On The Synthesis of Adaptive Parameter-Dependent Output Feedback Controllers Through LMI-Based Optimization

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***Abstract*—This paper addresses the problem of designing adaptive output feedback controllers for stabilizing plants affected by parameters. A novel approach is proposed that allows one to design a fixed-order fixed-degree adaptive parameter-dependent output feedback controller by solving convex optimization problems with Linear Matrix Inequalities (LMIs). The proposed approach is based on the construction of a function that provides a stability margin of the closed-loop system depending on the controller. The conservatism of the proposed approach can be reduced by increasing the size of the LMIs.**

Keywords*–*Parameter-dependent; Adaptive Controller; Stability; LMI.

I. INTRODUCTION

Ensuring stability is of fundamental importance in engineering. Given an unstable plant, this is generally achieved by designing a stabilizing output feedback controller, i.e., a controller that elaborates the output of the plant in order to provide an input to the plant such that the so obtained closed-loop system is stable. The design of such a controller is based on the model of the plant, and several techniques can be used.

Real plants are often affected by parameters. These can happen due to various reasons. One reason is that such parameters can represent quantities that the user can modify, such as the gain of an amplifier, in order to achieve a different performance. Another reason is that such parameters can represent quantities that are unknown or subject to changes, such as the mass, resistance, temperature, etc.

Whenever the plant is affected by parameters, the output feedback controller should be able to ensure stability for all admissible values of the parameters. For this, the controller should be dependent on the parameters in general, i.e., should be able to adapt to different plants corresponding to different values of the parameters. Such a controller would be, hence, adaptive, in particular parameter-dependent.

Unfortunately, the design of stabilizing output feedback controllers for plants affected by parameters is a difficult problem. Indeed, several conditions do exist in the literature for establishing stability of systems affected by parameters, in particular conditions based on convex optimization constrained by LMIs; see for instance [1]–[5]. However, such conditions lead to nonconvex optimization problems whenever a controller is searched for, generally due to the product of the Lyapunov function and the controller that generates Bilinear Matrix Inequalities (BMIs); see for instance [6] [7]. Also, several non-LMI strategies are available for the design of stabilizing feedback controllers for plants that are not affected by parameters, however, for plants affected by parameters, such strategies could not be used due to the lack of analytical expressions (such as factorizations dependent on the parameters) or could lead to controllers of unacceptable order and degree; see for instance [8].

This paper addresses the problem of designing adaptive output feedback controllers for stabilizing plants affected by parameters. A novel approach is proposed that allows one to design a fixed-order fixed-degree adaptive parameter-dependent output feedback controller by solving convex optimization problems with LMIs. The proposed approach is based on the construction of a function that provides a stability margin of the closed-loop system depending on the controller. The conservatism of the proposed approach can be reduced by increasing the size of the LMIs. A numerical example illustrates the proposed approach. This paper extends our previous work [9].

The paper is organized as follows. Section II introduces the preliminaries. Section III describes the proposed approach. Section IV presents an illustrative examples. Lastly, Section V concludes the paper with some final remarks. This work is supported in part by the Research Grants Council of Hong Kong under Grant HKU711213E.

II. PRELIMINARIES

Notation: $\mathbb{N}, \mathbb{R}, \mathbb{C}$: sets of nonnegative integers, real numbers, and complex numbers; $\mathrm{re}(A)$: real part of matrix A; A': transpose of matrix A; $A \geq 0$: symmetric positive semidefinite matrix A; $\mathrm{spec}(A)$: set of eigenvalues of A.

Let us consider the plant

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & A(p)x(t) + B(p)u(t) \\ y(t) & = & C(p)x(t) \end{array} \right. \tag{1}$$

where $t \in \mathbb{R}$ is the time, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $y(t) \in \mathbb{R}^q$ is the output, $p \in \mathbb{R}^q$ is the vector of parameters, and the matrices $A(p)$, $B(p)$ and $C(p)$ are given matrix polynomials. It is supposed that

$$p \in \mathcal{P} \tag{2}$$

where $\mathcal{P}$ is the set of admissible parameters. The plant (1) is controlled by the parameter-dependent output feedback controller

$$\left\{ \begin{array}{rcl} \dot{\tilde{x}}(t) & = & \tilde{A}(p)\tilde{x}(t) + \tilde{B}(p)y(t) \\ u(t) & = & \tilde{C}(p)\tilde{x}(t) + \tilde{D}(p)y(t) \end{array} \right. \tag{3}$$

where $\tilde{x}(t) \in \mathbb{R}^{\tilde{n}}$ is the state of chosen order $\tilde{n} \in \mathbb{N}$, and the matrices $\tilde{A}(p)$, $\tilde{B}(p)$, $\tilde{C}(p)$ and $\tilde{D}(p)$ are matrix polynomials to determine of chosen degree $\tilde{d} \in \mathbb{N}$.

