Image Restoration by Revised Bayesian-Based Iterative Method

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***Abstract*—A restoration method of the degraded images based on Bayesian-based iterative method is proposed. An iterative method is developed by treating images, point spread functions, and degraded images as probability measures and by applying Bayes' theorem. The method functions effectively in the presence of noise and is adaptable to computer operation.**

***Keywords*-point spread function, image restoration, Bayesian rule**

I. Introduction

To make a clear image from degraded image, many enhancement techniques have been developed until now. The main technique is to use one of sharpness filters that are based on derivatives of the image with respect to pixels [1]-[2]. These methods used one or more masks to approximate a derivative operation. Although they could enhance the image, they would enlarge also noises.

The present method for enhancement of images is to use the Bayesian rule. This method was first proposed by [3]. The Bayesian rule reflects optimal estimation in a sense to minimize the cost function under noisy observation and an iterative algorithm was proposed to find the optimal solution. The algorithm include two parts, the first one is to estimate a point spread function (PSF) from the estimated image and the second one is to estimate the original image by using the estimated PSF. Thus, this algorithm might be optimal when the observed image is similar to the original image, that is, in case of a high S/N ratio. Therefore, the results will depend on the initial guesses of PSF.

In this paper, we propose a new algorithm to speed up the convergence and find better restoration compared with the results of [3]. The idea is to select the observed image as an initial guess of the restored image and every iteration we use the observation image instead of an estimated image when we estimate the PSF.

First, we will show the principle of the Bayesian-based iterative method proposed by Richardson[3]. Then we will state the proposed method. After that the simulation results will be illustrated to show the effectiveness of the present method.

II. Principle of Image Restoration

In image enhancement, the ultimate goal of restoration techniques is to improve a given image in some sense. Restoration is a process that attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon. As shown in Fig. 1, the degradation process may be modeled as an operator $\mathbf{H}$ in case of noiseless situation. It operates on an input image $f(x,y)$ to produce a degraded image $g(x,y)$. For the sake of simplicity, we denote $f(x,y)$ by $f(x)$, $g(x,y)$ by $g(x)$, $h(x,y)$ by $h(x)$, etc. In equation form, we have

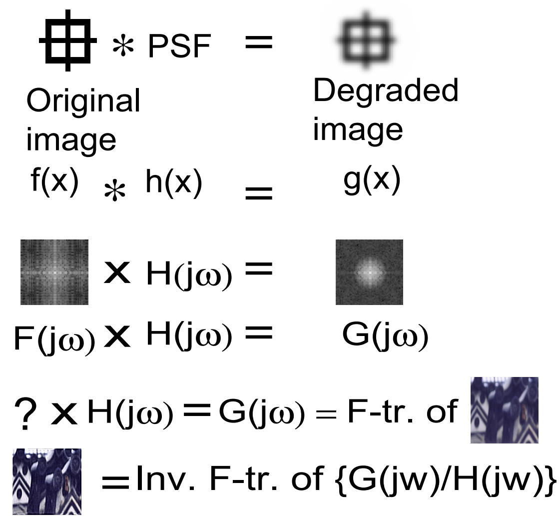

Fig. 1. The restoration principle.

$$g(x) = \mathbf{H}f(x) = h * f(x) = \sum_{y=-\infty}^{\infty} h(x-y)f(y). \quad (1)$$

where $h(x)$ is an impulse response and * denotes the operation of convolution.

Based on the convolution theorem, the frequency domain representation of Eq.(1) becomes

$$G(j\omega) = H(j\omega)F(j\omega) \quad (2)$$

where $G(j\omega)$, $H(j\omega)$, and $F(j\omega)$ are Fourier transforms of $g(x)$, $h(x-y)$, and $f(y)$, respectively. As shown in Fig.1,Given $G(j\omega)$ and some knowledge about $H(j\omega)$, the objective of restoration techniques is to recover $F(j\omega)$ which means to recover the original image $f(x)$ via the inverse Fourier transform.

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III. Richardson's Iterative Method

We will review the iterative method by Richardson [3] in this section. Given the degraded image g, the point spread function h and the original image f are estimated based on Bayes' theorem. It will be effective to estimate the original image f from the observed image g. It was assumed that g , h , and f are discrete and are not necessarily normalized. The numerical values of g , h , and f are considered as measures of the frequency of the occurrence of them at those points. h is usually in normalized form. Energy of f originating at a point is distributed as g at points according to the energy indicated by h. Thus, g represents the resulting sums of the energy of f originating at all points.

