Signal Processing in Vibration Analysis with Application in Predictive Maintenance of Rotating Machines

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***Abstract*—A general approach for change detection in vibration signals with application in predictive maintenance of rotating machines represents the object of the paper. After an overview of the maintenance approach, the condition monitoring in predictive maintenance is presented. Also, some vibration analysis techniques, making use of change detection, independent component analysis, time-frequency analysis and energy distribution, with application in predictive maintenance of rotating machinery, are discussed. They can be combined in a unified approach, offering new possibilities for more robust detection of changes in vibration signals, and assuring proactive actions in predictive maintenance. Finally, some experimental results for detection of faults in rolling element bearings and in a rotating machine operating, an industrial pump, are presented.**

***Index Terms*—Predictive maintenance; Change detection; Independent component analysis; Time-frequency analysis; Energy distribution; Rolling element bearings; Industrial pump.**

I. INTRODUCTION

Vibration analysis is the one of the most effective tool used to check the health of plant machinery and diagnose the causes. The health of machine is checked by routine or continuous vibration monitoring with sophisticated instruments, giving an early indication of a possible failure and offering countermeasures to avoid a possible catastrophic event. The paper presents some vibration analysis techniques, and different combination of these, in order to offer a framework for predictive maintenance of rotating machines and represents an extended and enhanced version of [1].

Vibration monitoring problem consists of machines condition and the change rate of its behavior. It can be ascertained by selecting of a suitable parameter for deterioration measuring and recording its value for further analysis. This activity is known as condition monitoring. The great parts of the defects encountered in the rotating machinery give rise to a distinct vibration pattern, or ”vibration signature”. Vibration monitoring has the ability to record and identify vibration ”signatures” for monitoring rotating machinery. Vibration analysis is applied by using transducers to measure acceleration, velocity or displacement, depending of the frequencies making the object of the analysis. Careful scrutiny and deep study of vibration ”signature” eliminate different fault possibilities and concludes to single fault. A logical and systematic approach has proved successful in diagnosing the basic causes. This applies to small, medium, large, direct coupled machines, motors, pumps, generators, turbo-machinery fans and compressors. Some machines are directly coupled to motor or some through gear boxes.

Sometimes, the vibration monitoring makes use of different change detection (CD) techniques. From statistical point of view, these techniques identify changes in the probability distribution of a stochastic process. The problem involves both detecting whether or not a change has occurred, or whether several changes might have occurred, and identifies the times of such changes produced. These techniques can be classified either as time, frequency or time-frequency domain based algorithms. They are based on distance measures, artificial intelligence, fuzzy logic, statistical differences, etc., applied on the original signals or on the preprocessed signals, in order to amplify the changes in their dynamics. Presently, the CD problem represents a key point, when preventive maintenance is replaced by predictive maintenance.

Some features, among the amplitude levels in the time domain, are easily extracted and classified, but they are affected by noise. Energy distribution in the time-frequency domain [2], involving more operations, can lead to more robust change detection in vibrating signal dynamics. Also, parametric signal processing algorithms can be used for change detection if there is an accurate model of the signal, in a selected representation space. However, the approach, based on modeling techniques, has limitations as well.

The time-frequency analysis (TFA) [3], in comparison with the time-domain analysis, usually provides a simpler interpretation and comprehension of nonstationary signals, with large application in vibration monitoring. The idea is to analyze the behavior of the energy distribution (ED), i.e., the distribution of energy at certain instant or certain frequency band or more generally [2], in some particular time and frequency region. The results can represent a starting point in solving CD problems. So, new analysis facilities in CD problem solving, are offered by the usage of the entropy based measures, such as Kullback-Leibler distance, Rényi distance, and Jensen difference, adapted to the time-frequency plane [4].

The paper is organized as follows. Section II refers to maintenance approach, while Section III presents the condition monitoring problem in predictive maintenance. Section IV has as subject change detection in vibration monitoring and is followed, in Section V, by a general view on the main signal processing techniques involved in vibration monitoring, with application in predictive maintenance of rotating machines. Section VI discusses different approaches in change detection of vibration signals based of signal processing techniques presented in Section V. Finally, in Section VII are discussed two case studies having as object fault detection in rolling element bearings (REB), and in a rotating machine, an industrial pump.

II. Maintenance Approach

Usually, the maintenance is performed as *preventive maintenance*, at fixed time intervals, or as *reactive maintenance*, when an actually fault produced. In the last case, it is necessary to perform immediately maintenance actions, while in the *predictive maintenance*, after a warning of a fault producing, the problem solving is carried out when necessary, so to avoid disruption of machine operating. A comparison of different maintenance types, with disadvantages and advantages is given in [5]. We present in the following some aspects concerning these approaches, to be taken into account, mainly in predictive maintenance of rolling element bearings.

A. Reactive Maintenance

This approach refers to machine running till a fault produced and involves fixing problems only the fault occurs. It represents the simplest and cheapest approach in terms of maintenance costs; often it implies additional costs, usually due to unplanned downtime. It can be seen as an easy solution to many maintenance strategies.

In rotating machines, rolling element bearings represent the most critical components, both in terms of initial selection and as well as in how they are maintained. Monitoring the condition of rolling bearings are essential and vibration based monitoring is frequently used to detect an early fault.

B. Preventive Maintenance

The preventive maintenance implies the scheduling of regular machine shutdowns, even if they are non required; this will increase the maintenance costs as some machine components are replaced, when this is not necessarily required. Some risks could appear due to replacing a defective machine part, incorrectly installing or reassembling parts. A frequently result of preventive maintenance consist of the fact that the maintenance is performed when there is nothing wrong in machine operating. Significant costs saving can be obtained by predictive maintenance.

C. Predictive Maintenance

The predictive maintenance refers to the process of monitoring the machine condition as it operates in order to predict which components are likely to fail and when. So, the maintenance can be planned and there is the possibility to change only those components that produce failure signs in its operating. The predictive maintenance principle consist of take measurements, to be used for prediction of the machine components behavior, susceptible of failure, and when these will be produced. Usually, these measurements include machine vibration, and machine operating parameters: flow, temperature, pressure, etc.

The continuous monitoring detects, in advance, the onset of component problems, so the maintenance is performed when needed. By this approach, unplanned downtime is reduced, as well as the risk of catastrophic failure. This will increase the efficiency and reducing of the costs. By predictive maintenance strategy, applied in rolling bearings, the costs can be avoid, giving in advance, a warning of a possible failure, enabling remedial action in advance.

III. Condition Monitoring

Condition monitoring consists of machine monitoring for early sign of failure so that the maintenance activity can be better planned, with reduced down time and costs.

The monitoring of vibration, temperature, voltage or current and oil analysis is frequently the most used. Vibration is the most widely used for its ability to detect and diagnose failure problems, but it offers also a prognosis on the useful life and possible failure mode of the machine. The prognosis is much more difficult to be performed and usually relies on continue monitoring of the fault to estimate the time when the machine will become unusable, taking into account the known experience in similar cases.

