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An Optimal PID Based Trading Strategy under the log-Normal Stock Market Characterization

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***Abstract*—Our paper proposes an optimal trading algorithm based on a novel application of the conventional Control Engineering (CE) to Algorithmic Trading (AT). We consider a fundamental CE concept, namely, the feedback control and apply it to the algorithmic trading (algo trading). The concrete feedback control strategy is designed here in a form of the celebrated Proportional-Integral-Derivative (PID) model. The highly frequent nature of the modern financial markets motivates the using of a model-free realisation of the generic PID framework. The control theoretical methodology we propose is additionally combined with some advanced statistical results for the historical market data. We obtain a specific log-normal probability distribution function (pdf) associated with the specific relative characteristics of the available stock data. This empirical pdf mentioned above provides a novel computational technique for the necessary PID gains optimization. For this aim we apply a suitable data driven optimization problem and also consider an alternative stochastic programming framework. The stochastic optimization naturally involves the Monte Carlo solution procedure. The optimized PID trading algorithm we propose is next represented in the frequency domain. This equivalent representation makes it possible to introduce a new concept in the financial engineering, namely, the "stock market energy" concept. Finally, we implement the resulting PID optimal trading algorithm and develop a Python based prototype software. We apply the corresponding prototype software to the Binance stock market. This practical example illustrates the proposed optimal PID trading scheme and also shows the effectiveness of the CE methods in the modern AT.**

Index Terms*—financial engineering; feedback algorithmic trading; model-free PID control; PID gains tuning; data driven optimization; forward testing; stochastic optimization; prototype software

I. Introduction and Motivation

Optimal design of profitable trading algorithms for financial markets constitutes a very challenging and technically sophisticated problem of the modern financial engineering (see e.g., [1],[3],[4],[5],[7],[8],[9],[10],[11],[31],[38],[41]). In this contribution, we study an idealized stock market and apply the CE based PID control design in the development of an optimization based trading algorithm. Recall that the PID synthesis constitutes one of the most powerful and successful real-world control algorithms (see e.g., [2],[27],[28],[34],[36]). The conventional model-based PID control implements a fundamental idea of the feedback action and is widely used in various engineering and applications.

The stock market idealization we follow in this paper usually involves some generic technical assumptions. We consider the situation characterized by "no transaction costs" and also assume the condition of "one stock portfolio". We also consider some further simplifying hypothesises, namely, the assumption of the "zero interest" and the "continuous trading" condition. Let us refer to [7],[8],[9] for the necessary technical details on the existing abstractions in the modelling of stock markets. Our paper deals with the discrete-time dynamics of the financial markets under consideration. This assumption is motivated by the generic stock tick dynamics and also by the corresponding decision making process. Recall that a tick is a measurement of the minimum upward or downward movement in the price of a security (see e.g., [24],[32] and the references therein). The given discrete dynamics of the stock market ticks naturally implies the discrete-time decision making in trading. We next use the generic notation of the stock ticks $t = 1,...,$ and the corresponding semi-open time-intervals of the trading buckets $[t, t+1)$.

The advanced financial engineering and financial economics constitute nowadays a powerful theoretical tool of the modern financial science (see [15],[16],[23],[40]). On the other hand, the mathematically rigorous time-series based financial theories can not be directly applied to an algorithmic generation of a profitable trading decision. This fact is a simple consequence of the highly frequent stock price dynamics that makes it impossible any adequate price forecasting. This fundamental property of the modern financial markets implies the so called High-Frequency Trading (HFT) approach. We refer to [16] for the corresponding concepts and useful information. A successful HFT strategy development is a very sophisticated challenging task. In this paper, we propose to use some fundamental aspects of the conventional CE design for this purpose. More specifically, we propose to use an optimized PID type control strategy for a profitable trading algorithm. The optimization techniques for this PID trading algorithm involve some novel optimal PID gains tuning strategies.

Recall that the control theoretic approach to the modern AT was originated in [7],[8],[9],[10],[11],[19],[31]. Let us also refer to [3],[4] for some novel PID related trading algorithms with a switched structure. Since the non-regular, highly frequent bid-ask spread behaviour on a stock does not admit a realistic forecasting model, we examine here a specific model-free version of the classic PID control. Our algorithmic approach proposes to react to the stock price variations instead of modeling them. We next combine the conventional PID control approach to AT with an advanced statistical characterization of some historical market data sets. We next identify a log-normal type probability distribution function (pdf) for some concrete quantities related to the data sets under consideration.

It is common knowledge that a classic and advanced PID control design incorporates a very important technical step, namely, the PID gains tuning [2],[27],[28],[34]. The established log-normal pdf for some market characteristics can next be used in the optimal PID gains tuning procedure. Note that various optimization techniques play an important role in the modern financial engineering (see e.g., [1],[4],[5],[14],[22],[26],[29],[30],[33],[42],[43],[44] and references therein). For the concrete PID gains optimization we use here two conceptually different approaches. The first approach constitutes a data driven regression based optimization. This useful approach can be combined with the well-known Forward Testing methodology and with the consideration of the corresponding In-Sample and Out-of-Sample concepts for the data sets. The second optimization approach we use is the classic stochastic optimization framework. This approach uses the established log-normal distribution of some market related quantities. It finally leads to the celebrated Monte Carlo solution technique (see e.g., [22],[39] and references therein).

The resulting PID trading algorithm with the optimized gains tuning procedure was applied to some concrete real-world examples. We illustrate the efficiency, profitability and practical implementability of the proposed algorithm and consider a trading application on the Binance BTC/USDT spot market. We also discuss shortly the necessary prototype software for implementation of the proposed optimal PID trading algorithm.