In the notation of this problem the usual form of the Bayes' theorem is stated as the conditional probability of f, given g. It was assumed that the degraded image g was of the form $g = h * f$, where * denotes the operation of convolution such that

$$g(x) = h * f(x) = \sum_{y=-\infty}^{\infty} h(x-y)f(y). \tag{3}$$

Note that f and g are intensity functions of original image and observed image, respectively and h is the weighting function depending on image measurement devices. We assume that the input image and the weighting function which means the restoration mechanism are unknown. The values of f, g, and h are not limited within [0,1]. We normalize and denote them by f', g', and h'. Thus, we have

$$f'(x) = \frac{f(x)}{\sum_{x=-\infty}^{\infty} f(x)} = \frac{f(x)}{F} \tag{4}$$

$$g'(x) = \frac{g(x)}{\sum_{x=-\infty}^{\infty} g(x)} = \frac{g(x)}{G} \tag{5}$$

$$h'(x) = \frac{h(x)}{\sum_{x=-\infty}^{\infty} h(x) == \frac{h(x)}{H}} \tag{6}$$

where F,G, and H could be equal since the restoration process is conservative. Note that f, g, and h are nonnegative and the total sums are equal to one. Thus, we could regard them as probability measures and $f'(x_1)$ as the probability measure of the original image $f(x_1)$ at x_1. This means that the possibility of the existing intensity of the original image $f(x_1)$ at x_1. Similarly, $g'(x_2)$ and $h'(x_1)$ mean the possibility of the existing intensity of the observed image $g'(x_2)$ at x_2 and the possibility of the transition weight from the input image $f(x_1)$ at x_1 to the output image $g(x_2)$ at x_2. Therefore, we have

$$P(g(x_2)|f(x_1)) = P(h(x_2 - x_1)) = h'(x_2 - x_1). \tag{7}$$

The above relation can be derived by using the following relation.

$$\begin{aligned} P(g(x_2), f(x_1)) &= P(g(x_2)|f(x_1))P(f(x_1)) \\ &= P(h * f(x_2), f(x_1)) \\ &= P(h(x_2 - x_1), f(x_1)) \\ &= P(h(x_2 - x_1))P(f(x_1)) \end{aligned}$$

where we have used independence assumption between original image and restoration mechanism.

Using the Bayes' theorem we have

$$\begin{aligned} P(f(x)|g(x_2)) &= \frac{P(g(x_2)|f(x))P(f(x))}{\sum_{x_1=-\infty}^{\infty} P(g(x_2)|f(x_1))P(f(x_1))} \\ &= \frac{f'(x)h'(x_2 - x)}{\sum_{x_1=-\infty}^{\infty} f'(x_1)h'(x_2 - x_1)}. \end{aligned} \tag{8}$$

If we multiply the both sides of Eq.(8) by $P(g(x_2)) = g'(x_2)$ and take the summation with respect to x_2, we get

$$\begin{aligned} P(f(x)) &= f'(x) \\ &= f'(x) \sum_{x_2=-\infty}^{\infty} \frac{h'(x_2 - x)g'(x_2)}{\sum_{x_1=-\infty}^{\infty} f'(x_1)h'(x_2 - x_1)}. \end{aligned} \tag{9}$$

Considering $F = G = H$ and multiplying them both sides of the above equation, we have

$$f(x) = f(x) \sum_{x_2=-\infty}^{\infty} \frac{h(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} f(x_1)h(x_2 - x_1)}. \tag{10}$$

Using the above equation, Richardson[3] proposed the following recurrence procedure to find the original image $f(x)$.

$$\begin{aligned} f_{n+1}(x) &= f_n(x) \sum_{x_2=-\infty}^{\infty} \frac{h_n(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} h_n(x_2 - x_1)f_n(x_1)} \\ n &= 0, 1, 2, \ldots. \end{aligned} \tag{11}$$

In order to derive the recursive equation of the PSF function $h(x)$, we will set $x_3 = x_2 - x$. Then from Eq.(8) we have

$$P(f(x_2 - x_3)|g(x_2)) = \frac{f'(x_2 - x_3)h'(x_3)}{\sum_{x_1=-\infty}^{\infty} f'(x_1)h'(x_2 - x_1)}. \tag{12}$$

