Code Verification of PECM with Strongly Discontinuous Flows

Zhibo Ma
Institute of Applied Physics and Computational Mathematics
Beijing, China
e-mail: mazhibo@iapcm.ac.cn

***Abstract*—The simulation of compressible flows with strong discontinuities is necessary to verify meshfree methods. In order to eliminate inconsistencies between mathematics and physics of existing meshfree methods exposed especially when discontinuities exist, a different set of kernel functions, modularizing factors and discrete equations is put forward by a new meshfree method named Physics Evoked Cloud Method (PECM). In this paper, PECM is briefly introduced and is verified by posteriori assessments based on comparisons between numerical and exact solutions. The assessments show that PECM has overcome shortcomings of existing meshfree methods and is able to accommodate various kinds of strong discontinuities including large density ratio, from which we see that PECM has an excellent robustness and a high accuracy.**

Keywords*—*PECM; RKPM; meshfree method; code verification.

I. INTRODUCTION

The ability and accuracy of numerical computation are the most important credibility indices for modeling and simulations [1][2]. For discontinuous problems such as shock waves or contact between different materials, spatial derivatives of pressure, density, and velocity etc. may not exist. Consequently the numerical methods are usually constructed according to the ideas put forwarded by Von Neumann or Godunov [3][4]. Neumann method captures physical discontinuities by adding artificial viscosity terms, whereas Godunov method does that by Riemann solutions. In this paper, we call the artificial terms modularizing factors in simulations, as they represent the microscopic phenomena through physical quantities corresponding to macroscopic discrete bodies and time steps.

As a purely Lagrangian meshfree method, Smoothed Particle Hydrodynamics (SPH) based on Neumann idea has undergone a long-term development and has earned very wide attentions [5][6]. Since it was put forward in 1977, efforts have been made to improve its accuracy, therefrom some derivative methods of SPH were proposed, such as Reproducing Kernel Particle Method (RKPM), Corrective Smoothed Particle Method (CSPM) and Modified Smoothed Particle Hydrodynamics (MSPH) [7]-[9]. Unfortunately, SPH and its derivative methods narrated above still display obvious shortcomings especially when they are applied in dynamic problems where strong discontinuities exist.

According to the idea of Taylor's series expansion, an arbitrary function can be approximated by polynomials. As the polynomial and its derivative are easily to be constructed, correction functions formed as polynomial have been generally used to modify the kernel functions of SPH. All the derivative methods including RKPM, CSPM, and MSPH use this mechanism to improve the accuracy of kernel estimation. However, these improvements originated just from mathematical ideas, so the physics laws have not been fully reflected by the numerical algorithms. For problems such as explosions or impacts with multi-materials, in which many kinds of discontinuity exist, these improvements are not remarkable, and the meshfree simulations are still difficult to avoid the large uncertainties, numerical oscillations or even nonphysics solutions.

In order to eliminate the deficiencies firmly tangled in the existing meshfree methods and greatly enhance the adaptability of numerical method, the author of this paper proposed a new method named Physics Evoked Cloud Method (PECM) and developed a software named How Are Universes Cuddling (HAUC) based on PECM [10]-[14]. In fact, PECM is a meshfree hierarchical methodology built according to the classification of multifarious materials. PECM has no specific restriction on the approach of meshfree approximation but six principles should be generally satisfied, one of which says that PECM should correctly reflect objective physics laws. At present, PECM temporarily use the approach of kernel approximation but the numerical algorithms were rearranged deeply in order to satisfy its general principles.

In this paper, RKPM is selected as a representative of SPH and its derivative methods. Preceding with description of governing equations and time stepping scheme in section II, RKPM and PECM for fluid dynamics are briefly described in section III and section IV. Section V addresses code verification aims to assess the credibility of PECM by a posteriori approach. Section VI gives conclusions and the article is closed with an acknowledgement. All the numerical and exact results were produced by HAUC.

