Empirical Heatmap Decomposition — A Fresh Look on Gaze Behavior

Saad Al-Baddai
Chair of Information Science, and CIML Group
Regensburg University
Regensburg, Germany
Email: saad.al-baddai@ur.de

Karema Al-Subari
and Elmar Lang
Institute of Biophysics
Regensburg University
Regensburg, Germany

Barbara Ströhl
and Bernd Ludwig
Chair of Information Science
Regensburg University
Regensburg, Germany

***Abstract*—The motivation for analyzing a person's gaze behavior stems from the eye-mind-hypothesis that correlates the observation of content by viewing it with its cognitive comprehension. A standard approach to quantitative analysis of gaze behavior is to analyze so called heatmaps generated by an eye tracker. This technical process is subject to noise. In order to compensate it, state-of-the-art eye tracking software smooths raw data using linear filters. In this paper, we present an alternative method based on interpolation. We provide empirical data that our method reproduces the actual gaze behavior more precisely. Furthermore, we introduce** *Empirical Heatmap Decomposition* **(EHD) to cluster eye movements into classes of similar frequency and amplitude. For evaluation, we present an analysis of gaze data that illustrates how EHD can uncover details in the observed gaze behavior that state-of-the-art heatmaps do not visualize.**

Keywords–Empirical Mode Decomposition; Eye tracking; Gaze Behavior; Sifting process

I. INTRODUCTION

Understanding the gaze behavior of users while they read multimodal documents containing text, as well as images, graphics, or sketches is of vital interest for anybody engaged in designing digital content, e.g., of web pages, electronic product catalogs, or search engine result pages. This understanding is of particular importance if the digital content is to be generated automatically and in real-time as necessary for, e.g., interactive web pages, digital guides for sightseeing or museums. For these applications it is crucial to ensure that all content is perceived by the users immediately and provides positive user experience and joy of use.

A. Fixations and Saccades

For empirical investigations of the user behavior in situations as sketched above, eye tracking is a state-of-the-art method for monitoring the gaze of users. The reading and comprehension of text has been investigated using eye tracking, e.g., by [1], [2]. The analysis of the gaze data allows to reconstruct in which way, in which order, with which velocity, and in which regions of the digital content users view the presented material. From these observations, it can be concluded how users process the observations cognitively. As reported by [3], eyes could be attracted by some part of a visual scene and fixate these parts — named *Areas of Focus* (AoFs) — for a longer period. The process of identifying AoFs, based on the duration of eye fixations, can help in advanced analyses of gaze behavior [4], among others in the analysis of so called distractors: Assuming that regular patterns for reading text do exist, how do images or even animated material (e.g., videos on a web page) influence and — in particular — disturb the comprehension of the presented text? This question was investigated — among others — by [5]. The authors report empirical results that pictures, and in particular those that are unrelated to the topic of the text, distract and slow down the standard reading behavior. A similar observation has been made by [6]. Unrelated material when included in the presented digital content provides negative user experience and leads to reading patterns that are significantly different from the standard ones. The authors even use this observation to construct an algorithm that predicts whether a user struggles with the displayed content due to the presence of distractors.

While these results are highly relevant for the purposes mentioned before, it has to be noted that most analyses of gaze are based on heatmaps for fixations and plots for saccades that visualize all movements at once. However, eye movements differ

- in their speed and
- in the fixation duration

between subsequent movements [1], [2], [6]. However, only few researchers develop mathematical models for a quantitative analysis of such types of gaze (e.g., [7]).

This fact is quite astonishing as in other research areas (e.g., in medical image processing), it is common practice to decompose raw data using transformations (e.g., Fourier or Wavelet) in order to identify different causes for observations in a data set. More recently, a more more data driven approach called Empirical Mode

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Decomposition (EMD) was pioneered by [8]. Based on an interpolation approach, EMD proved to reveal the characteristics of the textures in raw data sets more transparently and intuitively. As an elaboration of EMD, [9] applied Green's functions [10] for the interpolation problem (GiT-BEMD) in order to avoid artifacts and the immense computation load in the decomposition process that presented a major problem to the original version of EMD. GiT-BEMD works in real time and can therefore be used in real-time applications.