Vibration monitoring can be considered the most widely used predictive maintenance technique, and can be applied to a wide area of rotating machines. Machine vibration comes from many sources such as bearings, gears, unbalance etc., each sources having its own characteristic frequencies, manifesting as a discrete frequency, or as a sum and/or difference frequency. It can result complex vibration signals which put problems in vibration analysis, but some techniques, with a high sensitivity to faults, can reduce the complexity of the analysis. Bearing defects can affect higher frequencies, offering a basis for detecting incipient failure.

Usually, the detection uses the basic form of vibration measurement, where the vibration level is measured on a broadband basis (10-1000 Hz or 10-10000 Hz). The spikiness of the vibration signal, in machines with little vibration other than in the case of the bearings, is highlighted by the Crest Factor, indicating an incipient defect; also a great value of the energy, given by RMS level, indicates a severe defect.

Only this type of measurement offers limited information, but it can be useful for trend evaluation; increasing vibration level highlights the machine condition deterioration. Also, a comparison of the measurement level with some vibration criteria from literature proves to be useful in practice.

Generally, rolling bearings produce very little vibration in faults absence, and present specific frequencies when a fault produced. At the beginning of a fault, for a single defect, the vibration signals present a narrow band frequency spectrum. As the malfunction increases, it can be noted an increase in the characteristic defect frequencies and sidebands, with a drop in these amplitudes, broadband noise increasing and considerable vibration at shaft rotational frequency [5]. At very low machine speed, low energy signals are generated by the bearings, difficult to be detected. Also, bearings located within a gearbox are difficult to monitor, because of the high energy at the gear, which can mask the bearing defect frequencies.

IV. Change Detection in Vibration Monitoring

The CD problem is frequently present for continuous monitoring of systems like machinery, structure, process, equipment or plant, using data provided by the sensors. So, it is possible to anticipate the abnormal functioning or these systems, before it is produced and to reduce the maintenance costs. The normal behavior of a system can be described by a parametric model, without using artificial excitation, reducing the speed of the equipment or temporary stop. If such early detections are possible, large changes of the system can be prevented, and the effects of defects, mechanical fatigue, etc. can be quickly anticipate, raising the availability of the system.

The applications in this field make use of theories based on statistics, providing theoretical instruments to solve the early detection problem. Many industrial processes are based on known physical principles, with available analytical models, and for very complicated or unknown models, semi-physical or black-box models can be used. Vibrations analysis and surveillance of machinery or industrial equipments represent important cases of detection and diagnosis problems.

The CD problem refers to detection of the change (the alarm) and evaluation of the change (estimation), providing information, in some case, for diagnosis (source isolation). The performance criterion of a change detection algorithm consists of its ability to correctly detect the changes, with minimum delay and minimum probability of false decisions. So, it must respond to the small changes (sensitivity to changes), and does not be affected by the disturbances, noise or modeling errors (robustness of the algorithm). The sensitivity and robustness properties are usually in conflict, a good change detection algorithm must perform a compromise between the two aspects.

Two basic approaches in CD are reported as based on quantitative models (using analytical redundancy) and qualitative models, which can be conveniently combined to improve the robustness of the generation of the quantitative residuals. In the case of analytical exact models absence, learning models, such as fuzzy and neural models, can be used. More, the neural networks can be used for classification of the residuals, while fuzzy logic is useful for decision making. The methods based on quantitative models are oriented to identification (parameter estimation), observers (state estimation) and parity space. Some heuristics results, obtained from the previous experience, can be used for diagnosing the origins of the failure or change, based on the dispersion of the characteristics.

Almost all CD solutions assume that the monitored system can be described, with sufficient precision, by a finite-dimensional linear model. In practice, if the system is more complex than the structure, described by a finite-dimensional model, the parameter estimates will still converge, but their values can be strongly dependent on the experimental conditions. The algorithms will not be able to separate the changes determined by the external conditions from those produced by the internal defect of the investigated system, so the classical tests will fail. The problems mentioned above point out the requirement of the robust CD algorithms, able to separate the changes determined by the external conditions from the changes of the internal dynamics of the system.

The first generation of CD algorithms is based on strong hypotheses, or strong assumptions, which are difficult to verify in practice. So, a second generation of solutions was required, insensitive to the uncertainty of the system's dynamics, to the operating environment, and to large noise, statistically unknown. In our opinion, among the central problems to be addressed in the CD area refer to robustness, sensitivity and versatility. The lack of robustness of the classical algorithms concerns the failure of the detection, if one or more of the hypotheses assumed during the design are not verified in practice. The sensitivity concerns the ability of the algorithm to detect the change, even if there are small scale incipient changes. Finally, the versatility concerns the ability of the methods and techniques to solve more CD problems, using the same set of algorithms.

To solve the vibration monitoring problem different techniques have been developed, one can be mentioned: analysis of overall vibration level, frequency spectrum, envelope spectrum, cepstrum analysis, etc. [5]. The success of vibration monitoring, in many practical cases, requires specialized functions and tools. Simple application of CD techniques on original mono- or multivariate vibration signals can assure the successful of monitoring. Sometimes, it is necessary that some signal pre- or postprocessing procedures to be applied, to emphasize and highlight the characteristics of the vibration signals making the object of the analysis. So, some signal processing techniques can be used in conjunction with CD techniques: independent component analysis (ICA), time-frequency analysis (TFA), energy distribution (ED) evaluation in time-frequency domain. These techniques are implemented in a software toolbox, Matlab VIBROTOOL Toolbox [6], built as a set of programs that compute specific parameters and solve specialized tasks for vibrating monitoring.

The CD problem can be solved by change point estimation (mean change), change detection using one and two model approach, with different distance measures and stoping rules

[7], multiple change detection [8], detection and diagnosis of model parameter and noise variance changes [9], for mono- and multivariable vibration signals. Some algorithms, making the object of [10] and [11] in CD, represented the starting points in developing of these algorithms. The analysis of the behavior of the vibration signals reveals that most of the changes that occur are either changes in the mean level, variance, or changes in spectral characteristics.

The toolbox is used in the framework offered by an experimental model, VIBROCHANGE, for vibrational processes analysis using advanced measuring and signal analysis techniques [12]. The main modules involved in VIBROCHANGE include:

- VIBROSIG - vibration and other signals measurement module.
- VIBROTOOL - Matlab Toolbox for change detection and diagnosis, with modules dedicated to change detection and segmentation problem solving, among others.
- VIBROMOD - hardware module for change detection and diagnosis implementing some components of the VIBROTOOL module, for on-line analysis and monitoring of vibration processes [13].

For laboratory condition working, a generator of vibrations in controlled operation mode, for different electro-mechanical processes, VIBROGEN, has been developed. It includes an electrical motor, as well as bearings and other gearboxes, to emulate an industrial process. The system is an open one and different incipient faults can be generated.

V. Signal Processing in Vibration Monitoring

The success of vibration monitoring requires specialized functions and tools to compute specific parameters and solve specialized tasks for change detection using classical and recent techniques.

Sometimes, only simple application of CD techniques on original mono- or multivariate vibration signals can assure the successful of monitoring. Frequently, it is necessary to apply some signal pre- or postprocessing procedures, to emphasize and highlight the characteristics of the vibration signals making the object of the analysis. So, some signal processing techniques can be used in conjunction with CD techniques: independent component analysis (ICA), time-frequency analysis (TFA), energy distribution (ED) evaluation in time-frequency domain, etc. These techniques are briefly described in in the following.