Multiplying both sides of the above equation by $P(g(x_2)) = g'(x_2)$, we have

$$\begin{aligned} & P(f(x_2 - x_3)|g(x_2))P(g(x_2)) \\ &= g'(x_2) \frac{f'(x_2 - x_3)h'(x_3)}{\sum_{x_1=-\infty}^{\infty} f'(x_1)h'(x_2 - x_1)}. \end{aligned} \tag{13}$$

Using the Bayes' rule, we have

$$\begin{aligned} & P(f(x_2 - x_3)|g(x_2))P(g(x_2)) \\ & = P(f(x_2 - x_3), g(x_2)) \\ & = P(g(x_2)|f(x_2 - x_3))P(f(x_2 - x_3)) \\ & = h'(x_3)f'(x_2 - x_3). \end{aligned} \quad (14)$$

From Eqs.(14)and (14), we have

$$\begin{aligned} & h'(x_3)f'(x_2 - x_3) \\ & = g'(x_2)\frac{f'(x_2 - x_3)h'(x_3)}{\sum_{x_1=-\infty}^{\infty} f'(x_1)h'(x_2 - x_1)}. \end{aligned} \quad (15)$$

Taking the summation of both sides of Eq.(15) with respect to x_2, using the relation of Eqs.(4), (5), and (6), and noting that

$$\sum_{x_2=-\infty}^{\infty} f'(x_2 - x_3) = 1, \quad (16)$$

we have the following relation.

$$h(x) = h(x) \sum_{x_2=-\infty}^{\infty} \frac{f(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} f(x_1)h(x_2 - x_1)}. \quad (17)$$

Thus, using the same recursive relation as Eq.(11), we have

$$h_{m+1}(x) = h_m(x) \sum_{x_2=-\infty}^{\infty} \frac{f_n(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} f_n(x_1)h_m(x_2 - x_1)}. \quad (18)$$

In order to check the convergences of the recursive relations given by Eqs.(11) and (18), the following relations are used.

$$\sum_{x_2=-\infty}^{\infty} \frac{h(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} h(x_2 - x_1)f_n(x_1)} = 1, \quad (19)$$

$$\sum_{x_2=-\infty}^{\infty} \frac{f(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} f(x_1)h(x_2 - x_1)} = 1. \quad (20)$$

(21)

Thus, we use the following criteria to stop the iterations.

$$1 - \epsilon < \sum_{x_2=-\infty}^{\infty} \frac{h_m(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} h_m(x_2 - x_1)f_n(x_1)} < 1 - \epsilon, \quad (22)$$

$$1 - \epsilon < \sum_{x_2=-\infty}^{\infty} \frac{f_n(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} f_n(x_1)h_m(x_2 - x_1)} < 1 - \epsilon. \quad (23)$$

Using the above relations, Richardson has proposed the following iterative algorithm(Richardson's Iterative Method).

Step 1. Set $n = 0, m = 0$, the initial guesses of $h_0(x)$, and $f_0(x)$, and small positive number ϵ.

Step 2. Solve the following equations:

$$f_{n+1}(x) = f_n(x) \sum_{x_2=-\infty}^{\infty} \frac{h_m(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} h_m(x_2 - x_1)f_n(x_1)} \quad (24)$$

$$h_{m+1}(x) = h_m(x) \sum_{x_2=-\infty}^{\infty} \frac{f_n(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} f_n(x_1)h_m(x_2 - x_1)}. \quad (25)$$

Step 3. If the following inequalities hold

$$\left| \sum_{x_2=-\infty}^{\infty} \frac{h_m(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} h_m(x_2 - x_1)f_n(x_1)} \right| < 1 - \epsilon \quad (26)$$

and

$$\left| \sum_{x_2=-\infty}^{\infty} \frac{f_n(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} f_n(x_1)h_m(x_2 - x_1)} \right| < 1 - \epsilon, \quad (27)$$

then stop, otherwise $n \leftarrow n + 1, m \leftarrow m + 1$ go to Step 2.

The above iteration has no proof of convergence that means the results obtained by the above iteration may result in the good results or may not.