B. Gaze Decomposition

GiT-BEMD (as will be explained later) is interesting for gaze analysis as it provides an "all-in-one solution" for processing gaze data: While the standard EMD applies interpolation for separating sources in the signal, GiT-BEMD provides a substitute for linear filtering of raw gaze data. State-of-the-art commercial eye tracking software such as SMI's BeGaze applies Gaussian filters to reduce noise (see BeGaze Manual V 3.4, 2014, p. 204). However, this approach is problematic as the Gaussian kernel may blur away interesting data points. Recently, as an alternative, other filters, e.g., Savitzky-Golay filter [11], have been employed in eye-tracking area [12]–[14]. They are based on regression while GiT-BEMD interpolates data points and in this way tends to preserve original data as perfectly as possible.

C. Contribution

Even if we will provide data later that GiT-BEMD outperforms filters used in state-of-the-art software, adding the interpolation approach to smoothing gaze data is a side contribution of the present paper. Our main focus is on decomposing the smoothed signal. To the best of our knowledge, this is the first paper that investigates the value of decomposition methods to the analysis of gaze data. The algorithmic solution we developed is intended to be part of an open source tool box for analyzing eye tracking data. Similar other tool boxes have been implemented so far: *eSeeTrack* that focuses on the analysis of patterns of sequential gaze recordings [15], or $i$map [16] [17] which analyzes and compares eye movements under different conditions, iComp [18], ILAB [19], or GazeAlyze [20]. None of them however decomposes eye tracking data and therefore GiT-BEMD is an innovative extension to existing toolboxes.

To present our results, in Section II we introduce our empirical setup. Then we introduce the GiT-BEMD method in detail. In Section III we apply GT-BEMD on gaze data and provide first empirical results highlighting the performance of the new approach in Section IV. In Section V, we draw conclusions of the approach's impact on theoretical and practical issues of eye tracking for gaze and viewing analysis.

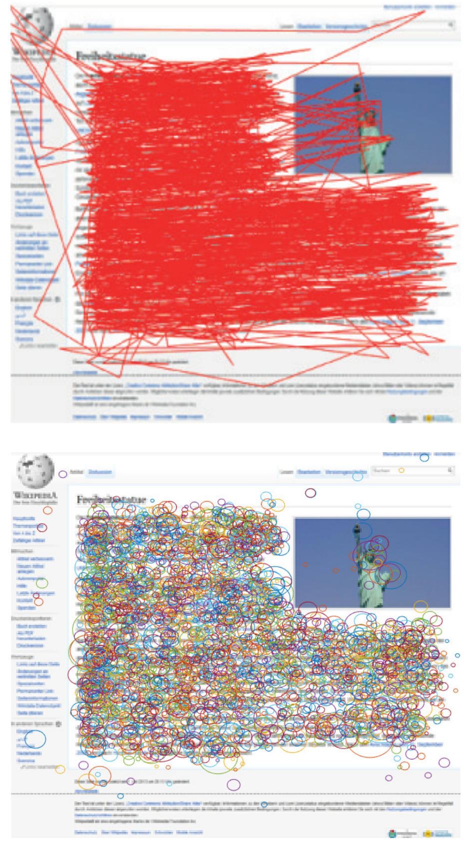

Figure 1. *Left*: Actual eye movement of participants during reading a Wikipedia page (scan paths observed by the eye tracker). *Right*: Fixations at the start and end point of a scan path (circle radius corresponds to the average fixation time)

II. Scenario

As [5], [6], we were interested in the effect of images on the reading behavior. An example of the digital content we investigated is the Wikipedia page in Figure 1. In particular, we wanted to know whether images that illustrate the textual information support the memorability of the digital content. According to the eye-mind-hypothesis [21], the gaze duration correlates with the cognitive processing time and depth of the perceived content. So it was surprising that participants mentioned details of an image on the Web page even if in the BeGaze heatmap no fixations on the image were registered (see Figure 4 left for an example).