A. Change Detection - CD

We present here only the framework in which the CD problem will be solved in the case studies presented in Section VII, using the Maximum A posteriori Probability (MAP) estimator [8].

CD allows for a first detection of changes in the original vibration signals, or in other signals, resulting after a possible preprocessing of these. A frequently used model, in this case, is a linear regression model with piecewise constant parameters [8],

$$y_t = \phi_t^T \theta(i) + e_t, \quad E(e_t^2) = R_t, \tag{1}$$

where y_t is the observed signal, $\theta(i)$ is the d-dimensional parameter vector in data stationary segment i, ϕ_t is the regressor; the noise e_t is assumed to be Gaussian with variance R_t.

The used model is referred to as changing regression, because it changes between regression models. Its important feature is that the jumps divide the vibration signals into a number of independent segments, since the parameter vectors in different segments are independent. Some important model derived from the model, where ϕ_t has different expressions [8]. In this framework, the problem of segmentation between ”homogenous” parts of the data arises more or less explicitly.

B. Independent Component Analysis - ICA

Independent Component Analysis (ICA) is closely related to the blind source separation (BSS) [14], offering new solutions for vibration and noise analysis [15]. The use of BSS techniques in conjunction with other techniques, such as CD and TFA, proved very useful in vibration monitoring. So, it is offered the possibility to translate the CD problem from the original space of the measurements to the space of the independent components (sources). The reduced number of components, in this case, will simplify the monitoring problem and the CD methods will be applied only for scalar signals; BSS also provides a mixing model of the independent sources, that point out how the source changes are reflected in the original vibration signals, for diagnosis purposes. When it comes to deal with mechanical signals, which are typically characterized by an excessive complexity, BSS faces a number of difficulties which seriously hinder its feasibility [15].

One of the frequently used model for BSS, assumes the existence of n independent signals $s_1(t), \ldots, s_n(t)$ and the observation of as many mixtures $x_1(t), \ldots, x_n(t)$, these mixtures being linear and instantaneous, i.e.

$$x_i(t) = \sum_{j=1}^{n} a_{ij} s_j(t) + n_i(t) \tag{2}$$

for each $i = 1, n$, and compactly represented by the mixing equation

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{s}(t) = [s_1(t), \ldots, s_n(t)]^T$ is an $n \times 1$ column vector containing the source signals, while vector $\mathbf{x}(t)$ contains the n observed signals and the square $n \times n$ ”mixing matrix” $\mathbf{A}$ contains the mixture coefficients.

The BSS objective is to recover the source vector $\mathbf{s}(t)$ using only the observed data $\mathbf{x}(t)$, the assumption of independence between the entries of the input vector $\mathbf{s}(t)$ and possible some

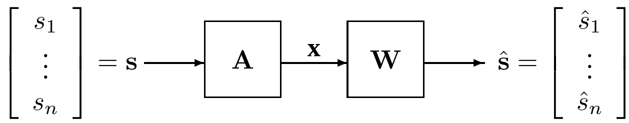

Fig. 1. Signal mixing and separating in BSS.

a priori information about the probability distribution of the inputs. It can be formulated as the computation of an $n \times n$ "separating matrix" $\mathbf{W}$ whose output $\hat{\mathbf{s}}(t)$ is an estimate of the vector $\mathbf{s}(t)$ of the source signals, and has the form:

$$\hat{\mathbf{s}}(t) = \mathbf{W}\mathbf{x}(t) \tag{4}$$

in the case of an instantaneous mixture (see Fig. 1).

For temporal coherent signals, the BSS problem can be solved using second and higher order statistics, the well known algorithms being SOBI (Second Order Blind Identification)) [16], and JADE (Joint Approximate Diagonalization of Eigen-matrices) [17], among others.

C. Time-Frequency Analyis - TFA

The analysis, processing, and parameter estimation of vibration signals whose spectral content changes in time are crucial in many CD applications. In this case, TFA can be of great interest, specially when the signal models are unavailable. In those cases, the time or the frequency domain descriptions of a signal alone cannot provide comprehensive information for change detection. The time domain lacks the frequency description of the signals. The TFA provides a proper description of the spectral content changes as a function of time.

The time-frequency representations (TFRs) can be classified according to the analysis approaches [18]. In the first category, the signal is represented by time-frequency (TF) functions derived from translating, modulating and scaling a basis function having a definite time and frequency localization. For a signal, $x(t)$, the TFR is given by

$$TF_x(t, \omega) = \int_{-\infty}^{+\infty} x(\tau)\phi^*_{t,\omega}(\tau)d\tau =< x, \phi_t, \omega >, \tag{5}$$

where $\phi_{t,\omega}$ represents the basis functions and $*$ represents the complex conjugate. The basis functions are assumed to be square integrable, i.e., they have finite energy. Short-time Fourier transform (STFT) [19], wavelets [20], [21], and matching pursuit algorithms [20], [22], are typical examples in this category.

The second category of time-frequency distributions (TFD), known as Cohen's shift invariant class distributions [3], characterizes the TFR by a kernel function. TABLE I gives the kernels used for main Cohen's class time-frequency distributions.

Some remarks on properties of the main Cohen's class time-frequency distributions from TABLE I could be made.

TABLE I. KERNELS USED FOR MAIN COHEN'S CLASS TIME-FREQUENCY DISTRIBUTIONS

Name	Kernel $\phi(\theta, \tau)$
SP	$\int h^*(u - \frac{1}{2}\tau) \exp^{-j\theta u} h(u + \frac{1}{2}\tau) du$
WVD	1
CWD	$\exp^{-\theta^2 t^2/\sigma^2}$
RID	2d Low pass filter in θ, τ space

The spectrogram (SP), suffers from the undesirable trade-off between the resolution and frequency resolution. On the other hand, the Wigner-Ville distribution (WVD) has a high time-frequency resolution, but is known to suffer from the presence of cross-terms. The Choi-Williams distribution (CWD) overcomes the WVD limitation suppressing to a large extent the cross-term interference, but some time-frequency resolution is lost. The last distribution belongs to the so-called Reduced Interference Distribution (RID), and also belongs to the Cohen's class, being an extension of the WVD.

Even if all TFDs tend to the same goal, each representation has to be interpreted differently, according to its own properties. For example, some of them present important interference terms, other are only positive, other are perfectly localized on particular signals, etc. The extraction of information has to be done with care, from the knowledge of these properties. We need a distribution that can reveal the features of the signal as clearly as possible without any "ghost" component and to apply a TFD that can get rid of the cross-terms while preserving a high time-frequency resolution.

D. Energy Distribution - ED

One of the simplest feature based signal processing procedures in TFA is via energy distribution. The idea is to analyze the distribution of energy at certain time instant or certain frequency band or more generally, in some particular time and frequency region. Such analysis is capable of revealing more information about a particular phenomenon [2], [18].