II. GOVERNING EQUATIONS AND TIME STEPPING SCHEME FOR SIMULATIONS

For fluid flows without external force, heat conduction and physics viscosity, the Lagrangian forms of differential governing equations are:

Mass $$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho \nabla \cdot \boldsymbol{v} \tag{1}$$

Momentum $$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla p \tag{2}$$

Energy $$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{p}{\rho}\nabla \cdot \boldsymbol{v} \tag{3}$$

Where ρ, $\boldsymbol{v}$, p, e and t indicate density, velocity, pressure, internal energy and time, respectively.

ISBN: 978-1-61208-737-5

During the time stepping as solving the unsteady flows corresponding (1) – (3), a two-step scheme consists of prediction and correction is adopted as in (4) – (5).

Prediction: $$\psi^{n+\frac{1}{2}} = \psi^n + 0.5\Delta t(\mathrm{d}\psi/\mathrm{d}t)^n \tag{4}$$

Correction: $$\psi^{n+1} = \psi^n + \Delta t(\mathrm{d}\psi/\mathrm{d}t)^{n+\frac{1}{2}} \tag{5}$$

Where ψ generally refers to ρ, e, $\boldsymbol{v}$, $\boldsymbol{r}$, h, and $\boldsymbol{r}$ is the position coordinates of discrete body, h is smoothing length for kernel approximation. By sound speed c and a factor τ related to CFL conditions, we have time step $\Delta t = \tau h/(c + |\boldsymbol{v}|)$, and p is obtained via equation of state $p = p(\rho, e)$ in each time step.

In this paper, the numerical simulations with RKPM or PECM are all based on the governing equations and time stepping scheme narrated above.

III. Description of RKPM

The ideas and schemes of RKPM are as follows.

A. Kernel Estimation

Kernel estimation of SPH for an arbitrary function $\boldsymbol{f}(\boldsymbol{r})$ can be described as

$$\langle f(\boldsymbol{r})\rangle = \int_\Omega f(\boldsymbol{r}')W^{(\mathrm{SPH})}(\boldsymbol{r}-\boldsymbol{r}',\ h)\mathrm{d}\boldsymbol{r}' \tag{6}$$

Where $W^{(\mathrm{SPH})}(\boldsymbol{r}-\boldsymbol{r}',\ h)$ is a compact kernel function with its support domain Ω and smoothing length h. $\boldsymbol{r}'$ and $\mathrm{d}\boldsymbol{r}'$ are respectively, the coordinates and volume of discretized micro-bodies, which is centered by coordinates $\boldsymbol{r}$ in Ω. With integration by part and Gaussian formula, estimations of derivatives can be shifted to the kernel function, such as

$$\langle \nabla \boldsymbol{f}(\boldsymbol{r})\rangle = \int_\Omega \boldsymbol{f}(\boldsymbol{r}')\nabla_r W^{(\mathrm{SPH})}(\boldsymbol{r}-\boldsymbol{r}',\ h)\mathrm{d}\boldsymbol{r}' \tag{7}$$

Where the operator $\nabla_{\boldsymbol{r}}$ represents the derivation with $\boldsymbol{r}$. RKPM has inherited all the SPH algorithms except the kernel function, which is corrected as

$$W^{(\mathrm{RKPM})}(\boldsymbol{r}-\boldsymbol{r}',\ h) = C^{(\mathrm{RKPM})}(\boldsymbol{r}-\boldsymbol{r}',\ h)W^{(\mathrm{SPH})}(\boldsymbol{r}-\boldsymbol{r}',\ h) \tag{8}$$

Briefly noting $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$ and b_j the volume of particle j, the n_c-order correction $C_{ij}^{(\mathrm{RKPM})} = C^{(\mathrm{RKPM})}(\boldsymbol{r}_{ij},\ h_i)$ in form of polynomial is solved by the consistency conditions as

$$\sum_{j=1}^N b_j C_{ij}^{(\mathrm{RKPM})} W_{ij}^{(\mathrm{SPH})} = 1 \tag{9}$$

$$\sum_{j=1}^N b_j C_{ij}^{(\mathrm{RKPM})} W_{ij}^{(\mathrm{SPH})} \boldsymbol{r}_{ij}^k = 0,\quad k = 1,\ 2,\cdots n_\mathrm{c} \tag{10}$$

