Reliability Assessment Based on Modelling and Simulations

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***Abstract*—Reliability assessments are generally carried out via tests. When a complex system has no opportunity to experience a system-level test, Modelling & Simulations (M&S) will be the only approach to assess the system reliability, and the Uncertainty Quantification (UQ) for M&S becomes one of the most important techniques. With a method named Quantification of Margin and Uncertainty (QMU), reliability could be assessed as to make decisions whether the indices have reached the demands. With our newly proposed UQ method, QMU could be effectively actualized based on M&S. With an example of reliability assessment for a stockpiled product, the main ideas and the implementation of QMU are demonstrated.**

Keywords- modelling & simulation;uncertainty quantification; calibration; verification & validation; reliability assessment.

I. INTRODUCTION

Reliability assessment can be regarded as a decision making process in which a decision should be reached whether the reliability of products meets the demands. Traditionally, such decision making is built on tests, which have a solid base of epistemology because of the natural creditability about test data. Owing to the impacts from politics, economy and security, etc., some complicated products can not be subjected to system-level test, so it is necessary to predict their performances by scientific computing [1]. After the moratorium on the testing of nuclear weapons in 1992, National Nuclear Security Administration (NNSA) of the United States developed a Stockpile Stewardship Program (SSP) to assess the nuclear stockpile without nuclear testing, and a new method named QMU was developed for SSP [2]. For QMU implementation, there is an intense divarication how to define the Margin (M) and Uncertainty (U). In 2009, we suggested to choose probability as QMU metric when failure can not be absolutely eliminated due to aleatory uncertainties in products, by which QMU gained an unified decision criterion as confidence factor C=M/U is greater than unity or not [3]. In 2011, Sentz et al. proposed a similar method when failures are inevitable owing to the unbounded stochastic variables [4]. After that, uncertainty quantification for M&S was left to be the main bottleneck in QMU application.

Many literature works exist on UQ of M&S, but the existing methods are roughly falling into two types. One is by comparing the simulation results and the test data [5], the other is by propagating uncertainties from the input variables to the simulation results [6][7][8]. Unfortunately, when system-level test is forbidden, the approach with comparison can not be used. Considering the probability that the systematic deviations may be neglected, and that the uncertainties may be overestimated especially when too many uncertain inputs exist, the propagated uncertainties can not be directly considered as the uncertainties of M&S used for prediction. In order to resolve this problem, we propose a new method to combine the information originating from comparison and propagation [9]. This method has the advantage to observe the true-value-covered and uncertainty-minimized principles of UQ, by which the uncertainties of the predictions made by M&S can be effectively quantified.

The new method includes two important steps, one is to extrapolate the uncertainties obtained by comparison from validation domain to application domain, the other is to fuse the uncertainty information originating from comparison and propagation. This paper aims to display the ideas of QMU, which is supported by the new methods of UQ.

In Section 2, a proposition of reliability assessment is described. Section 3 shows how to predict the performance by M&S. Section 4 shows the UQ of the prediction results. Reliability assessment is fulfilled by QMU in Section 5, and a conclusion is given in Section 6.

II. DESCRIPTION OF PROPOSITION

The example we discuss is about reliability assessment for a product which has five parts, namely priming-device, high-explosive, special-metal, frame-material and energetic-material. The lowest demands are $Y^{demand} = 500MJ$ for yield-energy Y and $R^{demand} = 0.99$ for priming-reliability R. Failures may be caused by random errors in the manufacturing process or by aging during stockpile. Available system-level test data are of stock time for 0 year, 3 years, 5 years and 10 years. Test data are unavailable for the products that are stocked for more than 10 years. The energetic-material is designed to be replaced every 5 years for its severe aging rate. We must decide on the proposition that both R and Y are greater than their lowest demands when the product sits in the stockpile for 15 years.

The priming-reliability R is obtained as follows

$$\begin{cases} R = \int_0^{\infty} I\left(\tau, \tau^{upper}\right) f_{\tau}(\tau) d\tau \\ I\left(\tau, \tau^{upper}\right) = 1, \quad \tau \le \tau^{upper} \\ I\left(\tau, \tau^{upper}\right) = 0, \quad \tau > \tau^{upper} \end{cases} . \tag{1}$$

 ISBN: 978-1-61208-506-7

where $f_\tau(\tau)$ is the probability density of characteristic time τ which has an average ${}^{\tau}\mu$. τ^{upper} is the upper limit of τ to guarantee successful priming. As a constant quantity lying on the configuration of products, τ^{upper} remains unchanged during stockpile and its best estimation is $\tau^{upper} = 1.2\mu s$ with an epistemic uncertainty ${}^{\tau^{upper}}U^{Modeling}_{whole\text{-}life} = 0.05\mu s$.

The epistemic uncertainties can be classified as "known unknowns" and "unknown unknowns", and the requirement to execute the decision making is no "unknown unknowns" existing in M&S. Although the material behaviors in the products stocked 15 years have a further change compared to that stocked less than or equal to 10 years, it can not bring essential changes in the detonation process and there is no "unknown unknowns" in M&S. So it is feasible to assess the reliability for the stock time of 15 years by M&S.

III. Prediction by Modelling & Simulations

M&S was first calibrated using the available test data, from which the computing parameters and the physics models including aging models of materials are determined. Then, in keeping with 0 year stock time and the medium values of machining tolerances, we have built a baseline entity model, for which we get the average values via M&S that $\tau^{M\&S}_{0year} = 0.364\mu s$ and $Y^{M\&S}_{0year} = 605.0MJ$. According to (1) and M&S results of the entity models that sampled from machining tolerances, we have $R^{M\&S}_{0year} = 0.99999996$.