The remainder of our paper is organized as follows: Section II contains a formal AT problem statement. This section includes a mathematical description of the conceptually novel model-free PID trading algorithm. In Section III we perform an advanced statistical analysis of the stock market data. We establish some log-normal probability distribution properties for some specific subsequential market characteristics. Section IV deals with the main problem of the PID trading, namely, with the optimal PID gains tuning. We consider the data driven optimization involving the historical data and the corresponding backtesting procedure. The corresponding regression analysis is considered in the general Forward Testing framework. In this section we also propose an alternative PID gains selection and use the scenario based stochastic optimization problem for this purpose. The resulting stochastic program is next solved by the classic Monte Carlo approach. In Section V we discuss a novel frequency domain interpretation of the proposed PID trading strategy. We apply the conventional z-transformation and the Fourier transformation for the constructive characterization of the PID trading scheme in the frequency space. Section VI contains a practically motivated application of the developed PID trading algorithm to a specific stock market, namely, to the Binance BTC/USDT futures stock. This section is also devoted to the prototype software design. Section VII summarizes our paper.

II. Model-Free PID Based Trading Algorithm

Consider a trading on an idealized stock market with an initial deposit (initial investment) $I(1) = I_1$ and introduce the current return

$$\Delta g(t),\ t = 1, ...,.$$

The current investment level at a time instant t is denoted by $\Delta I(t)$. In parallel with the current values introduced above we also consider the cumulative return and the cumulative investment $g(t)$ and $I(t)$, respectively. The nonlinear discrete-time PID trading strategy can be formalized as follows:

$$\begin{aligned} &\delta I(t+1) = K_P(t+1)\Delta g(t) + K_D(t+1)\dot{\Delta} g(t) + \\ &K_I(t+1)\int_{t-T}^{t} h(\tau)\Delta g(\tau) d\tau, \\ &\Delta I(t+1) = \chi(\delta I(t+1)), \quad \text{for} \quad t = 1, ... \end{aligned} \tag{1}$$

By $K_P(\cdot)$, $K_D(\cdot)$ and $K_I(\cdot)$ we denote here the necessary PID gains for the proportional, integral, and derivative terms of the classic regulator scheme (see [2],[27],[28]). Let us introduce the vector of these PID gains:

$$K(\cdot) := (K_P(\cdot),\ K_D(\cdot),\ K_I(\cdot))^T.$$

Note that the integral term in (1) incorporates the "process memory" on a given time interval $[t - T, t]$. The integral kernel $h(\cdot)$ in (1) is defined by a suitable "memory loss" function. We consider an exponentially weighted "memory loss" function $h(\cdot)$ with $h(t) = 1$. Note that the proposed scheme (1) is similar to the classic PID control design (see e.g., [28],[34]). The nonlinear function $\chi(\cdot)$ in 1 constitutes a "control saturation" and can naturally be defined as follows:

$$\chi(\delta I) := \begin{cases} \delta I, & \text{if } \delta I^{\min} \le |\delta I| \le \delta I^{\max}; \\ \pm\delta I^{\max}, & \text{if } |\delta I| > \delta I^{\max}; \\ 0, & \text{if } |\delta I| < \delta I^{\min}. \end{cases} \tag{2}$$

Here

$$\delta I^{\max},\ \delta I^{\min}$$

are prescribed maximal and minimal current investment levels, respectively. The above "saturated" investment model constitutes a formal mathematical condition and serves as a natural restriction for the investment volume.

Assuming the investment decision $\Delta I(t+1)$ in (2) and taking into consideration the stock price $p(\omega,t+1)$, we next calculate the current return $g(t+1)$:

$$\Delta g(t+1) = \frac{(p(\cdot,t+1) - p(\cdot,t))}{p(\cdot,t)} \Delta I(t+1) \quad (3)$$

If we use the CE analogy, we conclude that the proposed investment level $\Delta I(t+1)$ in (1) plays a role of a "control input". We next call it "investment decision". Note that the current investment decision is deployed at a present time instant t under the condition of a natural unknownness of the market price

$$p(\omega,t+1).$$

By $\omega \in \Omega$ and Ω we denote here a probability state space with a unknown probability measure. This stock price

$$p : \Omega \times \mathbb{Z}_+ \to \mathbb{R}$$

is assumed to be a measurable stochastic function. Note that the current return $\Delta g(t+1)$ is calculated an a posteriori value, where $p(\cdot,t+1)$ in (3) denotes a concrete realization of the stochastic price $p(\omega,t+1)$.

Since we have a CE analogy, we also can represent the above PID trading algorithm (1)-(3) using the block diagram. The corresponding block scheme of the proposed model-free PID trading strategy (1)-(3) is presented in Figure 1.

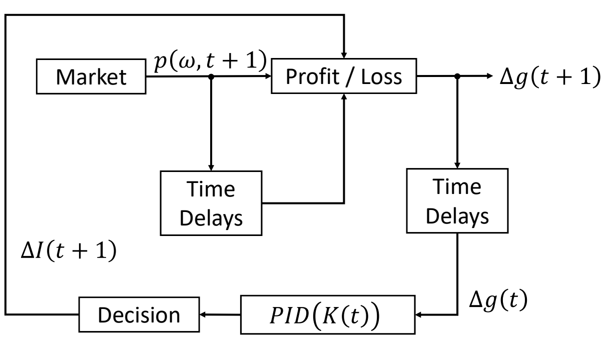

Fig. 1. Model-free PID based trading algorithm

Note that the feedback channel in (1)-(3) is implemented by the current return $\Delta g(t+1)$ in (3). In fact, the PID trading algorithm (1)-(3) and the corresponding block diagram (Figure 1) represent the so-called "delayed PID" scheme. We refer to [28] for details.