In order to deeper analyze the collected gaze data and understand whether the described phenomenon was an artefact of the BeGaze's method to generate heatmaps, we applied four different methods on the same data set:

- The first method is the same as used by SMI's BeGaze analysis tool: the raw data is smoothed with a Gaussian filter.

- The second method is based on EMD: after Gaussian filtering, the data is decomposed using EMD (see [9]). It produces three heatmaps. Each of them groups fixations that are similar in terms of frequency and amplitude.
- The third method is GiT-BEMD without decomposition (and therefore analogous to the first method).
- The fourth method is GiT-BEMD including decomposition (analogous to the second method).

Our hypothesis is that EMD and GiT-BEMD reveal the gaze behavior more precisely than pure filtering. In order to measure the effect of the decomposition, we compute those areas of neighboring pixels in which all pixels have an average fixation time higher than to be expected under the assumption of a uniform probability distribution for fixations over the whole digital content. This is an unsupervised way to identify AoFs (area of focus). Using this notion, our hypothesis can be stated quantitatively. The decomposition based methods are more precise than pure filtering if they

- detect significantly more AoFs and
- in each detected AoF, identify more pixels fixated longer than chance (i.e., with probability higher than $1/$pixels in image)

than the pure filtering methods. Before presenting in Section IV the results using the unsupervised metric, in the next section we introduce the mathematical foundations of our approach and the empirical data we used to analyze the performance of GiT-BEMD to detect AoFs.

III. METHOD

We collected gaze data using an SMI RED 250mobile eye tracker at 250 Hz. In a controlled lab experiment, BA students of an introductory course to information science were asked to read a Wikipedia page presented to them for two minutes. They knew in advance that they had to answer fact retrieval questions about the page's content afterwards. We randomly chose 10 data sets from the described experiment to test the GiT-BEMD method.

A. Smoothing Heatmaps

The raw data produced by an eyetracker contains a list of subsequently fixated pixels during recording eye movements. Saccades can be calculated as difference vectors of subsequently fixated pixels. Due to noise caused by technical constraints of the eyetracker, for each fixated pixel in the raw data there is a certain chance that the observed fixation is a artifact. To account for this issue, raw data is smoothed. As already discussed above, the standard way to smoothing in commercial eyetracking software is Gaussian filtering that essentially is a linear operation on data windows to compute a weighted average from all pixels in the window. A major disadvantage of averaging is that isolated peaks in a window are blurred away. While this effect is even welcome in image processing, in the analysis of gaze data important fixation are eventually discarded leading to an erroneous reconstruction of the actual gaze behavior.

As an alternative, GiT-BEMD instead of averaging data locally, interpolates data globally. In this way, smoothing is achieved by spline interpolation. Smoothing then basically aims at finding the smoothest envelope surface passing through a grid of irregularly spaced extrema (i.e., the fixation count or duration of pixels from the raw data; see [9], [22], among others). The boundary value problem for a spline that interpolates fixated pixels can be stated with appropriate conditions for the spline's derivatives [23]. The resulting system of equations can then be solved using the family of Green's functions [10]. An envelope surface that fulfills all stated conditions can be expressed as

$$s(\mathbf{x}) = \sum_{n=1}^{N} w_n \Phi(\mathbf{x}, \mathbf{x}_n) \qquad (1)$$

In this formula, $\mathbf{x}_u$ denotes any point where the surface is unknown, $\mathbf{x}_n$ represents the n-th recorded fixation, $\Phi(\mathbf{x}, \mathbf{x}_n)$ is the Green's function and w_n is the respective weight in the envelope representation. Calculating $s(\mathbf{x})$ for all pixels finally generates a smoothed heatmap.

An envelope surface is constructed in two steps: the first step estimates the weights $\mathbf{w} = [w_1 \quad w_2 \dots w_P]$: The surface values $[s(\mathbf{x}_1), \dots, s(\mathbf{x}_N)]^T \equiv \mathbf{c} = [c_1, c_2, \dots, c_N]^T$ are known in a total of N pixels $\mathbf{x}_n$. Employing (1) for each of the known points $\mathbf{x}_n$, a linear system of N equations is obtained:

$$\mathbf{G}\mathbf{w} = \mathbf{c}$$

where n-th row of matrix $\mathbf{G}$ is the evaluation of the Green function $\Phi(\mathbf{x}_n, \mathbf{x}_m), m = 1, 2 \dots N$. We solve the equation for the weights $\mathbf{w} = \mathbf{G}^{-1}\mathbf{c}$.