Once the local frequency content has been obtained, using TFA, an entropy measure can be evaluated for extracting the information containing in a given position of $t = n$. The Rényi entropy measures class [23], [24], with some significant contributions [25], offers new measures for estimating signal information and complexity in the time-frequency plane.

For a generic time-frequency distribution, $P_x(n, k)$, the Rényi entropy measure has the following form:

$$R_\alpha = \frac{1}{1-\alpha} \log_2 \left(\sum_n \sum_k P_x^\alpha(n, k) \right) \tag{6}$$

where n is the temporal discrete variable and k the frequency discrete variable, with $\alpha \geq 2$ being values recommended for time-frequency distribution measures [25]. The normalized Rényi entropy measures, with the normalization done in various ways, leads to a variety of possible measure definitions [2], [25]. Eisberg and Resnik [26], assimilate the time-frequency

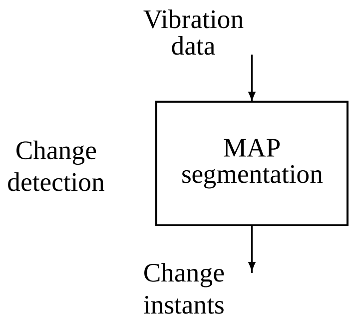

Fig. 2. First approach in machine health monitoring

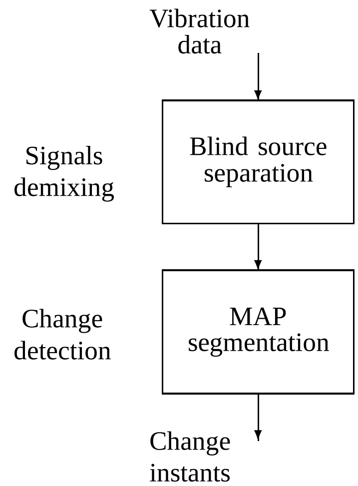

Fig. 3. Second approach in machine health monitoring

distributions at a given instant $t = n$ to a wave function and for $\alpha = 3$, resulting

$$R_3 = -\frac{1}{2}\log_2\left(\sum_n\sum_k P_x^3(n,k)\right) \tag{7}$$

The normalizing stage affects exclusively to index k, when the operation is restricted to a single position n to satisfy the condition $\sum_k P_x(n,k) = 1$ in such position.

The measure (7) can be rewritten for a given n as follows:

$$R_3(n) = -\frac{1}{2}\log_2\left(\sum_k P_x^3(n,k)\right) \tag{8}$$

Empirically the normalization proposed in [26] had shown to be most suitable for an application in vibration signal analysis. The values of $R_3(n)$ depend upon the size N of the window and it can be shown that they are within the interval $0 \leq R_3(n) \leq \log_2 N$. Hence, the measure can be normalized by applying $\hat{R}_3(n) = R_3(n)/\log_2 N$.

VI. General Approach for Change Detection

The signal processing techniques mentioned above can be used in different combinations to solve the problem of machine health monitoring. Three main approaches are discussed in the following.

A first approach simply consists of original signal segmentation (see Fig. 2), resulting the change points in vibration signal dynamics. The MAP algorithm [8], is one algorithm which can be used in this case, with good results for mono- and multivariate signals. Some experimental results, using this approach, in simulation and with real data, are presented in [8], [27].

A second approach (see Fig. 3) makes use of change detection of the signals resulted after blind source separation of independent vibration sources, starting from the original vibration signals. In this case, the problem is transferred from the original space of the measurements to the space of independent sources, where the reduced number of components will simplify the health monitoring problem, and the change detection methods will be applied for scalar signals. The assessment of the approach on a real machine is presented in [7].

The third approach, considered as a complex and general approach, practically includes all the signal processing techniques discussed above and is given in Fig. 4.

The approach makes use of time-frequency information content, the short-term time-frequency Rényi entropy, and a segmentation algorithm, based on MAP estimator. The segmentation algorithm operates on Rényi entropy, as a new space of decision. The procedure can be applied on the original vibration signals, or on the independent vibration sources resulted for these, after blind source separation. This approach enables more robust change detection in vibration signals. The application of the presented approach offers a simpler analysis and interpretation of the vibration signals behavior, providing new physical insight into vibrational processes. Same experimental results in simulation and with real data are given in [28], [29], [30].

VII. Case Studies

This section presents some experimental results obtained in two case study having as object fault detection in rolling element bearings (REB) and in a rotating machine, a pump, using the framework described in the previous sections of the paper.

A. Fault Detection in Rolling Elements Bearings

1) Test Data: The performed experiments use a data set from [31], with three faults having different locations: $F1$ (Inner race), $F2$ (Ball) and $F3$ (Outer race), and four sizes of the faults; $F0$ denotes no faults; only the data for the first case (06HH) have been used (see TABLE II).

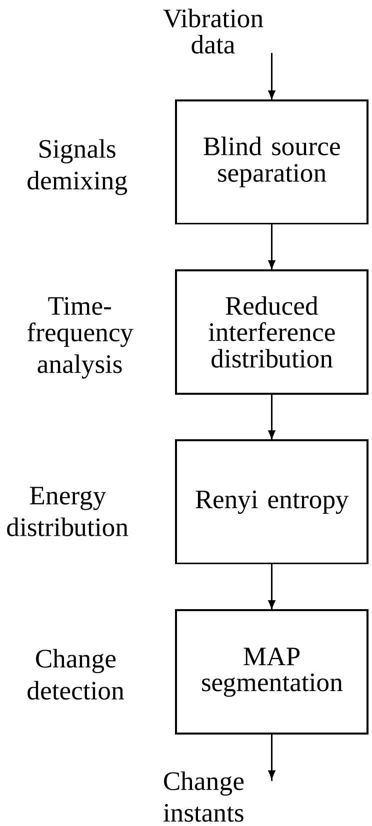

Fig. 4. Third approach in machine health monitoring

TABLE II. 1ST DATA TEST SET (6203 BEARING TYPE)

Fault size	$F0$ Free	$F1$ Inn. Race	$F2$ Ball	$F3$ Outer Race
0.000"	$y_0(t)$	-	-	-
0.007"	-	$y_1(t)$	$y_2(t)$	$y_3(t)$
0.014"	-	$y_4(t)$	$y_5(t)$	$y_6(t)$
0.021"	-	$y_7(t)$	$y_8(t)$	$y_9(t)$
0.028"	-	$y_{10}(t)$	$y_{11}(t)$	-

The signal $y_0(t)$ contains 4,096 samples recorded during normal conditions operating, while signals $y_i(t)$, $i = 1, \dots 11$ indicate files/vectors, containing each 4,096 samples, for the cases with faults; the sampling rate was of 12,000 samples/s.

2) Preliminary Analysis: For the signals mentioned above, some statistical features in time domain [32], have been computed, and are given in TABLE III, offering a general view of the signal characteristics.

The signals, making the object of the analysis, are simultaneously characterized in time and frequency domain using their mean localizations and dispersions. So, the averaged time and the time spreading, as well as the averaged frequency and the frequency spreading [33], are given in TABLE IV for signals analyzed.

3) Algorithm Description: The model used in the case study is a linear regression model with piecewise constant parameters (1).

