Mathematical Modeling Approach to the Design and Analysis of Measurement and Control Systems

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***Abstract*—Mathematical modeling represents system behaviors in mathematical terms. In system analysis, a descriptive model can be done using different control laws or fitting experimental measurements or other empirical data. This work presents an analysis based on mathematical modeling as the behavior observation representations for measurement and control systems. Some adaptable control methods based on multi-function optimizations from the linear and nonlinear characteristic measurements became a step to ensure stable system performance. Developing a mathematical model for changing the regenerative parameters within the limit of the feasible structure is a synthesis of measurement and control systems. An application for calculating energy from power demand associated with the switching delays depending on the charge controlled capacitor properties is constructed. The process of developing a suitable mathematical model from experimental measurements and control gives the key for the principle practice modeling design system.**

Keywords- mathematical model; system analysis; measurement; control system.

I. INTRODUCTION

In the traditional measurement system engineering, the role of a model is not only important for advanced system design, but also to measure the viability and reliability of the system. Traditional model approaches to system engineering have always relied on mathematical modeling. Mathematical models are usually used in different engineering disciplines such as electrical engineering, computer sciences, nature sciences, or social sciences to explain how a system works and to predict the behavior in the future.

The energy resources in the world are exploited with the use of traditional energy, causing an environment problem as well as global warming. Renewable energy sources are clean and available in nature, but the problem is, it is difficult to use energy economically and efficiently. Reasonable exploitation and use of renewable energy requires a sustainable optimal synthesis method. The preferred solution is to select the optimal mathematical model for forecasting techniques. According to the classification criteria of mathematical models, nonlinear systems tend to be studied with strongly nonlinear models. There are two main approaches: the model free approach and the model based approach. The model based approach has a fundamental role in determining time, size and cause of a characteristic in a dynamic system operation. The model based approach is done by the structure of the model and the existing input-output relationships.

The model free approach is used in real time measurements and process history, when the model based approach is unavailable or not useful. In dynamic systems, the inner parameters of the system will modify its dynamics. If a characteristic property or parameter of the system from the standard condition changes, there are some blocking situations of disconnects between system components or structural damage inside. The pre-knowledge of a healthy system usually uses observer-based methods or parameter estimation techniques. However, some nonlinear characterizations might not be observed and estimated because of the inaccurate values of the physical variables or environmental conditions.

The nonlinear characteristic approach in a system is critical to the quality of measuring the production processing capability. This characterization is done by using a phenomenological or general basis function, which is the observed behavior of the system. This shows that changes in the inputs of a model affect the output, but sometimes produce un-observed states. Because the characteristics of each part or each component are completely different, from random fault as variations of supply voltage, temperature changes or inaccurate instrument reading, improving safety and reliability for the automatic recovery, a nonlinear system needs a specific control technique with a low cost and high effectiveness. Nonlinear characterization measurement can be suggested as a method in fault detection for operating constraints in process system than the corporation with the potential inaccuracy of the model or model mismatch. A mathematical model built from this characterization is to select in design and analysis the measurement and control system. The rest of the paper is structured as follows. In Section II, we provide an overview to improve the analysis and design approach with mathematical modeling. In Section III, an adaptive

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development method based on control theory allows to develop simulation nonlinear characteristics as a virtual tool for ensuring stable system performance. The conclusion presents a collection from the series references for using mathematical modeling for design and analysis of measurement and control systems.

II. MAIN PROBLEM

In each structure system, the classification criteria are linear or nonlinear mathematical model, static or dynamic model, explicitly or implicitly calculated, deterministic or probabilistic model, deductive, inductive or floating model. This work concentrates on linear and nonlinear mathematical models. A mathematical model describes the system by establishing the relationship between the variables from a set of variables and equations. Some properties of the system are the set of functions between the different variables and building the equations of the described system.

A. Model based and model free

The first group is the model-based method, also known as knowledge based dynamic model. The behavior of the system when modeled as a set of differential equations is usually in state space form. The nonlinear state space model has the general structure:

$$\begin{cases} \underline{\dot{x}} = f\left(\underline{x}, \underline{u}\right) & \underline{x} \in R^n, \ \underline{u} \in R^m \\ \underline{y} = g\left(\underline{x}\right) & \underline{y} \in R^p \end{cases} \quad (1)$$

The measurement vector y is provided by the available sensors, and u is the control input vector, n, m, p are the dimensions of the vector x, u, y [10]. Under classical assumptions about control inputs, this general representation rewrites in an input affine form:

$$\begin{cases} \underline{\dot{x}} = \left(\underline{x}\right) + G\left(\underline{x}\right).\underline{u} & \underline{x} \in R^n, \ \underline{u} \in R^m \\ \underline{y} = g\left(\underline{x}\right) & \underline{y} \in R^p \end{cases} \quad (2)$$

This model retains the nonlinear global behavior of the system while benefiting from interesting results in nonlinear control theory. A further step towards simplification is to linear equation (2) around an operating point or a reference trajectory. The linear model has the form:

$$\begin{cases} \dot{x} = Ax + Bu \\ \quad y = Cx \end{cases} \quad (3)$$

Where A, B, C are possibly varying time. We suppose A and B are controllable for the linear conditions in which the criterion optimization stands. The computation to solve around equilibrium uses the matrices between themselves as takes into account the nonlinearities of function f. g corresponds to the global behavior of the system.