Corresponding slopes s_m in directions $\hat{\mathbf{n}}_m$ can be obtained by evaluating the relations

$$s_m = \sum_{m=1}^{N} w_m \nabla \Phi(\mathbf{x}_m - \mathbf{x}_n) \cdot \hat{\mathbf{n}}_m \qquad m = 1, \dots, N.$$

The second step estimates the interpolating envelope surface: Using the weights $\mathbf{w}$, the value $s(\mathbf{x}_u) \equiv c_u$ of the envelope surface can be estimated at any point $\mathbf{x}_u$ by solving (1), which can be re-written as

$$c_u = \mathbf{w}^T \mathbf{\Phi}. \qquad (2)$$

The vector $\mathbf{\Phi} = [\Phi(\mathbf{x}, \mathbf{x}_1) \ \Phi(\mathbf{x}, \mathbf{x}_2) \ \dots \ \Phi(\mathbf{x}, \mathbf{x}_N)]^T$ contains the Green's function values of all distances between the N data constraints and the considered location.

GiT-BEMD implements the concept of smoothing

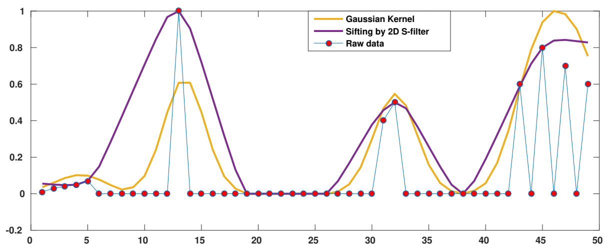

Figure 2. The effect of Gaussian smoothing and the S-filter in 1D. The y axis shows normalised fixation durations.

described above with one extension: As eyetracking data normally is sparse (i.e., for many pixels there is no fixation at all), adding white noise is required to decrease the computational load of building an envelope surface by interpolating from the extracted extrema. Without added noise, zero crossings would be considered as local minima and maxima that actually do not exist. After a surface has been constructed by applying the described concept, the artificially added noise can be ignored and does not disturb the further analysis.

Figure 2 illustrates the approach and shows a first comparison to Gaussian smoothing. It is easy to see that the GiT-BEMD aims to actually interpolate the raw data points while the Gaussian smoothing calculates sort of a "compromise". This is true for regions where a few pixels have many fixations as in the case of the leftmost extremum. Such situations are typical for eyetracking data and therefore we conclude that GiT-BEMD has the potential to outperform Gaussian smoothing.

For the two dimensional case, the same effect is illustrated in Figure 3. In the top left, one can see the (sparse) map of actual fixations while in the top right for some examples the fixation durations are plotted. Some extrema are given explicitly. The map in the bottom left illustrates how Gaussian smoothing influences the relevance of these extrema while on the right the map computed by GiT-BEMD is displayed. It is obvious that Gaussian smoothing tends to build large prominent regions (as the 70.2 ms) when the neighbors of a pixel show similarly high fixation durations. On the other hand, isolated pixels such as the 59.6 ms are "smoothed away" and considered as noise. GiT-BEMD however preserves large, but also detects small regions.

B. Decomposition of Heatmaps

Gaze data is produced by eye movements of different velocity and fixations (i.e., eye movements with velocity $v = 0$) of different durations. Therefore, it is reasonable to separate the gaze data into several components for movements of similar velocity (i.e., frequency) and analyze the durations (i.e., amplitudes). Such separations are common and well-known algorithms for their