TABLE III. TIME-DOMAIN STATISTICAL FEATURES OF THE SIGNALS $y_0(t), y_1(t), \dots, y_{11}(t)$ IN TIME DOMAIN

Signal	RMS	Mean	Var.	Cres. fact.	Skew.	Kurt.
$y_0(t)$	0.999	-0.002	0.998	3.796	-0.094	2.890
$y_1(t)$	0.992	0.007	0.985	5.145	0.124	5.456
$y_2(t)$	1.007	0.021	1.014	3.720	0.003	2.997
$y_3(t)$	0.997	0.016	0.995	5.189	0.088	7.698
$y_4(t)$	0.997	-0.001	0.995	4.016	0.067	4.281
$y_5(t)$	1.013	0.013	1.027	5.299	0.012	7.032
$y_6(t)$	0.987	0.078	0.974	9.747	-0.144	22.505
$y_7(t)$	0.724	0.001	0.525	6.937	-0.066	5.775
$y_8(t)$	0.978	0.046	0.958	3.779	0.023	2.982
$y_9(t)$	1.018	0.011	1.037	6.495	0.315	6.868
$y_{10}(t)$	0.981	0.019	0.963	4.378	0.043	3.457
$y_{11}(t)$	0.955	0.002	0.913	9.992	-0.086	21.255

TABLE IV. TIME-FREQUENCY STATISTICAL FEATURES OF THE SIGNALS $y_0(t), y_1(t), \dots, y_{11}(t)$

Signal	Aver. time	Time spread	Aver. freq.	Freq. spread
$y_0(t)$	2.104e+003	4.251e+003	-8.197e-009	0.287
$y_1(t)$	2.032e+003	4.155e+003	-2.359e-008	0.850
$y_2(t)$	2.026e+003	4.103e+003	-1.035e-006	0.906
$y_3(t)$	2.090e+003	4.167e+003	-2.206e-008	0.969
$y_4(t)$	1.944e+003	4.157e+003	-5.457e-009	0.804
$y_5(t)$	2.082e+003	4.247e+003	-3.880e-008	0.983
$y_6(t)$	1.954e+003	4.099e+003	-1.229e-008	0.920
$y_7(t)$	1.993e+003	4.843e+003	-1.134e-008	0.820
$y_8(t)$	2.057e+003	4.187e+003	-1.800e-007	0.968
$y_9(t)$	2.054e+003	4.273e+003	-1.604e-007	0.857
$y_{10}(t)$	2.006e+003	4.184e+003	-1.435e-007	0.909
$y_{11}(t)$	2.085e+003	4.081e+003	-9.584e-010	0.911

To solve the segmentation problem, all possible segmentation k^n are considered, estimate one linear regression model in each segment, and then choose the particular k^n that minimizes an optimality criteria of the form:

$$\widehat{k^n} = \arg \min_{n \geq 1, 0 < k_1 < \ldots < k_n = N} V(k^n) \tag{9}$$

For the measurements in a i-th segment, $y_{k_{i-1}+1}, \dots y_{k_i} = y_{k_{i-1}+1}^{k_i}$, results the least square estimate and its covariance matrix:

$$\hat{\theta}(i) = P(i) \sum_{t=k_{i-1}+1}^{k_i} \phi_t R_t^{-1} y_t, \tag{10}$$

$$P(i) = \left(\sum_{t=k_{i-1}+1}^{k_i} \phi_t R_t^{-1} \phi_t^T \right)^{-1}. \tag{11}$$

The following quantities are used in optimal segmentation algorithm:

$$V(i) = \sum_{t=k_{i-1}+1}^{k_i} (y_t - \phi_t^T \hat{\theta}(i))^T R_t^{-1} (y_t - \phi_t^T \hat{\theta}(i)) \tag{12}$$

$$D(i) = -\log \det P(i) \tag{13}$$

$$N(i) = k_i - k_{i-1} \tag{14}$$

where $V(i)$ - the sum of squared residuals, $D(i)$ - $-\log\det$ of the covariance matrix $P(i)$ and $N(i)$ - the number of data in each i segment, and represent sufficient statistics for each segment. The data and quantities used in segmentation k^n, having $n-1$ degrees of freedom are given in TABLE V.

TABLE V. DATA AND QUANTITIES USED IN OPTIMAL SEGMENTATION PROCEDURE

Data	$y_1, y_2, \ldots, y_{k_1}$	$\ldots$	$y_{k_{n-1}+1}, \ldots, y_{k_n}$
Segment	Segment 1	$\ldots$	Segment n
LS est.	$\hat{\theta}(1), P(1)$	$\ldots$	$\hat{\theta}(n), P(n)$
Statistics	$V(1), D(1), N(1)$	$\ldots$	$V(n), D(n), N(n)$

To solve the optimal segmentation procedure, different types of optimality criteria have been proposed [11]. In the following we use MAP criterium [8]. The number of segmentations k^n is 2^N (can be a change or no change at each time instant), and this put problems concerning the dimensionality.

The conceptual description MAP estimator, for the data and quantities given in TABLE IV, is given in Fig. 5, for three different assumptions on noise scaling: (i) known $\lambda(i) = \lambda_0$, (ii) unknown but constant $\lambda(i) = \lambda$ and (iii) unknown and changing $\lambda(i)$, where q is the change probability at each time instants ($0 < q < 1$).

Data: Vibration signal $y_t, \quad t = 1 \ldots N$

Step 1: Examine every possible segmentation, parameterized in the number of jumps n and jump times k^n, separately.

Step 2: For each segmentation, compute the best models in each segment parameterized in the least square estimates $\hat{\theta}(i)$ and their covariance matrices $P(i)$.

Step 3: Compute in each segment:

$$
\begin{aligned}
V(i) &= \textstyle\sum_{t=k_{i-1}+1}^{k_i} (y_t - \phi_t^T \hat{\theta}(i))^T R_t^{-1} (y_t - \phi_t^T \hat{\theta}(i)) \\
D(i) &= -\log\det P(i) \\
N(i) &= k_i - k_{i-1}
\end{aligned}
$$

Step 4: MAP estimate, $\widehat{k^n}$, for the three different assumptions on noise scaling

(i) known $\lambda(i) = \lambda_0$,

$$\widehat{k^n} = \arg\min_{k^n,n} \textstyle\sum_{i=1}^{n} (D(i) + V(i)) + 2n \log \frac{1-q}{q}$$

(ii) unknown but constant $\lambda(i) = \lambda$,

$$
\begin{aligned}
\widehat{k^n} = {} & \arg\min_{k^n,n} \textstyle\sum_{i=1}^{n} D(i) + (Np - nd - 2) \times \\
& \times \log \textstyle\sum_{i=1}^{n} \frac{V(i)}{Np-nd-4} + 2n \log \frac{1-q}{q}
\end{aligned}
$$

(iii) unknown and changing $\lambda(i)$,

$$
\begin{aligned}
\widehat{k^n} = {} & \arg\min_{k^n,n} \textstyle\sum_{i=1}^{n} (D(i) + (N(i)p - d - 2) \times \\
& \times \log \frac{V(i)}{N(i)p-d-4}) + 2n \log \frac{1-q}{q}
\end{aligned}
$$

Results : Number n and locations k_i , $k^n = k_1, k_2, \ldots, k_n$

Fig. 5. MAP segmentation algorithm.