B. Modularizing Factors

Modularizing factors of RKPM are the same of those in SPH, such as artificial pressure

$$\Pi_{ij}^{(\mathrm{SPH})} = \begin{cases} (-\alpha c_{ij}\mu_{ij} + \beta\mu_{ij}^2)/\rho_{ij} & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} < 0 \\ 0 & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} \ge 0 \end{cases} \tag{11}$$

And artificial heat flux is

$$\boldsymbol{H}_{ij}^{(\mathrm{SPH})} = \begin{cases} \dfrac{2\zeta_{ij}(e_i - e_j)\boldsymbol{r}_{ij}}{\rho_{ij}(\boldsymbol{r}_{ij}^2 + \eta h_{ij}^2)} & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} < 0 \\ 0 & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} \ge 0 \end{cases} \tag{12}$$

Where particle j is in the support domain Ω_i of particle i, and $\boldsymbol{v}_{ij} = \boldsymbol{v}_i - \boldsymbol{v}_j$, $h_{ij} = (h_i + h_j)/2$, $c_{ij} = (c_i + c_j)/2$, $\rho_{ij} = (\rho_i + \rho_j)/2$, $\mu_{ij} = h_{ij}\boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij}/(\boldsymbol{r}_{ij}^2 + \varepsilon h_{ij}^2)$, $\zeta_{ij} = (\zeta_i + \zeta_j)/2$, $\zeta = g_1 hc + g_2 h^2(|\nabla\cdot\boldsymbol{v}| - \nabla\cdot\boldsymbol{v})$, α, β, ε, η, g_1, g_2 are constants.

C. Discrete Equations

The standard discrete equations of RKPM for mass, momentum and energy, are written in (13) – (15).

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = \sum_{j=1}^N m_j \boldsymbol{v}_{ij}\cdot\nabla_i W_{ij}^{(\mathrm{RKPM})} \tag{13}$$

$$\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = -\sum_{j=1}^N m_j\left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij}^{(\mathrm{SPH})}\right)\nabla_i W_{ij}^{(\mathrm{RKPM})} \tag{14}$$

$$\frac{\mathrm{d}e_i}{\mathrm{d}t} = \frac{1}{2}\sum_{j=1}^N m_j\left[\left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij}^{(\mathrm{SPH})}\right)\boldsymbol{v}_{ij} + \boldsymbol{H}_{ij}^{(\mathrm{SPH})}\right]\cdot\nabla_i W_{ij}^{(\mathrm{RKPM})} \tag{15}$$

Where $W_{ij}^{(\mathrm{RKPM})} = W^{(\mathrm{RKPM})}(\boldsymbol{r}_{ij}, h_i)$, N is the total particle number in Ω_i, the operator ∇_i means the derivation with $\boldsymbol{r}_i$.

IV. Description of PECM

The ideas and schemes of PECM are as follows.

A. Kernel Estimation

PECM calls the micro-body as cloud other than particle in SPH, and uses a new kernel for momentum equation as

$$W^{(\mathrm{PECM})}(\boldsymbol{r}-\boldsymbol{r}',\ h) = C^{(\mathrm{PECM})}(\boldsymbol{r}-\boldsymbol{r}',\ h)W^{(\mathrm{SPH})}(\boldsymbol{r}-\boldsymbol{r}',\ h) \tag{16}$$

The correction $C_{ij}^{(\mathrm{PECM})}$ observes new consistencies

$$\sum_{j=1}^N b_j\rho_j C_{ij}^{(\mathrm{PECM})} W_{ij}^{(\mathrm{SPH})} = 1 \tag{17}$$

$$\sum_{j=1}^N b_j\rho_j C_{ij}^{(\mathrm{PECM})} W_{ij}^{(\mathrm{SPH})} \boldsymbol{r}_{ij}^k = 0,\quad k = 1,\ 2,\cdots n_\mathrm{c} \tag{18}$$