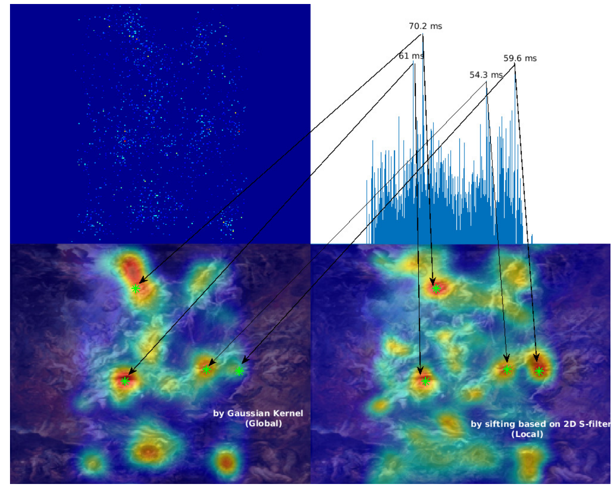

Figure 3. Effects of Gaussian smoothing and the GiT-BEMD. *Top left:* map of actual fixations. *Top right:* some fixation durations. *Bottom left:* Gaussian smoothing. *Bottom right:* GiT-BEMD

computation are Fourier or Wavelet transforms. However, as with Gaussian smoothing, Fourier or Wavelet transforms are classical approaches, but not suited optimally for eyetracking data. Instead, the GiT-BEMD decomposes data by constructing a series of surface envelopes for a given data set. In essence, EHD iteratively applies the GiT-BEMD smoothing explained above on the raw data without adding any artificial noise. Once a surface function has been constructed, it gets subtracted from the initial data, and the procedure is repeated on the pixelwise difference. In this way, EHD can decompose a heatmap $H(m,n)$ into several component heatmaps. This kind of decomposition can be computed effectively by using a so called sifting process [8], [24]–[27] as follows:

$r_{-1}(m,n) := H(m,n)$;
$k := 0$;
while $r_{k-1}(m,n) \neq 0$ *or* $r_{k-1}(m,n)$ *is not monotone* **do**
  $i := 0$;
  $I_{k,i}(m,n) := r_{k-1}(m,n)$;
  while $I_{k,i}(x)$ *has non-negligible local mean* **do**
    $U(m,n)$ is a cubic spline through all local maxima of $I_{k,i}(m,n)$;
    $L(m,n)$ is a cubic spline through all local minima of $I_{k,i}(m,n)$;
    $\text{mean}_{k,i}(m,n) := \frac{1}{2}\left(U(m,n) + L(m,n)\right)$;
    $I_{k,i}(m,n) := I_{k,i}(m,n) - \text{mean}_{k,i}(m,n)$;
    $i := i + 1$;
  end
  $\text{IMF}_k(m,n) := I_{k,i}(m,n)$;
  $r_k(m,n) := r_{k-1}(m,n) - \text{IMF}_k(m,n)$;
  $k := k + 1$;
end

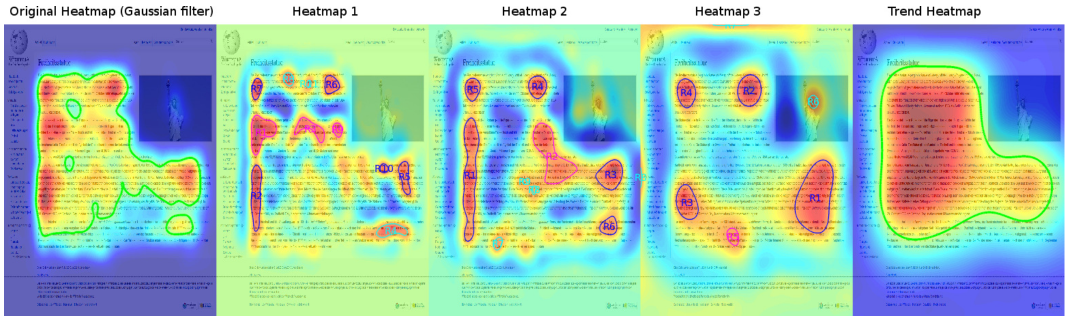

Figure 4. Heatmap (Gaussian smoothing) Components 1, 2, and 3 extracted by EHD. Boundary lines surround detected AoFs.