In a practical problem, only one of the equations from **Step 4** (see Fig. 5) is evaluated, according with the assumption on noise scaling of the procedure.

For the exact likelihood evaluation, can be implemented recursive local search techniques and numerical searches based on dynamic programming or MCMC (Markov Chain Monte Carlo) techniques [11], [8].

Starting from the optimal segmentation results it is possible to analyze the data resulted for each stationary data segment to locate and diagnose the produced fault or change in the REB: outer race, inner race, bearing cage, ball (roller), according with the frequency area where it was produced.

4) Multiple Fault Detection: Started from the data given in TABLE II data sequences with multiple faults have been generated, for 3 types of events: inner race faults, ball faults and outer race faults, with different fault size: 0.007”, 0.014”, 0.021”, 0.028”, for the first two cases, and 0.007”, 0.014”, 0.021” for the third case. The following data sets have been used in the analysis, for fault detection:

$$
\begin{aligned}
s_1(t) &= [y_0(t), y_1(t), y_4(t), y_7(t), y_{10}(t)] \\
s_2(t) &= [y_0(t), y_2(t), y_5(t), y_8(t), y_{11}(t)] \\
s_3(t) &= [y_0(t), y_3(t), y_6(t), y_9(t)]
\end{aligned}
$$

resulting data sequences of 20480 values for signals $s_1(t), s_2(t)$ and 16384 for signal $s_3(t)$. The real faults instants were 4097, 8193, 12288 and 16384. These data sets offer the possibility to fault detection of a graduate size of fault, for the cases mentioned above.

The experimental results refer to the signals $s_1(t), s_2(t), s_3(t)$ and the segmenting algorithm presented above with unknown and constant noise scaling, and MCMC algorithm [8], with a value of jump probability, $q = 0.3$ and appropriate design parameters in search scheme, for different model order, na. The fault instants detected for different model orders na are presented in TABLE VI, TABLE VII and TABLE VIII for $s_1(t)$, $s_2(t)$ and $s_3(t)$, respectively.

The signal $s_1(t)$, making the object of the analysis, and the estimated multiple fault times for the inner race, $na = 20$ and $q = 0.3$, are presented in Fig. 6, while the signal $s_2(t)$ and the estimated multiple fault times for ball, $na = 20$ and $q = 0.3$ are given in Fig. 7. The signal $s_3(t)$ and the estimated multiple fault times for the outer race, $na = 60$ and $q = 0.3$ make the object of Fig. 8.

TABLE VI. FAULT DETECTION IN SIGNAL $s_1(t)$ USING DIFFERENT MODEL ORDER

Model order	Fault detection instants
$na = 10$	4096, 8687, 9501, 10684, 11322, 11500, 12570, 12627, 12967, 13068, 13961, 14527, 14627, 14777, 15964, 16384.
$na = 15$	4096, 8687, 9502, 10684, 11501, 12570, 14777, 16384.
$na = 20$	4096, 8195, 8687, 11502, 13026, 16384.

The changes in signals $s_1(t)$, $s_2(t)$ and $s_3(t)$, resulted after the data concatenation, are gradual, whose effect may

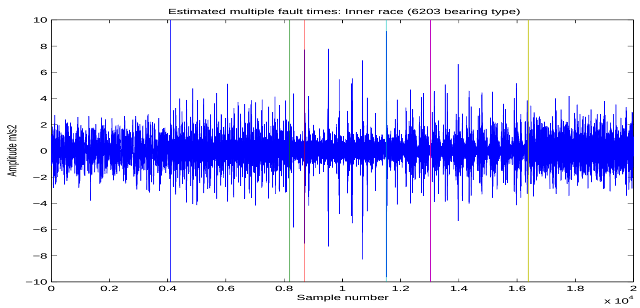

Fig. 6. The signal $s_1(t)$ and estimated multiple fault times for inner race, $na = 20$, $q = 0.3$.

TABLE VII. FAULT DETECTION IN SIGNAL $s_2(t)$ USING DIFFERENT MODEL ORDER

Model order	Fault detection instants
$na = 10$	4096, 8191, 8497, 8614, 9305, 9929, 11946, 16385, 16711, 16901, 18065, 18129.
$na = 15$	4096, 8190, 11946, 16385, 16719, 18108, 18128.
$na = 20$	4096, 8190, 11945, 16385, 16751, 18233.

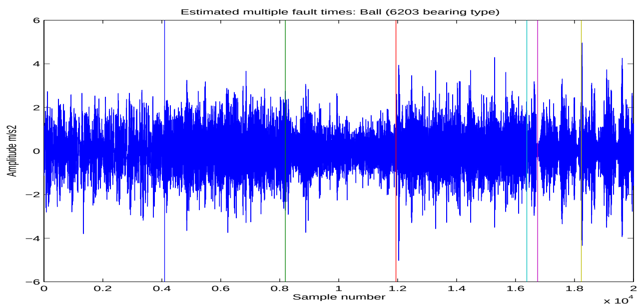

Fig. 7. The signal $s_2(t)$ and estimated multiple fault times for the ball, $na = 20$, $q = 0.3$.

TABLE VIII. FAULT DETECTION IN SIGNAL $s_3(t)$ USING DIFFERENT MODEL ORDER

Model order	Fault detection instants
$na = 10$	4096, 4383, 7081, 7170, 7897, 7950, 8192, 12298, 12367, 12480, 12982, 13151, 13260, 13407, 13596, 14042, 14179, 14378, 14489, 14668, 14823, 15169, 15271, 15575, 15605, 16050, 16229.
$na = 15$	4096, 8192, 12296, 12368, 12479, 12669, 12813, 13261, 13455, 13596, 14042, 14173, 14378, 15015, 15164, 15271, 15469, 15605, 16051, 16346.
$na = 20$	4096, 8192, 12293, 12367, 12479, 12669, 12813, 13261, 13460, 13594, 14042, 14189, 14378, 15271, 15473, 15604, 16051.
$na = 60$	4096, 8198, 12287, 12352, 14057.

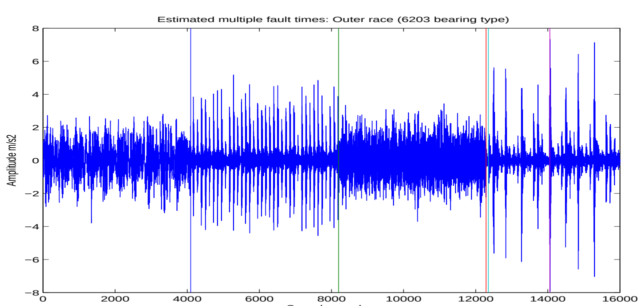

Fig. 8. The signal $s_3(t)$ and estimated multiple fault times for outer race, $na = 60$, $q = 0.3$.