The general integral formula for

$$\delta I(t+1)$$

in (1) can be concretized in the discrete-time:

$$\begin{aligned} &\delta I(t+1) = (K_P(t+1) + K_I(t+1) + K_D(t+1))\Delta g(t) + \\ &(K_I(t+1)h(t-1) - K_D(t+1))\Delta g(t-1) + \\ &K_I(t+1) \sum_{\tau=t-T}^{t-2} h(\tau)\Delta g(\tau). \end{aligned} \quad (4)$$

Recall that the main CE problem of the conventional PID control approach consists in defining the adequate PID gains tuning rules (see e.g., [27],[28] and references therein). In the generic real-world application fields of the classic PID controllers the tuning techniques are well established [2],[34]. For the conceptually sophisticated model-free PID algorithm under consideration, an adequate design of the PID gains tuning schemes constitutes a challenging theoretic problem. The suitable PID gains selection immediately determines the main trading signal by the rule (4). In fact, an adequate (optimal) PID gains tuning scheme development is a key problem of a profitable PID based investment decision.

Let us also note that the model-free character of the proposed PID scheme (1)-(3) and (4) indicates the application possibility of the modern ML approaches to an optimal PID gains tuning problem. We refer to [12],[13],[26] for the technical details related to the RL approach and to some applications of the ML techniques in AT.

III. Advanced Statistical Description of the Stock Market Data

We now study the important PID gains tuning problem and apply an additional statistical information related to the stock market. Introduce the following "price/volume" ratio:

$$\theta(\omega,t+1) := \frac{p(\omega,t+1)}{v(t+1)}. \quad (5)$$

Here $p(\omega,t+1)$ is an unknown (stochastic) price at the time instant $t+1$ and $v(t+1)$ is an investment volume to the same time:

$$v(t+1) := \frac{\Delta I(t+1)}{p(\cdot,t)}. \quad (6)$$

Note that (6) expresses the entities number of a traded financial instrument (for example, futures trading). The comprehensive (a posteriori) statistical analysis of a wide spectrum of stock instruments demonstrates that the pdf of the above value

$$\theta(\omega,t)$$

constitutes a specific log-normal distribution (see e.g., [18],[20],[35],[44] and references therein). Clearly, the real investment volume of a hedge fund, a private trader or of a bank is usually a restricted. In the case of the PID trading algorithm (1)-(3), the maximal investment volume is a direct consequence of the boundedness of $\Delta I(t+1)$ in (1) and (6).

The log-normal pdf related to the financial markets has mainly been studied for option prices. In this connection we also refer to the celebrated Black and Scholes model [15]. A useful discussion on this subject can also be found in [40]. Let us also mention the log-normal pdf's of some volatility involved quotients and other stock market parameters and indices (see e.g., [1],[5],[18],[23]). We next analyze the stationary statistical distribution $\rho_1(\cdot)$ of the quantity $\theta(\omega,t+1)$ introduced in (5):

$$\begin{aligned} \rho_1(\theta) = &\frac{a}{\sqrt{2\pi}\sigma(\theta - s)} \times \\ &\exp -(0.5\sigma^2) \times (\ln(\theta - s) - \mu)^2. \end{aligned} \quad (7)$$

Here $\mu \in \mathbb{R}$ denotes a statistical mean, $\sigma \in \mathbb{R}_+$ is a dispersion and $s \in \mathbb{R}$ denotes a shifting. Note that (7) is often called "three-parameters"

$$\{\mu, \sigma, s\}$$

log-normal distribution (see e.g., [1],[18]). The concrete parameter values of the log-normal pdf $\rho(\theta)$ can be calculated using the backtesting on the historical stock market data. As mentioned above, (7) constitutes an adequate model distribution for the introduced price/volume ratio $\theta(\omega, t+1)$. The quality of this statistic model can be established by application of the Chi-Quadrat test for distributions qualification. We refer to [35] for necessary technical details. For example, one can consider the normalized value

$$\chi^2/q$$

for the above test. Here q is the number of degrees of freedom. The concrete stock market histogram for a monthly Binance BTC/USDT price/volume ratio $\theta(\omega, t+1)$ is depicted in Figure 2.

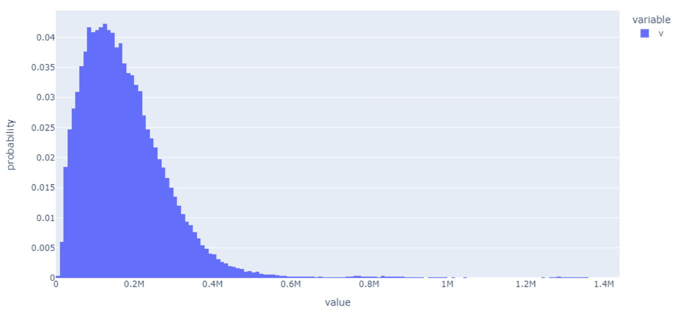

Fig. 2. The log-normal like histogram of the ratio $\theta(\cdot, t)$ for Binance BTC / USDT spot market

In parallel to (7) we also consider the log-normal pdf for the following price/price ratio:

$$\vartheta(\omega, t+1) := \Big(\frac{p(\omega, t+1)}{p(\cdot, t)}\Big), \tag{8}$$

where

$$\{p(1), \ldots, p(T)\}, \ \{v(1), \ldots, v(T)\}.$$

are the stock prices and investment volumes data for the time instants $t = 1, \ldots, T$. The log-normal probability distribution for the value

$$\vartheta(\omega, t+1)$$

in (8) was established in [1]. We denote this pdf by $\rho_2(\cdot)$. We also refer to [5],[16] for the necessary statistical and econometrical details.