Table 1 shows the aging effects of each part after 15 years stocked, from which we have the predictions as $\tau^{M\&S}_{15years} = 0.364 + 0.16 = 0.524(\mu s)$, $R^{M\&S}_{15years}$ =0.999995, and $Y^{M\&S}_{15years} = 605.0 - 55.0 = 550.0(MJ)$ by M&S and sampling.

IV. Quantification of Uncertainties

We have some repeated test data for each stock time such as 0 year, 3 years, 5 years, and 10 years. According to the formula in [9] as $U^{M\&S} = \left|y^{M\&S} - \bar{y}^{test}\right| + t_{(1-\beta)/2,\nu} \cdot s/\sqrt{n}$, and the confidence β=0.95, uncertainties of M&S in validation domain can be quantified by comparison as $U_\tau(t)$=0.123μs, 0.128μs, 0.132μs, 0.143μs and $U_Y(t)$ =9.05MJ, 9.53MJ, 10.20MJ, 12.17MJ for each stock time.

Through optimal square approximations, we have got the relationships between M&S-uncertainties and stock time such as $U_\tau(t) = 0.123 + 0.00158t + 0.0000427t^2$ and $U_Y(t) = 9.033 + 0.132t + 0.0183t^2$, from which the uncertainties corresponding to 15 years are extrapolated as ${}^{\tau}U^{extrapolaton}_{15years} = U_\tau(15) = 0.156(\mu s)$ and ${}^{Y}U^{extrapolaton}_{15years} = U_Y(15) = 15.13(MJ)$.

As stock time increases from 10 years to 15 years, there will be additional epistemic uncertainties in the physics models. From ${}^{\tau}U^{\Delta}_{15years} = \sum_{i=1}^{N} \left|\partial\tau / \partial\xi_i\right| \left({}^{\xi_i}U^{\Delta}_{15years}\right)$ and ${}^{Y}U^{\Delta}_{15years} = \sum_{i=1}^{N} \left|\partial Y / \partial\xi_i\right| \left({}^{\xi_i}U^{\Delta}_{15years}\right)$, we have ${}^{\tau}U^{\Delta}_{15years} = 0.05\mu s$ and ${}^{Y}U^{\Delta}_{15years} = 20.0MJ$ of additional propagated uncertainties in Table 2.

The UQ method proposed in [9] shows the total uncertainty of prediction should be quantified by fusing the information corresponding to comparison and propagation as $U^{M\&S}_{15years} = U^{extrapolaton}_{15years} + U^{\Delta}_{15years}$. So we have ${}^{\tau}U^{M\&S}_{15years} = 0.156 + 0.05 = 0.206(\mu s)$ and ${}^{Y}U^{M\&S}_{15years} = 15.13 + 20.0 = 35.13(MJ)$.

Using (1), the uncertainty of priming-reliability can be obtained as ${}^{R}U^{M\&S}_{15years} = 0.003$ just by propagation of the input epistemic uncertainties ${}^{\tau_\mu}U^{M\&S}_{15years} = {}^{\tau}U^{M\&S}_{15years} = 0.206\mu s$ and ${}^{\tau^{upper}}U^{Modeling}_{whole\text{-}life} = 0.05\mu s$.

V. Reliability Assessment with QMU

The margins of priming-reliability and yield-energy are ${}^{R}M_{15years} = R^{M\&S}_{15years} - R^{demand} = 9.995 \times 10^{-3}$ and ${}^{Y}M_{15years} = Y^{M\&S}_{15years} - Y^{demand} = 50.0(MJ)$ with their uncertainties ${}^{R}U^{margin}_{15years} = {}^{R}U^{M\&S}_{15years} + {}^{R}U^{demand} = 0.003 + 0 = 3.0 \times 10^{-3}$ and ${}^{Y}U^{margin\,Y}_{15years} = U^{M\&S}_{15years} + {}^{Y}U^{demand} = 35.13 + 0 = 35.13(MJ)$.

Finally, we have ${}^{R}C_{15years} = {}^{R}M_{15years} / {}^{R}U_{15years} = 9.995 \times 10^{-3} / 3.0 \times 10^{-3} = 3.33$ and ${}^{Y}C_{15years} = {}^{Y}M_{15years} / {}^{Y}U_{15years} = 50.0 / 35.13 = 1.42$, from which we affirm that the demands to products could be satisfied as the confidence factors $C = M/U$ are all greater than unity.

VI. Conclusion

The QMU methods proposed here fit engineering systems in which aleatory and epistemic uncertainties might coexist. Supported by the new methods of UQ for M&S prediction, reliability assessment with unified criterion could be fulfilled based on M&S even when the new system-level test is unavailable. These methods have already been applied in some engineering systems.

Acknowledgment

This research is supported by the National Nature Science Foundation (Grant No. 11371066, 11272064) of China.

TABLE I. Aging Effects to System Performances

Parts or system	$\Delta\tau(\mu s)$	$\Delta Y(MJ)$
Priming-device	0.03	0.0
High-explosive	0.05	-15.0
Special-metal	0.01	-2.5
Frame-material	0.02	-5.5
Energetic-material	0.05	-32.0
Whole system	0.16	-55.0

TABLE II. Additional Uncertainties from Propagation

Parts or system	${}^{\tau}U^{\Delta}_{15years}(\mu s)$	${}^{Y}U^{\Delta}_{15years}(MJ)$
Priming-device	0.01	0.0
High-explosive	0.02	5.5
Special-metal	0.01	9.5
Frame-material	0.01	5.0
Energetic-material	0.00	0.0
Whole system	0.05	20.0

 ISBN: 978-1-61208-506-7

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 ISBN: 978-1-61208-506-7