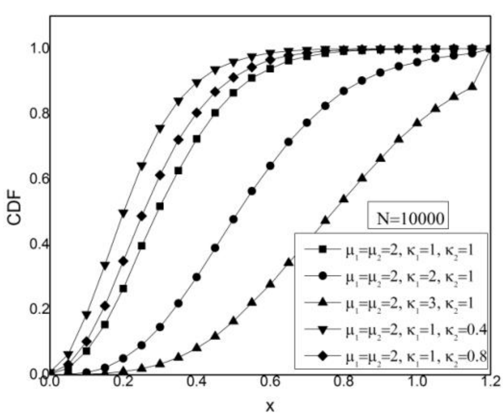

Figure 2. The cumulative distribution function of the product of two κ-μ random variables.

Since Rician factor is the ratio of dominant component power and scattering components power, it is evident that bigger dominant component powers and smaller scattering components give better system performance.

V. Conclusion

In this article, the communication relay radio system with two sections is analyzed. Both sections are exposed to κ-μ multipath fading.

The outage probability is defined in two ways. For the first, the outage probability is probability that signal envelope at one of sections becomes less than the specified threshold. The outage probability for this case can be calculated as a CDF of minimum of signal envelopes from both sections. The outage probability is calculated in the closed form. The influence of Rician κ factors on the outage probability is then analyzed.

For the second way, the outage probability is presented as probability that communication relay system output signal envelope drops below the defined threshold. The outage probability for this case is equal to the CDF of product of signal envelopes at sections. The fading parameters influence on the outage probability is described also.

The obtained formulas for the outage probability for relay (κ-μ)-(κ-μ) channels could be used for calculation the outage probability of other relay channels. For $\kappa_1=0$ and $\kappa_2=0$, (κ-μ)-(κ-μ) relay channel reduces to Nakagami-Nakagami relay channel; for $\mu_1=1$ and $\mu_2=1$, (κ-μ), (κ-μ) distribution becomes Rician-Rician channel [18]. Because of that, this article has general importance.

Results of this analysis can be used by designers of relay systems in terms of the presence of fading with the κ-μ distribution. The designers of these systems can choose optimal parameters for given value of the outage probability. Because of the generality of the results, the presence of other types of fading can also be covered with this investigation.

In our further work, the other system characteristics can be determined. Further, the outage probability for relay radio system with three sections can be derived. Besides, the performance for similar relay systems in the presence of other small-scale general fading distributions, such as *α-μ*, *α-η-μ*, or *α-λ-μ*, can be calculated too.

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