We next obtain

$$\ln\big(\theta(\omega, t+1)/\theta(\cdot, t)\big) = \ln\Big(\frac{p(\omega, t+1)}{p(\cdot, t)}\Big) - \ln\Big(\frac{v(t+1)}{v(t)}\Big). \tag{9}$$

and

$$\ln\Big(\frac{v(t+1)}{v(t)}\Big) = \ln\Big(\frac{p(\omega, t+1)}{p(\cdot, t)}\Big) + \ln\theta(\cdot, t) - \ln\theta(\omega, t+1). \tag{10}$$

Expressions (9)-(10) make it possible to generate a part of the complete investment decision, namely, the decision about the investment volume $v(t+1)$ to the time instant $t+1$:

$$v(t+1) = \exp\Big[\ln\Big(\frac{p(\omega, t+1)}{p(\cdot, t)}\Big) + \ln\theta(\cdot, t) - \ln\theta(\omega, t+1) + \ln v(t)\Big]. \tag{11}$$

Observe that $\theta(\cdot, t)$ and $v(t)$ in (11) are known values. Additionally, the price/volume ration $\theta(\omega, t+1)$ and the price/price ratio

$$\frac{p(\omega, t+1)}{p(\cdot, t)}$$

can effectively be simulated using the corresponding log-normal probability distribution functions. We next can also forecast the value

$$\ln\big(p(\omega, t+1)/p(\cdot, t)\big)$$

in (11) using the corresponding log-normal distribution (see [1],[5],[18]). We assume here that the necessary parameters of the log-normal distributions of the values $\theta(\omega, t+1)$ and

$$p(\omega, t+1)/p(\cdot, t)$$

are previously determined by the generic backtesting technique. For example, for (7) we need to identify a, μ, σ, s. This identification procedure is performed using the available statistics in form of a histogram (see e.g., Figure 2).

Assuming the investment volume decision (11), we immediately get a tuning scheme for the PID gains $K(\cdot)$. Combining (6) and (11) we finally obtain

$$\chi(K_P(t+1)\Delta g(t) + K_D(t+1)\dot{\Delta} g(t) + K_I(t+1)\int_{t-T}^{t} h(\tau)\Delta g(\tau) d\tau) = p(\cdot, t)\exp\Big[\ln\Big(\frac{p(\omega, t+1)}{p(\cdot, t)}\Big) + \ln\theta(\cdot, t) - \ln\theta(\omega, t+1) + \ln v(t)\Big]. \tag{12}$$

Since the investment decision $\Delta I(t+1)$ is designed by the PID rule (1)-(2), the obtained formulae (12) constitutes a specific tuning restriction for the PID gains $K(\cdot)$. This novel tuning approach involves the advanced statistical (log-normal) analysis of the financial market behaviour.

IV. Optimization Approaches to the PID Gains Tuning

A credible anticipating strategy for the behaviour of stock markets (using a certain amount of historical data) constitutes the main conceptual problem of the algorithmic trading. In the framework of the proposed PID trading algorithm (1)-(3), we have to incorporate into the PID methodology an additional optimization procedure for a ”best choice” of the PID gains. In this section, we describe two advanced optimization approaches to the optimal PID gains tuning. The resulting PID based strategies (1)-(3) with an optimal gain selection are next called optimal PID trading algorithms. Let us note that the ”advanced” character of the optimal tuning mentioned above corresponds to the model-free structure of the proposed PID trading scheme. The model-free nature of an optimal PID implementation (1)-(3) also indicates the usability of the modern ML and RL approaches for optimal PID gains selection problems. Let us refer to [12],[13],[26] for some technical results and novel ideas.

A. Data Driven Gains Optimization

In this section we use a conventional regression based optimization framework applied to the historical stock data. Our aim is to combine the least square optimization with the generic Forward Testing (FT). Note that the FT technique is a common methodology of the modern AT (see e.g., [23],[26]). It includes the so called In-Sample and Out-of-Sample data subsets of the initially given historical data set. The In-Sample data set is part of historical data on which the optimization is performed. The subset of historical data that has been reserved for a possible validation of the optimized trading algorithm is known as an Out-of-Sample data set.

We now choose a number $M_1 \in \mathbb{N}$ as a cardinal number of the In-Sample set and consider the corresponding investment volumes:

$$v^j(t+1), \ j = 1, ..., M_1 - 1.$$

Let M_2 denotes a cardinal number of the Out-of-Sample sat such that the complete historical data set under consideration has

$$M := M_1 + M_2$$

elements. Using the given (nonlinear) structure of the PID trading algorithm (1)-(3), we next introduce the following main optimization problem

$$\begin{aligned} &J_1(K(t+1)) := \sum_{j=1}^{M_1} \big(\chi(\delta I(t+1)) - \\ &v^j(t+1)p(\cdot, t+1)\big)^2 \to \min_{K(t+1)} \\ &\text{subject to } (1)-(3), (12) \end{aligned} \tag{13}$$