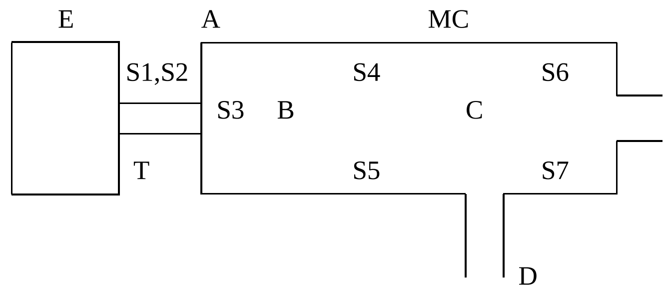

Fig. 9. Schematically multichannel measurement

increase, producing new changes in the signal dynamics that can be detected by the algorithm. The further deterioration of the rolling element bearing during operating produces new fault instants, different from 4096, 8192, 12288 and 16384 instants. According with data from TABLE VI, TABLE VII and TABLE VIII one can notice that in all the cases the main faults are detected. Also, it can be noted that for the models with high order ($na = 20$, $na = 20$ and $na = 60$, respectively), only the main faults are detected at instants 4096, 8192, 12288 and 16384 or near instants. The models, of high order, can increase the robustness of the optimal segmentation algorithm to gradual, or small changes in signal dynamics. Different values of q offer similar results, but a higher order of the model leads to a better fault detection, the model being more able to approximate the signal dynamics.

B. Industrial Pump Monitoring

The machine under investigation is an industrial pump. The used data set consists of multichannel measurements for 7 channels repeated for two identical machines: the first is virtually fault free and the second shows a progressed pitting in both gears [34]. The data were selected from the high-frequency measurements, digitized at 12800 Hz, a data segment of 4096 values, 2048 from the fault free machine, and last 2048 from the machine with a progressed pitting in gears, both for minimum load. The data have been low-pass filtered to 5000 Hz.

A scheme of the machine with its components and sensor position is given in Fig. 9, with the following legend:

E	=	electromotor
A	=	incoming shaft (driving shaft)
MC	=	machine casing
T	=	tachometer
B	=	first delay (gear-combination)
C	=	second delay (gear-combination)
D	=	outgoing shaft (to vane in water)
S1-S7	=	position of sensors 1,7

The rotating speed of the driving shaft is measured with a tachometer. This measurement is done synchronously with

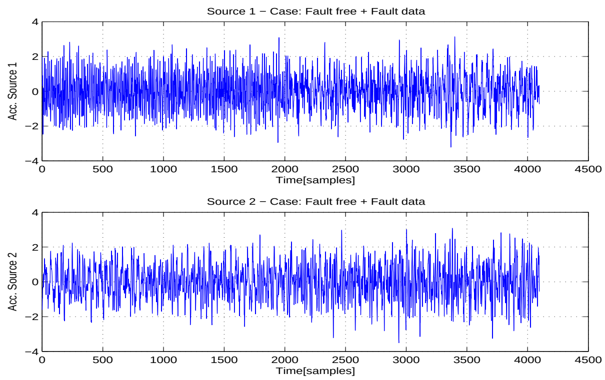

Fig. 10. Independent vibration sources in normal operating and fault conditions of the pump

7 accelerometers used in the following manner: sensor S1,S2 are radially mounted near the driving shaft, with an angle of 90 degrees between them, sensor S3 is axially mounted near the driving shaft, and sensor S4-S7 are radially mounted on different parts of the machine.

The data represented the object of the analysis in [7], where the blind source separation (BSS) and change detection in source signals, according with the approach presented in Fig. 3. The case study, making the object of this section, used the general approach, given in Fig. 4, for the data set mentioned above. It includes vibration signal demixing, time-frequency analysis, energy distribution evaluation using short-term Rényi entropy, and its segmentation, based MAP estimator. The segmentation algorithm operates on Rényi entropy, as a new space of decision. We discuss in the following this approach and present the experimental results.

1) Blind Source Separation: The acceleration measurements for 7 channels and 4098 values, from the fault free machine and progressed pitting in gears machine, represented the input data for SOBI algorithm [16], when 2 independent vibration sources and an instantaneous mixture model have been considered. The number of the sources resulted via eigendecomposition of the sample covariance matrix [35]. The independent vibration sources are presented in Fig. 10.

2) Time-Frequency Rényi Entropy: Fig. 11 shows the reduced interference distribution (RID) [36], of S1 source, computed with a kernel based on the Hanning window [33]. In Fig. 11 at linear scale, it can be noted a change in the spectral content of the source, in the second part of the signal.

Results of the TFD analysis for S2 source are presented in Fig. 12 for RID, with a same Hanning window. Similar conclusions, as in the previous analyzed case, concerning TFD properties, could be established. From Fig. 12 it can be noted a reduced change in the spectral content of the source, in the second part of the signal, in comparison with the S1 source.

A first conclusion, in this stage of time-frequency analysis, could be that S1 source has been induced by the fault in pump gears, but because the source separation is not perfect, due to

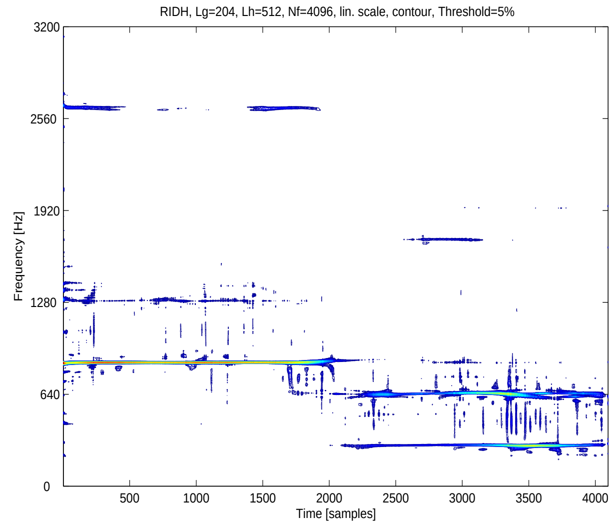

Fig. 11. Reduced interference distribution for vibration source S1 in normal and fault operating conditions

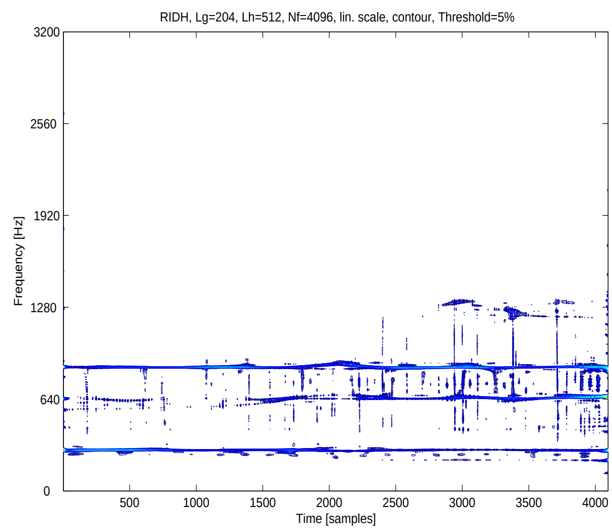

Fig. 12. Reduced interference distribution for vibration source S2 in normal and fault operating conditions

a possible lack of the BSS method robustness, the real change can also induced in other sources, in our case in the S2 source. The differences between the changes in spectral content in both sources point out this fact.