IV. Proposed Algorithm

In order to get the better results compared with Richardson's algorithm, we consider a new method based on the property of degraded images such that the blurred images are similar to the original images. In the Richardson's algorithm, if the bad estimation of $h_m(x)$ at the beginning stage, corresponding recovered images would become different images. After obtaining the bad estimation of recovered images, worse estimation of the point spread function. As a result, the iteration will produce worse and worse estimation of the point spread function and recovered images. Assuming the degraded images are not so far from the original images, we use the blurred image to estimate the point spread function $h_m(x)$ instead of the recovered image that is the estimated image. Therefore, we have proposed the following algorithm:

Step 1. Set $n = 0, m = 0$, small positive number ϵ, and $f_0(x) = g(x)$. Set the initial guesses of $h_0(x)$.

Step 2. Solve the following equations:

$$f_{n+1}(x) = f_n(x) \sum_{x_2=-\infty}^{\infty} \frac{h_m(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} h_m(x_2 - x_1)f_n(x_1)} \quad (28)$$

$$h_{m+1}(x) = h_m(x) \sum_{x_2=-\infty}^{\infty} \frac{g(x_2 - x)g(x_2)}{\sum_{x_1=-\infty}^{\infty} g(x_1)h_m(x_2 - x_1)}. \quad (29)$$

Step 3. If the following inequalities hold

$$\left|\sum_{x_2=-\infty}^{\infty}\frac{h_m(x_2-x)g(x_2)}{\sum_{x_1=-\infty}^{\infty}h_m(x_2-x_1)f_n(x_1)}\right|<1-\epsilon \quad (30)$$

and

$$\left|\sum_{x_2=-\infty}^{\infty}\frac{f_n(x_2-x)g(x_2)}{\sum_{x_1=-\infty}^{\infty}f_n(x_1)h_m(x_2-x_1)}\right|<1-\epsilon, \quad (31)$$

then stop, otherwise $n \leftarrow n+1$ and go to Step 2.

V. Variation Using the Reverse Function

We consider more simple form of the proposed algorithm. We set the dominator of Eq.(11) by

$$L_{nm}(x_2)=\sum_{x_1=-\infty}^{\infty}h_m(x_2-x_1)f_n(x_1). \quad (32)$$

It is the convolution sum between the original image $f_n(x_1)$ and the point spread function $h_m(x_2-x_1)$. Therefore, if $L_{nm}(x_2)=g(x_2)$, then the estimated image $f_n(x_1)$ becomes the true original image. Furthermore, we have

$$f_{n+1}(x)=f_n(x)\sum_{x_2=-\infty}^{\infty}\frac{h_m(x_2-x)g(x_2)}{L_{nm}(x_2)}. \quad (33)$$

We define $r_{nm}(x_2)$ by

$$r_{nm}(x_2)=\frac{g(x_2)}{L_{nm}(x_2)} \quad (34)$$

which means the ratio between the observed degraded image and the degraded image obtained by using the estimated point spread function. Then Eq.(33) becomes

$$f_{n+1}(x)=f_n(x)\sum_{x_2=-\infty}^{\infty}h_m(x_2-x)r_{nm}(x_2). \quad (35)$$

We define the reverse function of $k(x)$ by $k(x)=h(-x)$. Then Eq.(**??**) becomes

$$f_{n+1}(x)=f_n(x)\sum_{x_2=-\infty}^{\infty}\frac{k_m(x-x_2)r_{nm}(x_2)}{.} \quad (36)$$

If we represent the convolution sum by Fourier transform, we have

$$\begin{aligned} f_{n+1}(x) &= f_n(x)FT^{-1}(FT(k_n(x-x_2))FT(r_{nm}(x_2))) \\ &= f_n(x)FT^{-1}(K_n(j\omega)R_{nm}(j\omega)) \quad (37) \end{aligned}$$

where FT and FT^{-1} denote the Fourier transform and inverse Fourier transform. Since $K_m(j\omega)=H_m(-j\omega)$, we could save the computational time by half.