Developing an improved mathematical model is estimating the accuracy of the estimated parameter and identification as characterization of the model system. To obtain the suitable mathematical model for the system with special characterizations by all techniques in computer sciences, an example is presented in [8].

A dynamic system model can be categorized by many factors, such as genetic and non-genetic aging factors. Genetic factors can not be easily controlled, while non-genetic can be controlled and include poor maintenance, high loads and stress in the operation of the system. Some parameters make the model can't be observed and controlled in the domain stabilization system. These parameters need to be identified to ensure an accurate action processing or make the move to protection mode and automatic mode. The algorithm for estimate these parameters is based on aging, such as mechanical, electrical and structural conditions, which are important for most of existing model designs. Beside the estimation parameters, processing signal by image is often used in any fields where the quick identification of an object is necessary. This work is as empirical model for the model based software and system engineering.

The model free approach is also used when no explicit dynamical model of the control system is available. It exploits measurements acquired in real time, or available in a previously constructed database, using the behavior of the signal model. Some nonlinear characterizations in the system might not be observed and estimated. The predicted solution is suitable operating in the control system. For the vehicle system, the model free approach is preferred for the trajectory control, represented by differential equations or difference equations. The model free approach is useful for a scenario in which the system depends on other random variables relating inputs to outputs for the role of control. An example of a model free approach is as follows. A system presents a model free by general state equation [10]:

$$\underline{\dot{x}} = \left(\underline{x}, \underline{u}\right) \qquad \underline{x} \in R^n, \ \underline{u} \in R^m \quad (4)$$

The output of this system y, y $\in R^m$ that vector output can be rewritten in the form:

$$\underline{y} = (y_1, y_2, y_3 \dots, y_m) \quad (5)$$

$$\underline{y_i} = h_i(\underline{x}, u_1, u_2, \dots, u_m) \quad i = 1 \text{ to } m \quad (6)$$

The output components of y are analytically independent, the state components *x* and the input components *u* are the function of the output components and a known number of their derivative, written as follows:

$$\begin{cases} x_i = \Phi_i(y_1, y_1^{(1)} \dots, y_1^{(\mu_i,1)}, y_2, \dots, y_2^{(\mu_i,2)} \dots, y_m, \dots, y_2^{(\mu_i,2)}) \\ u_j = \psi_j(y_1, y_1^{(1)} \dots, y_1^{(\upsilon_j,1)}, y_2, \dots, y_2^{(\upsilon_j,2)} \dots, y_m, \dots, y_2^{(\upsilon_j,2)} \end{cases} \quad (7)$$

The functions $\underline{\Phi}$ and $\underline{\Psi}$ are identified by the equation:

$$\underline{\dot{\Phi}} = f\left(\underline{\Phi}, \Psi\right) \quad (8)$$

Consider a nonlinear system whose discrete time dynamics from an initial state are given by:

$$\underline{X}_{k+1} = \underline{X}_k + f\left(\underline{X}_k, \underline{U}_k\right) \quad (9)$$

with k ∈ N where $\underline{X}_k \in R^n$, $\underline{U}_k \in R^m$, f is a smooth vector field of $\underline{X}_k$ and $\underline{U}_k$ is, respectively, the state and the input vectors of this system at time k. We suppose here that each input has an independent effect on the dynamic state:

$$\frac{\partial f}{\partial u_i} \neq \frac{\partial f}{\partial u_j} \quad i \neq j \text{ with } i, j = \{1,\ldots, m\} \qquad (10)$$

The discrete dynamic model in a nonlinear system can be rewritten as:

Model continue → Discretion → Discrete Model

$$\underline{\dot{X}} = f(X, U)\underline{X}_{k+1} \Longrightarrow \underline{X}_k + f(\underline{X}_k + \underline{U}_k)$$
$$\exists Y_k |\ Y_k = h(\underline{X}_k) \qquad (11)$$

With $k \in N$ where $\underline{X}_k \epsilon R^n$, $\underline{U}_k \epsilon R^m$

$\underline{X}_k$: is in respect to the state vector of this system at time k.

$\underline{U}_k$: is the input vector of this system k to k+1.

f: is a smooth vector field of $\underline{X}_k$ and $\underline{U}_k$.

$\underline{Y}$: the output vector measurement.