Evidently, (13) constitutes a specific nonlinear regression determined on the In-Sample data set. Note that problem (13) involves the previously obtained statistical characterisation (12). This statistical result for the PID gains constitutes a natural restriction in the minimization problem (13). The optimization problem (13) can be solved by some known numerical methods (see e.g., [3],[21],[29] and references therein). It finally leads to the optimal PID gains

$$K^{opt}(t+1) := \{K_P^{opt}(t+1), \ K_D^{opt}(t+1), \ K_I^{opt}(t+1)\}.$$

The Out-of-Sample data set can next be used for the validation procedure of optimal solution $K(t+1)$ obtained using the In-Sample set. This validation is based on the comparison of two values of the same objective functional from (13), namely the In-Sample optimal value

$$J_1(K^{opt}(t+1))$$

and the following Out-of-Sample optimal value

$$\begin{aligned} &J_2(K^{opt}(t+1)) := \sum_{j=1}^{M_2} \big(\chi(\delta I^{opt}(t+1)) - \\ &v^j(t+1)p(\cdot, t+1)\big)^2. \end{aligned} \tag{14}$$

Note that the Out-of-Sample optimal investment

$$\delta I^{opt}(t+1))$$

in (14) is calculated using the optimal gains $K^{opt}(t+1)$ (obtained on the In-Sample data set) and the basic expression from (1). Moreover, the set of investment volumes $v^j(t+1)$ in (14) corresponds to the Out-of-Sample set:

$$v^j(t+1), \ j = 1, ..., M_2 - 1.$$

The above optimization problem (13) will be considered as a consistent (optimal) PID gains selection procedure if

$$|J_1(K^{opt}(t+1)) - J_2(K^{opt}(t+1))| \leq \varepsilon$$

for a sufficiently small prescribed $\varepsilon > 0$. Let us note that an alternative validation procedure for the PID gains optimization can involve the celebrated Monte Carlo approach (see e.g., [22],[37]).

We now apply the validated optimal gains vector

$$K^{opt}(t+1)$$

for defining the deployed (optimal) investment level

$$\Delta I^{opt}(t+1)$$

using the PID algorithm (1)-(3). The resulting optimal investment volume

$$v^{opt}(t+1)$$

for the time instant $(t+1)$ has the following expression:

$$v^{opt}(t+1) := \frac{\Delta I^{opt}(t+1)}{p(\cdot, t+1)}.$$

Note that the complete trading decision at the time instant $(t+1)$ generated by the proposed algorithm can be formalized using the following signal / volume pair:

$$\begin{aligned} &\{\text{trading}-\text{signal}, \ \text{trading}-\text{volume}\} := \\ &\{\text{sign}[\Delta I^{opt}(t+1)], \quad |v^{opt}(t+1)|\}. \end{aligned}$$

Here sign$[\cdot]$ is the signum function. Moreover,

$$\text{trading}-\text{signal} := \text{sign}[\Delta I^{opt}(t+1)]$$

denotes the optimal trading signal and

$$\text{trading}-\text{volume} := |v^{opt}(t+1)|$$

is the absolute value of the optimal investment volume. The above trading signal / trading volume pair definitively determines a $(t+1)$-trading decision of proposed PID trading algorithm. The proposed nonlinear optimization approach (13) implies that the optimal investment decision

$$\Delta I^{opt}(t+1)$$

constitutes an a priori optimized value in the above signal/volume pair. The corresponding optimal volume $v^{opt}(t+1)$ is in fact an a posteriori value that can be computed after the stock prise $p(\cdot,t+1)$ is known.

B. Stochastic Optimal Gains Tuning

The optimal gains selection problem (13) from the previous section constitutes a data driven regression-like approach. This generic optimization approach leads to the constrained nonlinear optimization and is based on the given historical stock market data. The additional FT technique discussed in the previous section is in fact an adequate expert driven clustering of the complete historical data set. Note that this separation of the data set is methodologically similar to the main idea of the celebrated Monte Carlo method (see [14],[22],[37] and references therein). Let us recall that the classic Monte Carlo method contains the so called "training" and "validation" steps (see [14],[17],[42] and references therein).

In this section we will formulate the stochastic optimization problem for an adequate gains selection in the proposed PID trading algorithm (1)-(3). The stochastic programming problem we consider is conceptually different to the regression-like problem (13). It involves the advanced statistical description of stock markets discussed in Section III. Using (11), we next introduce the following minimization problem:

$$\begin{aligned}
&J(\omega,K(t+1)) := \sum_{j=1}^{M} \big(\chi(\delta I(t+1)) - \\
&\exp\big[\ln\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big) + \ln\theta(\cdot,t) - \\
&\ln\theta(\omega,t+1) + \ln v(t)\big] \times p(\omega,t+1)\big)^2 \to \min_{K(t+1)} \\
&\text{subject to } (1)-(3),(12)
\end{aligned} \tag{15}$$

The above optimization problem constitutes a nonlinear stochastic program (see e.g., [14],[42]). Clearly, problem (15) now contains a probabilistic costs functional

$$J(\omega,K(t+1)).$$

The stochastic program (15) can be interpreted as a two stage or a multi stage problem. However, we consider here the scenarios based Monte Carlo optimization approach (see e.g., [14]). In this section, we study this problem only from a conceptual point of view and discuss the corresponding Monte Carlo sampling solution scheme.