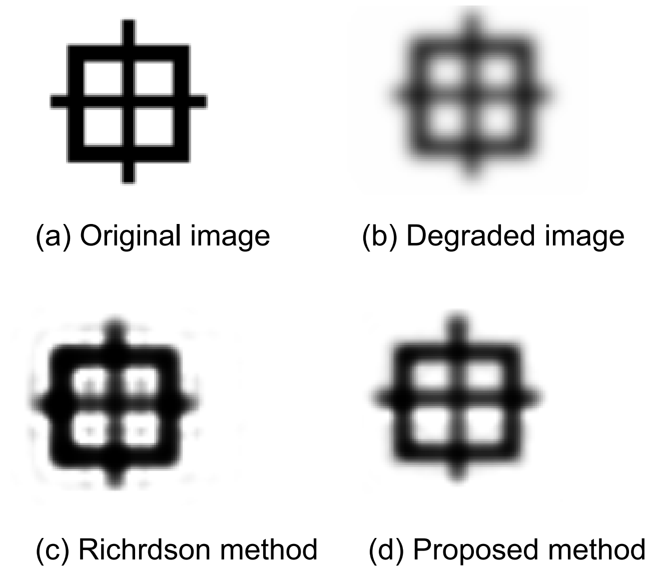

Fig. 2. The comparison for Example 1.

VI. Simulation Results

In order to show the effectiveness of the proposed method, we will consider gray image (Example 1) and color image(Example 2). The computer specification used here is shown in TABLE I. In Example 1 the gray image of 64×64 was made using Photoshop. The color image of512×512 of Example 2 was cropped from the standard sample data of high-resolution color images [4]. The degraded images are made by using Gaussian filters with the standard deviation $\sigma=2.0$. We used the stopping parameters of m and n when maximum iteration number k is given. In these simulations, we changed those parameters (m,n,k) in three cases, that is, (10,100,10), (5,100,10), and (5,5,100). TABLE II shows simulation results for three cases with PSNR (Peak Signal-to-Noise Ratio). In Fig. 2 shows the simulation results of the gray image with (10,100,10). From this Fig. 2 the proposed method restored more clear image compared with the results by Richardson's method [3]. In Figs.3-6 we show the simulation results of Example 2. The original image is shown in Fig. 3 and the degraded image is shown in Fig. 4. The restored images by [3] and the proposed method are shown in Fig. 5and Fig. 6.

TABLE I
Computing environment

OS	Windows XP
CPU	AMD Athlon(tm)64 X2 Dual Core
Memory	2GB

TABLE II
PSNR between original image and restored image

Threshold value(m,n,k)	Degraded image	Richardson	Authors
(10,100,10)	14.5	15.7	16.6
(5,100,10)	14.5	16.5	16.0
(5,5,100	14.5	9.2	16.2

Fig. 3. Original image for Example 2.

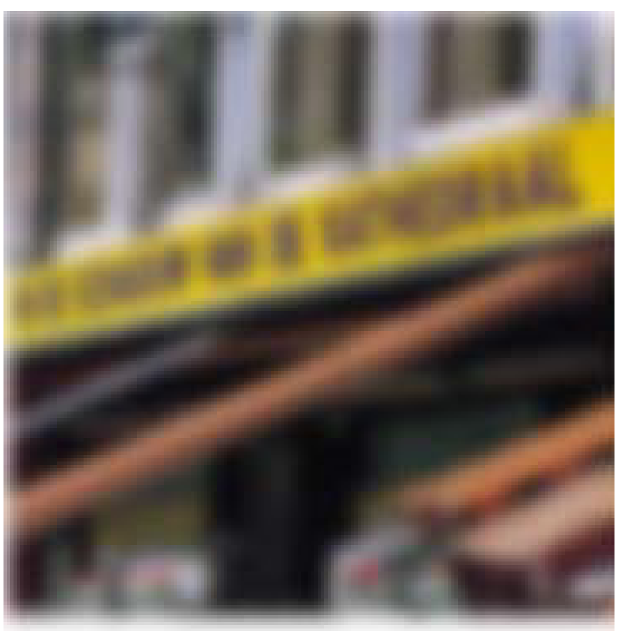

Fig. 4. Degraded image for Example 2.

Fig. 5. Richardson’s method for Example 2.

Fig. 6. Proposed method for Example 2.

VII. Conclusions

In this paper, a method of restoration of the degraded images by using Bayesian-based iterative method. The simulation results showed that the proposed method could restore the degraded images more clearly compared with the Richardson's method while the threshold values of (n, m, k) must be determined by trial and error. Furthermore, the computation load has been decreased by half by introducing the ratio between the observed degraded image and the degraded image.

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