Problem 1. The problem addressed in this paper consists of determining a fixed-order fixed-degree output feedback controller (3) such that the closed-loop system (1)–(3) is asymptotically stable for all $p \in \mathcal{P}$. □

Let us observe that the plant (1) can represent the model of a nonlinear system that has been linearized for an equilibrium point of interest. In this case, the matrices in (1) are obtained by evaluating the derivatives of the vector field and of the output function of the nonlinear system at the equilibrium point and corresponding input.

III. PROPOSED APPROACH

The first step of the proposed approach is to express the closed-loop system (1)–(3) as

$$\dot{z}(t) = E(p, v)z(t) \tag{4}$$

where $z(t) \in \mathbb{R}^{n+\tilde{n}}$ is the state, $v \in \mathbb{R}^w$ is the vector of design variables in the controller, and $E(p, v)$ is a matrix polynomial in p and v. This can be simply done from (1)–(3) defining, for instance, $z(t) = (x(t)', \tilde{x}(t)')'$.

The second step of the proposed approach is to introduce a function, denoted by $\xi(v)$, that provides a stability margin of the closed-loop system depending on the controller. To this end, $\xi(v)$ could be defined under the constraint that $\xi(v) > 0$ if and only if the closed-loop system (1)–(3) is asymptotically stable for all $p \in \mathcal{P}$. Moreover, larger (respectively, smaller) values of $\xi(v)$ should correspond to more (respectively, less) stable systems. For instance, a possibility is given by

$$\xi(v) = - \sup_{p \in \mathcal{P}, \lambda \in \mathrm{spec}(E(p,v))} \mathrm{re}(\lambda). \tag{5}$$

Another possibility consists of exploiting the Hurwitz's determinants; see for instance [10]. Let us observe that one can introduce acceptable stability margins by requiring that $\xi(v)$ is greater than a specific positive value, whose definition depends on the problem requirements and on the choice of $\xi(v)$.

The third step of the proposed approach is to search for a polynomial $\zeta(v)$ that approximates $\xi(v)$ from below to a desired accuracy. This could be done by imposing

$$\begin{cases} \xi(v) & > & \zeta(v) \\ \xi(v) & < & \zeta(v) + \varepsilon \end{cases} \tag{6}$$

where $\varepsilon > 0$ is the desired accuracy.

The fourth step of the proposed approach is to search for a value of v that makes $\zeta(v)$ positive, i.e.,

$$v^* : \ \zeta(v^*) > 0. \tag{7}$$

In fact, from (6), it would follows that

$$\xi(v^*) > 0, \tag{8}$$

i.e., v^* solves Problem 1.

The search for $\zeta(v)$ satisfying (6) and the search for v^* satisfying (7) can be addressed through convex optimization problems with LMIs. Moreover, under some assumptions on the data, the conservatism of these procedures can be decreased by increasing the size of the LMIs involved.

IV. ILLUSTRATIVE EXAMPLE

For simplicity, let us consider the plant (1) with

$$\begin{cases} A(p) = \begin{pmatrix} -1 & 0 & 1-p \\ 0 & -1 & 1 \\ 1+p & 0 & 0 \end{pmatrix}, \ B(p) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \\ C(p) = \begin{pmatrix} 1 & p & 0 \end{pmatrix}, \ \mathcal{P} = [-2, 2]. \end{cases}$$

This plant is unstable depending on p. Indeed,

$$p = 0 \quad \Rightarrow \quad \mathrm{spec}(A(p)) = \{-1.618, -1, 0.618\}.$$

Also, it can be verified that there does not exist any stabilizing controller of order 0 and degree 0 for this plant.

Hence, we consider the problem to find an adaptive parameter-dependent output feedback controller (3) of order 0 and degree 1 that stabilizes the plant.

Let us express the matrix $\tilde{D}(p)$ as $\tilde{D}(p) = v_1 + v_2 p$, where $v_1, v_2 \in \mathbb{R}$ are the design variables. We search for a polynomial $\zeta(v)$ as described in Section III, finding

$$\zeta(v) = 0.139v_1^3 - 0.921v_1^2v_2 - 1.275v_1^2 - 0.605v_1v_2^2 \\ -0.667v_1v_2 - 1.482v_1 + 0.39v_2^3 - 2.833v_2^2 - 6.262v_2 - 4.52.$$

Hence, we search for v^* satisfying (7), finding

$$v^* = (-2, -1.660)'.$$

Therefore, we conclude that the controller (3) with $\tilde{D}(p) = -2 - 1.660p$ stabilizes the plant for all $p \in \mathcal{P}$.

V. CONCLUSION

A novel approach has been proposed for designing a fixed-order fixed-degree adaptive parameter-dependent output feedback controller for stabilizing plants affected by parameters. Future work will analyze its computational burden.

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