The mathematical model by state equation (11) is structured from the output function, the state component and input component. It makes more clearly about the characteristic set to function dynamic system, using model output obtained by simulation and prediction when the process noise enters the system. Input noise is an important role defined non-functional properties in a systematic manner when not well established domain. The simulation is developed by the non-functional properties tool under the function properties parameters from the model free method. An application for building using the model based approach is shown in references [8][9], in which energy performance diagnosis is done based on a mathematical model from non destructive measures, inverse method and artificial neural networks. Another application for associated model free can be found in reference [10,] that improves the model based approach by a dynamic model with characteristic matrix fault effect to flight trajectory. These works provide an overview in using a model as the best way for measurement and control advance methods.

B. Maintaining the measurement and control system

Modern control theory approaches not only the model based approach but is also based on different methods like the model free approach in which the output has a characteristic prediction of the input. Analytical redundancy means to exploit mathematical relations between measured or estimated variables to detect possible dysfunction. This should be understood as knowledge based dynamic model [10]. Residual generation uses a model of the system in which the control inputs sent to the actuators and the system outputs as measured by the sensors injected to predict the behavior of the system, or part of it, and compare this prediction to the actual action. The associated algorithms are located functional and non-functional properties of models to performance benchmarking and optimization**.**

Based on the difference between performance level-PL and safety integrity level SIL, each architecture model is associated with three parameters: frequency and duration of exposure Fr, probability of an event occurring Pr, probability of avoiding or limiting the harm Av [11]. For the particular applications, the model choices such as thermal and ageing models are required by prior experience using the equipment in process industries where the transformer thermal loss of life related with changing the regenerative parameters within the limit of the feasible structure [3]-[6]. Thermo physical parameters are the important parameters in the transformer thermal for building, vehicle, etc. Building a model based on [4] by chosen thermal, loss of life model, the load and ambient temperatures are the factors for long term planning to realistic conditions. A method and a model presented in [3] show model parameterization by the variability of operating conditions to achieve nonlinear loads. For insulating materials, the degradation model Arrhenius from the chemical kinetic law of Arrhenius presented in [5] to obtain some results in [6].

III. APPLICATIONS

Some approaches to system design and analysis selected mathematical models, such as the works illustrated in [1][2][7]. A reconstruction performance of the process analysis is shown in [1] by computation tomography. An acoustic emission technique is applied for ceramic material in [2]. A nondestructive control method to have the characterization put in for plastic composite is presented in [7]. The way to ensure stable system performance becomes a step in the control based mechanism such as the energy from the associated power demand [8]-[10]. An unmanned vehicle as drone in a complex system has been applied as a mathematical model approach and user centered methodology in Samanta method for the designed human factors based on the analysis and development of a specific Web based tool [12]. This methodology also involved the model free approach for different support stages. To achieve the innovation criticized for expected structure achieved on dynamic switching, the nonlinear differential equation expressions based on the form fracture cracks growth structure and data prediction tendency levels are used [13]. Figure 1 illustrates the idea of building smart models based on mathematical modeling for design and analysis of measurement and control system.

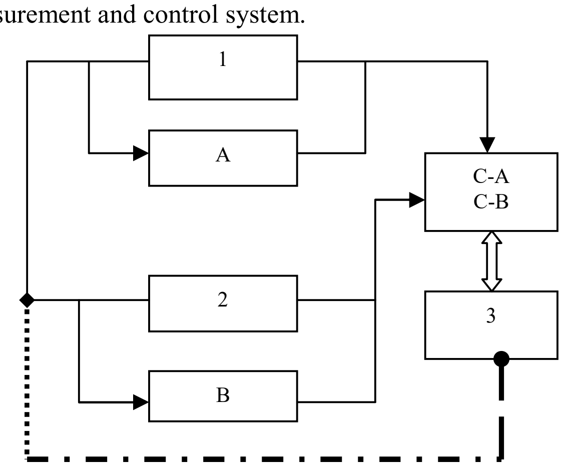

Figure 1. Example of the framework for developing a mathematic model.

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1- The model free approach as the tendency equation with nonlinear dynamic equation for control system.
2- The model based approach as the database theory with the foundation database.
C- Connection equations.
C-A – Following the design and control prediction, the strategy dynamic trajectory path is calculated.
C-B - The approximate equivalent analytical solution based on foundation database.
3- States transforming equation to the adaptive smart model.

IV. Conclusion

Mathematical models are composed of relationships and variables, which can be described as operators and parameters, or multi-functions, according to their structure. In a complex dynamic system, experimental measurements and control with linear and nonlinear characterizations leads to important advances. Such as the useful theories, are developed a suitable mathematical model in the process system to solve the mismatch model problem. Training and tuning to obtain the model evaluation provides a given mathematic model that described the system accurately. The idea of this paper is a new solution for developing a mathematical model based on smart model adaptive from regenerative parameters within the limit feasible structure by renews switch delay connections. The future work continues present more detail about the floating model in catastrophe theory to obtain natural dynamic nonlinear equations.

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