Taking into consideration the mean approach we replace (15) by the following deterministic program:

$$\begin{aligned}
&J_E(K(t+1)) := \sum_{j=1}^{M} E_{\rho_1(\cdot),\rho_2(\cdot)}\big(\chi(\delta I(t+1)) - \\
&\exp\big[\ln\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big) + \ln\theta(\cdot,t) - \\
&\ln\theta(\omega,t+1) + \ln v(t)\big] \times p(\omega,t+1)\big)^2 \to \min_{K(t+1)} \\
&\text{subject to } (1)-(3), \\
&\chi(K_P(t+1)\Delta g(t) + K_D(t+1)\dot{\Delta} g(t) + \\
&K_I(t+1)\int_{t-T}^{t} h(\tau)\Delta g(\tau)d\tau) = \\
&E_{\rho_1(\cdot),\rho_2(\cdot)}\big(p(\cdot,t)\exp\big[\ln\big(\frac{p(\omega,t+1)}{p(\cdot,t)}\big) + \ln\theta(\cdot,t) - \\
&\ln\theta(\omega,t+1) + \ln v(t)\big]\big).
\end{aligned} \tag{16}$$

Here

$$E_{\rho_1(\cdot),\rho_2(\cdot)}$$

is the mathematical expectation operator with respect to the (joint) pdf for the pair

$$(\theta(\omega,t),\vartheta(\omega,t))$$

of quotients θ and ϑ introduced in Section III. Note that the log-normal characterization of the probability distribution functions $\rho_1(\cdot)$ and $\rho_2(\cdot)$ implies a specific insufficiency of the use a mathematical expectation in (16). The mean value optimization problem (16) does not possesses the necessary robustness property. This fact can imply some losses of deposit in the case problem (16) is directly applied to trading. In this situation one can replace the mean $E_{\rho_1(\cdot),\rho_2(\cdot)}$ in (16) by a known robust statistical characteristic (median and ctr.) associated with the pdf of the pair $(\theta(\omega,t),\vartheta(\omega,t))$.

Let us also note that the initial stochastic problem (15) as well as the auxiliary (deterministic) problem (16) constitute non-data driven optimization. These abstract problems use the model-based approach that involves the log-normal probability distribution functions $\rho_1(\cdot)$ and $\rho_2(\cdot)$. This fact constitutes a conceptual difference of between the data driven PID gains optimization (13)-(14) and the alternative PID tuning strategy based on the stochastic optimization problem (15).

The auxiliary problem (16) provides a possible approach for the numerical treatment of the initial problem (16). Following the Monte Carlo methodology, we define some probabilistic scenarios

$$P(\omega=\omega_i), i=1.,,,N$$

for some probabilities $P(\cdot)$ associated with the realizations of the stochastic variables ("events")

$$\Gamma := \{\omega_1,\ldots,\omega_N\}.$$

The celebrated scenarios based approximating problem for (16) can now be formalized as follows:

$$
\begin{aligned}
&J_E(K(t+1)) := \sum_{j=1}^{M}\sum_{i=1}^{N} P(\omega=\omega_i)\big(\chi(\delta I(t+1))- \\
&\exp\big[\ln\big(\frac{p(\omega_i,t+1)}{p(\cdot,t)}\big)+\ln\theta(\cdot,t)- \\
&\ln\theta(\omega_i,t+1)+\ln v(t)\big]\times p(\omega_i,t+1)\big)^2 \to \min_{K(t+1)} \\
&\text{subject to } (1)-(3), \\
&\chi(K_P(t+1)\Delta g(t)+K_D(t+1)\dot{\Delta} g(t)+ \\
&K_I(t+1)\int_{t-T}^{t} h(\tau)\Delta g(\tau)d\tau) = \\
&\sum_{i=1}^{N} P(\omega=\omega_i)\big(p(\cdot,t)\exp\big[\ln\big(\frac{p(\omega_i,t+1)}{p(\cdot,t)}\big)+\ln\theta(\cdot,t)- \\
&\ln\theta(\omega_i,t+1)+\ln v(t)\big]\big).
\end{aligned}
\tag{17}
$$

Evidently, the finite sum in (17)

$$\sum_{i=1}^{N} P(\omega=\omega_i)g(\omega_i,\cdot),$$

where $g(\omega_i,\cdot)$ denotes the corresponding function in (17) for a concrete event ω_i, approximates the mathematical expectation $E_{\rho_1(\cdot),\rho_2(\cdot)}$ in the auxiliary problem (16).

Similar to the FT methodology used in the previous section, we now divide the above N-dimensional event set Γ into two subsets and define a suitable N_1-dimensional training set and an additional N_2-dimensional validation set

$$\Gamma_1 := \{\omega_1,...,\omega_{N_1}\},$$
$$\Gamma_2 := \{\omega_1,...,\omega_{N_2}\}.$$

Here

$$N_1+N_2=N.$$

with

$$N \gg N_1.$$

The above division of the initially given N-dimensional scenarios set Γ makes it possible to consider the basic Monte Carlo approximating problem (17) on the training set Γ_1 and then on the full scenarios set Γ. In the case of an admissible mismatch in the values of objective functionals of problem (17) considered on Γ_1 and on the full set Γ the optimization procedure is successfully completed. Otherwise, one needs to increase the dimensionality of the historical data set and repeat the above training and validation steps in the Monte Carlo framework.

V. Frequency Domain Representation of the PID Trading Algorithm

A formal frequency domain involved description of the feedback based trading approach can be found in [4]. Let us recall that the frequency domain analysis constitutes a generic approach of the classic CE. We refer to [34] for the corresponding mathematical foundations and computational details of the Laplace and Fourier transforms in the CE framework.

In this section, we discuss the frequency domain analysis of the proposed PID trading algorithm (1) - (3). Let us also observe that the above PID trading algorithm can be interpreted as a control design with time delays. Taking into consideration the main profit formulae (3), the linear expression $\delta I(t+1)$ in (1) can be considered as an ARMA-like formal model. Hence the final expression for $\Delta I(t+1)$ in the PID structure (1) constitutes in fact a nonlinear ARMA abstraction. Note that the consideration of the classic PIDD regulator in the frequency domain constitutes a standard approach of the conventional control systems theory. Let us also observe that the discrete time PID control (4) can be naturally interpreted as a delayed proportional control.