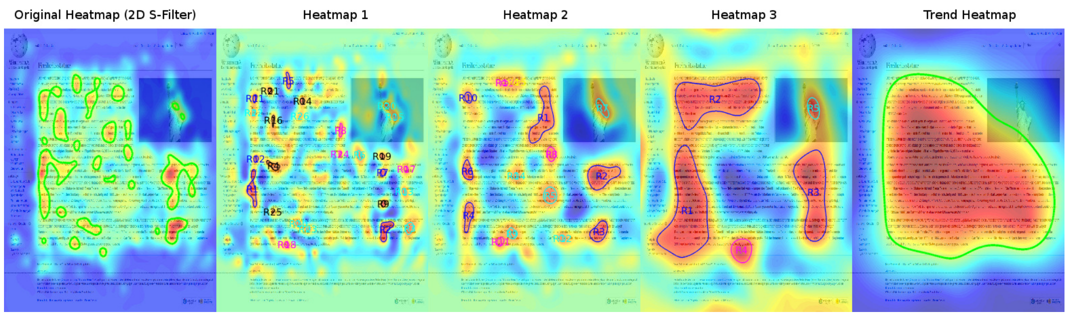

Figure 5. Heatmap (GiT-BEMD). Components 1, 2, and 3 extracted by EHD. Boundary lines surround detected AoFs.

The result of this process is the following decomposition of the original heat map $H(m,n)$ into the approximative decomposed heatmap $\hat{H}(m,n)$:

$$\hat{H}(m,n) = \sum_{j=1}^{k} \mathrm{IMF}_j(m,n) \qquad (3)$$

Note that $\hat{H}(m,n)$ depends on the parameters and stop criteria. For any $\hat{H}(m,n)$, in each extracted component $\mathrm{IMF}_j(m,n)$ different AoFs can be detected.

IV. EVALUATION

In this section, we will present an example which can illustrate the efficiency of the approach we are advocating in this paper. In our Wikipedia experiment, we are interested in analyzing the reading behavior. In particular, we want to know whether test persons fixate the image that is a distractor for reading the text. This issue has been investigated already in previous work [5], [6], as outlined in the introduction.

A. *Baseline: State-of-the-Art Eyetracking Software*

In Figure 1, one can observe that there are fixations on the statue of liberty, but not many. We use this data as ground truth in our evaluation. We first normalize all fixation durations to the interval $[0;1]$. While the average duration on the whole Wikipedia page is 0.55, the highest duration in the area of the statue of liberty is 0.1869 and the fixated pixels are quite isolated in comparison to the pixels in the text area. However, as we know from the post-test-questionnaires, all test persons remembered the statue on the page although they could not know about the image as the page was designed just for the experiment imitating a Wikipedia page. Therefore, according to the eye-mind hypothesis, the test persons must have seen the image. This is in line with the fixations in the raw data, but contradicted by the results of the BeGaze filtering. It becomes obvious that state-of-the-art heatmaps eventually provoke researchers to draw wrong conclusions.

B. *Smoothing by Interpolating raw data*

However, as can be seen in Figure 4 on the left, after Gaussian smoothing there is no AoF detected in the area of the statue as the highest fixation duration is lower than elsewhere and the surrounding pixels do not contribute fixations. So, the information is regarded as noise and discarded even if the area must have been fixated. GiT-BEMD's output is shown in Figure 5. Here, AoFs are detected (see the areas enclosed by a green line). In order to analyze this intuitive observation systematically, for each of the ten test persons, we computed the number of pixels fixated longer than by chance (i.e., their fixation duration is 0.5 or more) and how many of them had still a duration of 0.5 or more after filtering. In an ideal heatmap, the difference between both numbers should be 0. However, for Gaussian smoothing the difference is 19.7 while for GiT-BEMD it is 6.9 on average over all 10 trails. The difference between both averages is significant ($p < 0.01$): Even without applying EHD, GiT-BEMD better preserves fixation details while still reducing noise in the raw data.