B. Modularizing Factors

PECM has two pressure factors as

$$\Pi_{ij,1}^{(\mathrm{PECM})} = \begin{cases} (-\alpha c_{ij}\mu_{ij} + \beta\mu_{ij}^2)\rho_{ij} & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} < 0 \\ 0 & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} \ge 0 \end{cases} \tag{19}$$

$$\Pi_{ij,2}^{(\mathrm{PECM})} = \begin{cases} (-\alpha c_{ij}\mu_{ij} + \beta\mu_{ij}^2)\hat{\rho}_{ij} & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} < 0 \\ 0 & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} \ge 0 \end{cases} \tag{20}$$

The heat flux factor is modified as

$$\boldsymbol{H}_{ij}^{(\mathrm{PECM})} = \begin{cases} \dfrac{\zeta_{ij}\rho_{ij}(e_i - e_j)\boldsymbol{r}_{ij}}{\boldsymbol{r}_{ij}^2 + \eta h_{ij}^2} & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} < 0 \\ 0 & \boldsymbol{v}_{ij}\cdot\boldsymbol{r}_{ij} \ge 0 \end{cases} \tag{21}$$

A new velocity factor is leaded in as

$$\boldsymbol{\Phi}_{ij}^{(\mathrm{PECM})} = \delta h_{ij}(p_i - p_j)\boldsymbol{r}_{ij}/(\boldsymbol{r}_{ij}^2 + \theta h_{ij}^2)/c_{ij}/\rho_{ij} \tag{22}$$

Here α, β, c_{ij}, ρ_{ij}, h_{ij}, μ_{ij}, ζ_{ij} are the same in SPH, δ, θ are constants and $\hat{\rho}_{ij} = \rho_i\rho_j/(\rho_i + \rho_j)$.

C. Discrete Equations

The discrete equations of PECM for mass, momentum and energy, are written in (23) – (25).

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = \rho_i\sum_{j=1}^N b_j\left(\boldsymbol{v}_{ij} + \boldsymbol{\Phi}_{ij}^{(\mathrm{PECM})}\right)\cdot\nabla_i W_{ij}^{(\mathrm{RKPM})} \tag{23}$$

 ISBN: 978-1-61208-737-5

$$\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = -\sum_{j=1}^{N} b_j \left(p_j + \Pi_{ij,1}^{(\mathrm{PECM})}\right) \nabla_i W_{ij}^{(\mathrm{PECM})} \tag{24}$$

$$\frac{\mathrm{d}e_i}{\mathrm{d}t} = \frac{1}{\rho_i} \sum_{j=1}^{N} b_j \left[\left(p_i + \Pi_{ij,2}^{(\mathrm{PECM})}\right)\left(\boldsymbol{v}_{ij} + \boldsymbol{\Phi}_{ij}^{(\mathrm{PECM})}\right) + \boldsymbol{H}_{ij}^{(\mathrm{PECM})}\right] \cdot \nabla_i W_{ij}^{(\mathrm{RKPM})} \tag{25}$$

D. Better Consistencies with Physics

Aimed at essentially improving the consistencies of mathematics with physics, from which an intensive robustness and a high accuracy can be expected, the numerical scheme of PECM has been reconstructed. Comparing with SPH and its derivative methods, PECM has three kinds of modifications such as:

1) *Kernel functions:* Kernels for equations of mass and energy keep unchanged, whereas for momentum equation, a new kernel is introduced considering the harmonious movements of the material cluster comprised of neighbouring clouds, which may have great differences in density with each other. In fact, there is an equation of strain compatibility for continuous mediums, but it does not appear in the governing equations. For mesh-based methods, it is automatically observed as the moving of meshes is dominated by nodes. For the existing meshfree methods like SPH, the particles with smaller density have greater trend to change their velocities than those with larger density, so the equation of strain compatibility is easily to be destroyed especially when strong gradients of density and pressure exist. With the new kernel, containing density, PECM does not update velocity of each cloud in an isolated manner, but felicitously reflets the moving consistency of the neighboring clouds, this may be the uppermost contribution for PECM to have exellent performances in robustness and accuracy.