We next apply the celebrated Discrete Signal Processing (DSP) approach and interpret the linear expression for $\delta I(t+1)$ in (1) as a linear filter. Application of the Z-transform to the discrete PID (4) implies the following generic form of the causal discrete-time Finite Impulsive Response (FIR) filter (see [35]):

$$
\begin{aligned}
&Y(z) = (K_P(s+1)+K_I(s+1)+K_D(s+1))z+ \\
&[(K_I(s+1)h(s)-K_D(s+1))z^{-1}+ \\
&K_I(s+1)h(s-1)z^{-2}+K_I(s+1)\sum_{\tau=s-T}^{s-2} h(\tau)z^{-\tau}].
\end{aligned}
\tag{18}
$$

Here $Y(\cdot)$ denotes the resulting Z-transform of the specific output signal $\delta I(t+1)$. Note that z is the proper variable in (18) and $s+1$ indicates the current PID gains. We next can easily obtain the Frequency Response (FR) function for the above FIR filter (18). Recall that FR of the filter (18) is a result of a the formal application of the DTFT (Discrete Time Fourier Transform) to the FIR (18). Following [35] we put

$$z \equiv \exp(-j\omega),$$

where j in (18) denotes the imaginary unit. The FR of the PID trading algorithm (1)-(3) can now be written as follows:

$$
\begin{aligned}
&FR(f) = (K_P(s+1)+K_I(s+1)+K_D(s+1))\times \\
&\exp(-jf)+[(K_I(s+1)h(s)- \\
&K_D(s+1))\times\exp(jf)+ \\
&K_I(s+1)h(s-1)\times\exp(2jf)+ \\
&K_I(s+1)\sum_{\tau=s-T}^{s-2} h(\tau)\times\exp(\tau jf)].
\end{aligned}
\tag{19}
$$

Here f denotes the frequency of a dynamic process. Note that the obtained FR expression (19) for the proposed PID trading algorithm makes it possible to apply the frequency domain methodology for the necessary tuning of the PID gains. The historical stock market data can be used here for an optimal selection of the above gains.

Let us give an illustrative computational example of the FR function. Consider an one day spot rate dynamics of the Binance BTC/USDT exchange (Figure 3).

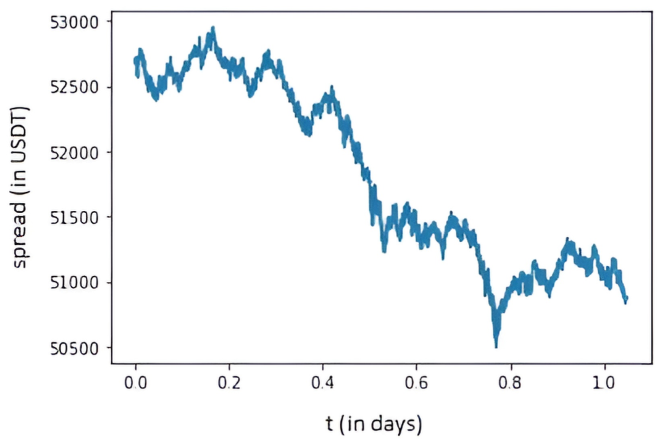

Fig. 3. The one-day spot rate dynamics on the Binance BTC / USDT exchange

The corresponding DTFT of the one day FIR filter dynamics (18), namely, the resulting FR (19) is presented on Figure 4.

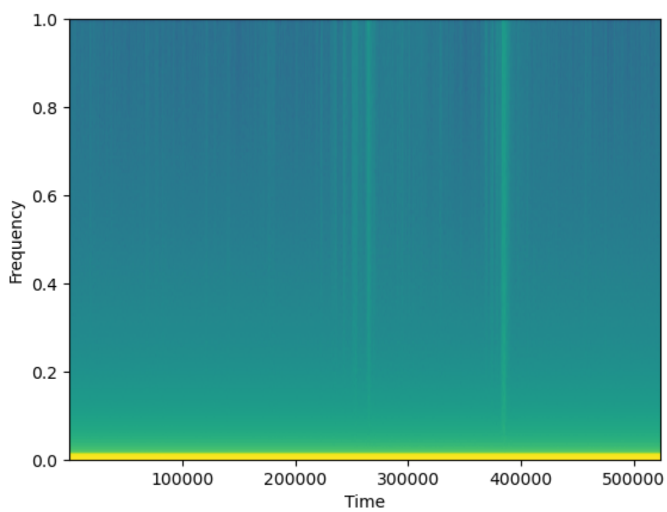

Fig. 4. The FR characteristic of the Binance BTC / USDT spot rate

In fact, the resulting frequency diagram, namely, Figure 4 represents a specific "market energy" distribution by the corresponding frequencies. This energetic interpretation of the presented DTFT is based ob the fundamental Parseval´s theorem in from the classic Fourier analysis (see e.g., [35]). Motivating from this consideration we introduce the following price energy concept associated with the historical stock market data:

$$\mathscr{E} := \sum_{i=1}^{N} FR^2(f),$$

where N corresponds to the dimension of one day data set under consideration and $FR(f)$ is the corresponding Frequency response associated with this data set.