C. Decomposition of Heatmaps

The benefit of the new EHD can be understood when considering the three components displayed in the middle of Figure 4. They are ordered from high to low velocity of the eye movements. Component 2 in the middle already highlights relevant fixations on the statue's head. However, they are not significant as other areas in this component (see the bounding lines around the named AoF). Component 3 then reveals a significant AoF on the head when only slow eye movements are considered. We conclude that EHD allows us to identify different for of gaze behavior caused by the nature of the observed object: Recognizing text requires many fast movements while identifying a known object by retrieving it from memory can only be achieved by slow movements scanning an AoF in detail.

EHD provides an even more exact analysis (see Figure 5): significant AoFs are found in all three components. The new approach is obviously more reliable in detecting as many actual fixations as possible. This claim can be backed by statistics: in component 1 (fast eye movements) the difference between actual and detected fixations is -1.7 for Gaussian smoothing and -30.0 for GiT-BEMD ($p < 0.001$). Independently of the filter used, EHD detects more fixations than could be expected assuming the same chance for each pixel. Note however, that in this component the fixation duration is very low. Consequently, pixels can be fixated longer than by chance relatively easily. This component therefore gives an overview of the parts of the page the test persons fixated while scanning the complete digital content superficially. For component 2 (average velocity of eye movement) the differences are 12.7 and -13.8. Gaussian smoothing again detects fewer fixations ($p < 0.001$) and now already misses fixations that are relevant actual fixations. Finally for component 3 the difference for Gaussian smoothing is 16.2, while for GiT-BEMD it is 5.9. Again, Gaussian smoothing misses more fixations, however in this component the difference is no longer significant ($p = 0.07687$). In this component, slow movements in particular during reading the text are observed. Saccades are short and therefore the Gaussian kernel removes fewer fixations as noisy compared to both other components. The comparison of the trend of both approaches reveals no significant differences (25.7 vs. 24.3 with $p = 0.7872$).

Tab. I summarizes the comparison of both approaches. The row *Original* provides evidence that the third method outperforms the first one as hypothesized earlier in Section II. The columns *Gaussian* and *GiT* indicate that the fourth method is superior to the second.

TABLE I. NUMBER OF DETECTED AoF PER COMPONENT

Heatmap	**p-value**	**Gaussian**	**GiT**	
Original	$p < 0.001$	7.70	20.50	$***$
Component 1	$p < 0.001$	29.10	57.40	$***$
Component 2	$p < 0.001$	14.7	41.2	$***$
Component 3	$p < 0.01$	11.20	21.50	$**$
Trend	$p > 0.05$	1.70	3.10	

V. Discussion

In this paper, we presented EHD as an approach to decompose gaze heatmaps according to the velocity of eye movements. We showed that the approach can be implemented effectively and even works in real-time — an interesting fact for pervasive computing. We validated the approach by analyzing gaze data from an experiment in which users had to read a simulated Wikipedia page and after the were tested which objects on the page they could remember. For a quantitative evaluation, we applied an unsupervised approach to detect AoFs. We chose this metric, as it does not require an expert to label AoFs. In our view, in this way we could avoid biases stemming from the expert's valuation of the digital content. Assuming instead that each pixel has the same chance to attract the focus of test persons, therefore enables a fairer evaluation of different methods to analyze gaze data. In future work, we will measure the effects of EHD by comparing the ability of test persons to recall certain details of the digital content and correlate their performance with the results produced by each analysis method. Assuming the eye-mind hypothesis to be valid, better recall performance o content must correlate statistically with a higher probability of the respective AoF to have been viewed.

The evaluation results show that EHD outperforms commercial state-of-the-art software packages for gaze behavior. It can identify AoFs which are not present in the heatmap if raw data is smoothed only, but not decomposed. Furthermore, EHD can distinguish different types of gaze behavior. Depending on the speed of the eye movement and the duration of fixations, EHD detects different AoFs. Therefore, EHD allows us to better understand how persons perceive content and in which chronological order they process it cognitively.

VI. Future Work

In the future, we will compare GiT-BEMD filtering to regression approaches such as the Savitzky-Golay filter [11] and continue the comparison with the state of the art in order to implement new toolboxes for gaze data analysis which we will make available to the public and use ourselves to investigate human reading behavior.

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