To evaluate the TFD resulted for S1 source we present in Fig. 13 the short-term Rényi entropy as measure of time-frequency distribution, computed for RID. It was used a sliding window of $N = 64$ values and a constant bias to be added to signal of 1.

For S2 source, we present in Fig. 14 the short-term Rényi entropy, as measure of time-frequency distribution, computed for RID, with the same values for the sliding window and constant bias added to signal.

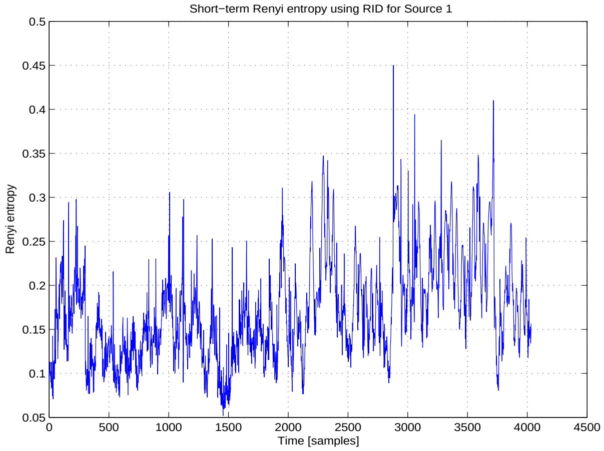

Fig. 13. Short-term Renyi entropy using RID for source S1

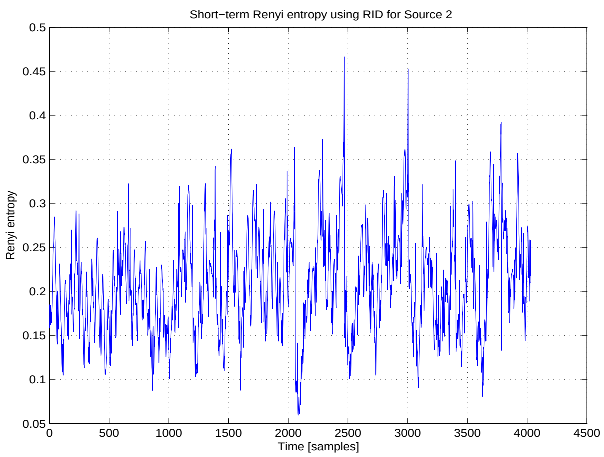

Fig. 14. Short-term Renyi entropy using RID for source S2

3) MAP Segmentation of Rényi Entropy: Visual inspection for the Rényi entropy of both sources, shows that the onset time is clearly visible as a change in energy and frequency content. Our experience is that, for this problem, as for many other signal processing ones, a piecewise constant model (1), could lead to a satisfactory trade-off between complexity and efficiency of the corresponding algorithms for the off-line estimation of the change time. The segmentation procedure has been performed using an autoregressive model (AR) of order 1, the unknown and constant noise scaling assumption and MCMC algorithm.

The parameter and variance estimates resulted in MAP segmentation are presented in Fig. 15 and Fig. 16 for Rényi entropies, obtained for S1 and S2 sources, respectively.

The variance traces of the piecewise constant model show, for both sources, significant jumps in the second part of the signals, and that a main distinct rupture event occurred. The proposed procedure assures more robust change detection in vibration signal analysis, than in the case of change detection in the estimated sources in time domain, see [7].

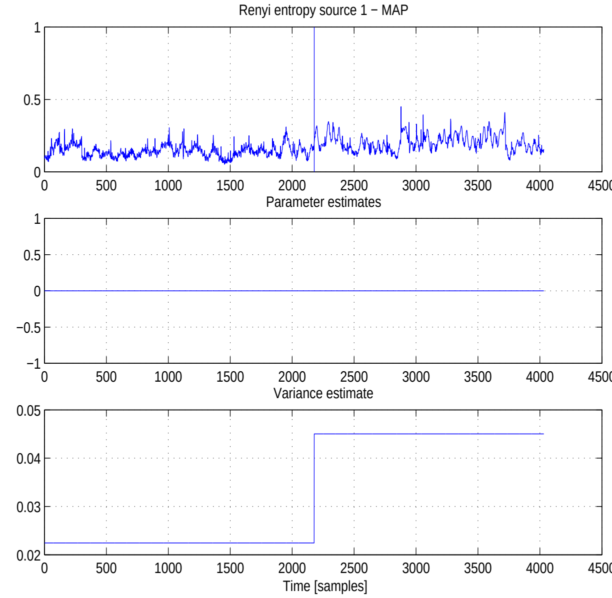

Fig. 15. MAP segmentation of short-term Renyi entropy for source S1

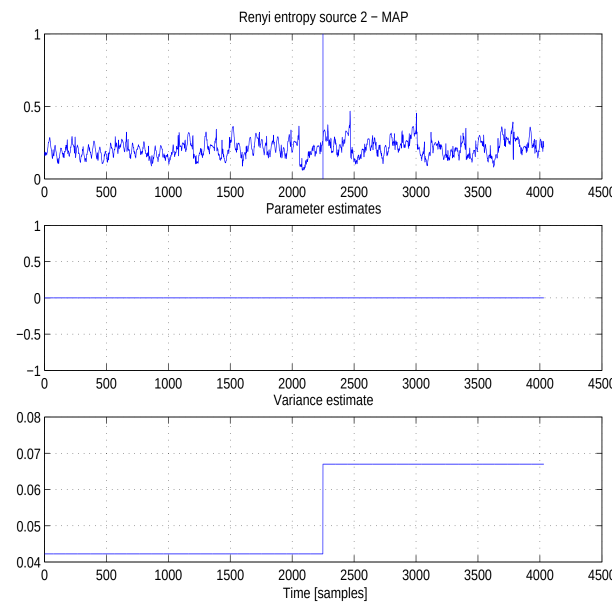

Fig. 16. MAP segmentation of short-term Renyi entropy for source S2

VIII. Conclusions

The paper considers the problem of change detection in vibration signals, with application in predictive maintenance of rotating machines, integrating some signal processing techniques, mainly independent component analysis, time-frequency analysis, energy distribution evaluation in time-frequency domain, and a change detection algorithm based on MAP estimator.

The case studies making the object of the paper prove the effectiveness of the proposed approach. The first case study, having as subject detection of faults in REB, uses a segmentation algorithm based on MAP estimator, directly applied to vibration signals, while the second, for monitoring of an industrial pump, makes use of time-frequency Rényi entropy segmentation, applied to independent vibration sources of the pump.

The general approach offers new possibilities for more robust detection of changes in vibrating signals and assures proactive actions in vibration monitoring. It offers a simpler analysis and interpretation of the vibration signals behavior, providing new physical insight into vibration processes for predictive maintenance. It can also be used for other domains that require change detection and diagnosis, such as biomedical signal processing (EEG, EKG, and MEG), seismic signal analysis, infrastructure monitoring, speech analysis, communication systems, video surveillance, transportation systems, etc.

Acknowledgment

The paper was partially supported by 2019-2022 Romanian Core Program, Project RO-Smart Ageing, funded by Ministry of Research and Innovation.

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