Finally, note that the proposed PID trading algorithm (1)-(3) can also be combined with the several well-known momentum trading strategies (see e.g., [31],[32]). In the framework of a stock market we a are looking for a function

$$S : \Omega \times \mathbb{Z}_+ \to \mathbb{R},$$

the trading strategy. For example, the celebrated fixed-mix strategy $S_{const}(\cdot,\cdot)$ that keeps the value fraction of a risky asset constant has the following easy formalization:

$$S(\omega, t+1) = \frac{c}{p(\omega, t+1)},$$

where c is a given constant. The celebrated (random) trend-following momentum strategy $S(\cdot,\cdot)$ can also be easy represented:

$$S(\omega, t+1) = \text{tr}\{S\}(t+1) + \zeta(\omega, t+1). \quad (20)$$

Here

$$\text{tr}\{S\}(t+1)$$

denotes a trend of the stochastic process $S(\omega, t+1)$. By $\zeta(\cdot,\cdot)$ we denote here the stochastic nois model of the generic market fluctuations. The trend following model (20) provides a future promising tool for an optimal FR based PID gains selection procedure. In fact, a frequency domain analysis provides a natural analytic CE tool for the HFT algorithmic trading.

VI. Prototype Software for Implementation of the PID Trading Algorithm

We now present an application of the developed PID based AT technique to a real-world stock market data. Consider the Binance Bitcoin/USDT futures stock and apply the proposed optimal PID trading algorithm (1)-(3). Consider the BTC/USDT futures price dynamics mentioned above as presented in Figure 5.

Fig. 5. Binance BTC / USD one day price index

We have applied the novel OPID trading algorithm to the above example. The PID gains optimal tuning procedure for this example was determined as a data driven optimization discussed in Section IV. The corresponding (positive) dynamics of the return is now presented in Figure 6.

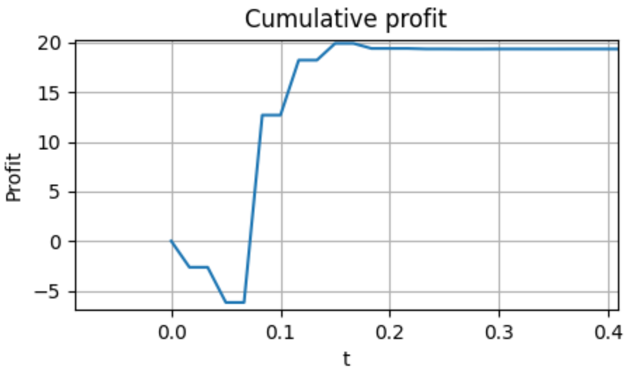

Fig. 6. Binance BTC / USD one day price index

Note that the operation time for every subsequent trading decision of the OPID in this example is timely restricted. The stock market under consideration has an obvious high-frequency behaviour. This fact naturally implies some expected difficulties of the common trading algorithms. In particular, this concerns the widely used moving average trading and the general trend following trading strategies. As one can see, the developed OPID constitutes an adequate approach to the HFT. The time ticks in the above trading example under consideration are significatively dense. The length of the corresponding intervals of trading buckets is equal to 1.728 sec.

Let us also observe the typical "hyperregulation and stabilization" dynamics of the proposed PID trading algorithm (see Figure 3). This dynamic behaviour is a characteristic one for the conventional PID controller (see e.g., [1],[3],[27],[28],[34]). The presented practical example illustrates the implementability of the developed PID trading algorithm AT. Moreover, as we can see the resulting financial behaviour of the return is a profitable behaviour.

VII. Concluding Remarks

In this paper, we developed a novel trading algorithm that involves the model-free PID control methodology and some well established statistical characteristics of the stock market data. The proposed analytic approach can naturally be extended to the real-time multi-asset trading. Moreover, it can also be efficiently combined with some classical trading strategies. The main idea of the proposed PID control approach to the algorithmic trading consists in modern data involved optimization techniques. These advanced optimization procedures determine an optimal calibration (optimal tuning) of the main PID trading parameters, namely, of the PID gains. The resulting optimal PID gains calculated for every stock price tick define a current trading decision. The optimization procedures mentioned above involve the conventional FT techniques in combination with the regression analysis as well as the Monte Carlo method from stochastic optimization.

The given historical stock market data, the statistical properties mentioned above and the data driven optimization techniques are constructively used for an adequate calibration (tuning) of the PID trading algorithm gains. This calibration involves the advanced backtesting procedure. The resulting optimal trading PID strategy generates at every subsequent time instant a profitable decision of the "buy/sell/hold" type for the stock market orders. In fact, the proposed CE like approach to the AT involves a combination of some mathematically rigorous tools, namely, the classic PID control methodology, applied statistics and computational optimization. This interconnected structure of the obtained trading algorithm and the model-free character of the PID scheme indicate the application possibility of the modern ML approaches to the design of the PID type trading algorithms (see e.g., [12],[13],[26]). The above combination of different mathematical tools finally leads to a novel and very promising trading strategy. The resulting implementable PID trading algorithm, the discussed initial software prototypes and the corresponding real-world scenario based simulations extend the family of the feedback based trading algorithms. We also expect a profitable application of the proposed optimized PID trading methodology in the High-Frequency Trading. Note that the developed PID based trading approach can also be involved (as an additional tool) into the several mathematical concepts of the modern financial engineering. For example, it can be considered in the framework of the well established financial time series analysis and Kalman filter techniques (see e.g., [3],[6],[29]). The proposed PID trading algorithm is also compatible with the generic price prediction and trend following techniques (see e.g., [24]).

Finally note that the algorithmic trading strategy proposed in our paper constitutes an initial theoretical development. We are mostly concentrated here on the mathematical and algorithmic aspects of the proposed technique. The prototypes of the financial solutions for the stock market proposed in our contribution need the comprehensive additional analytic development, the corresponding simulations, further backtesting, adequate optimization approaches and practical applications to the real stock markets.

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