2) *Modularizing factors:* Being firstly created by PECM, the velocity factor and the second pressure factor were designated according to the laws of interaction between clouds, in which the pressure and velocity should keep continuity macroscopically and the impact force that contribute to inner energy should be dominated mainly by the matter with lower density. These two factors can effectively eliminate the nonphysics results such as numerical oscillations and wall heating.

3) *Discrete equations:* In equations of mass and energy, variables like mass, density and pressure of clouds are withdrawed out of the acting domain of kernel operator. This can help PECM to prevent numerical dissipations in density or energy etc., and the numerical computations can accurately predict the flow fields even they objectively have very abrupt distributions in space.

V. CODE VERIFICATION OF PECM

The code verification of PECM is depicted as follows.

A. Techniques of Code Verification

Stability and accuracy are the most important concerned of numerical methods. In code verification, we assess the credibility of PECM with three modes: the first is to observe the computing stability based on the dynamic videos formed by numerical results at different times; the second is to inspect the accuracy by comparison between numerical and exact results, good agreement means high accuracy; the third is to quantify the accuracy order based on the decreasing trends of numerical errors when discrete scales diminish. As the space limit, this paper just shows the agreements between the numerical and exact results via static figures.

B. Results of Code Verification

In this paper, five models are typically selected to exhibit the advantages of PECM by comparing its numerical results with those of RKPM and the exact solutions. The parameters for initial states in entity models, physics models and numerical computations are listed in Table Ⅰ - Ⅲ, respectively, where ΔX is the initial discrete scale of clouds and EOS is the abbreviation for equation of state. The numerical results corresponding these parameters are showed in Figure 1-5, where different colors are just used to help the readers easily distinguishing the left or right district each cloud belongs to.

Model 1 aims to verify if the codes are correctly compiled. As all the left and right districts are of same material forming a properly symmetric impact, this model is fit to diagnose code bugs as it is not apt to deeply reveal shortcomings of numerical methods. Figure 1 shows both RKPM and PECM give results that generally agree with exact results, but PECM is more accurate than RKPM, especially in the space near impact interfaces where the shock may lead to discontinuities in density and pressure. Although discontinuity exists just in velocity, we can still observe the deficiencies of RKPM from the numerical results, in which the density decline induced by the wall heating could not be compensated quickly, and the effect of energy smearing made by artificial heat flux can lead to overestimated energy behind the shock wave. In the numerical results, we also see that the velocity factor in (22) can help PECM to eliminate wall heating where strong impact exists.

Model 2 corresponds to a state of dynamic equilibrium and based on physics theory, this equilibrium will keep unchanged forever. This model is built aiming to prove the PECM kernel is more reasonable than RKPM kernel for momentum equation and, the kernel approximation should not act on mass, density and pressure in equations of mass and energy like RKPM does. As the initial state has unified distributions in pressure and velocity, there is no driving force to make material accelerated, so the discontinuities exist in density, energy and EOS can be used exclusively to assess if the numerical schemes have the ability to restrain the errors from generating and developing. Figure 2 shows that PECM gives results free of any errors whereas RKPM leads to illusive error waves.

Model 3 imitates a high pressure explosive impacting a low pressure heavy metal, which is a typical situation in anti-armor weapons or detonation systems especially when nuclear fissions exist. In this model the initial ratio between pressure and density, namely p_0/ρ_0, is $(3.5\times10^{10})/(2.5\times10^{3})$=

1.4×10^7 for left district and $0/(2.0\times10^4) = 0$ for right district, the ratio of p_0/ρ_0 between the two sides is $1.4\times10^7/0=\infty$. The extremely large discrepancy of p_0/ρ_0 across the material interfaces may induce severe instability for RKPM. We have tried many ways to control the evolution of smoothing length in RKPM but interface separation occurs inevitably. In Figure 3 we can see the interface separation and very large errors of RKPM, whereas PECM gives the results well agreed with exact solutions. The new kernel for momentum equation contributes the most to improve the stability and accuracy of PECM; in addition, the modularizing factor for velocity in (22) benefits the PECM to restrain the numerical oscillations.

Model 4 imitates a high pressure heavy metal impacting a low pressure thin gas, which is another typical situation in detonation systems especially when nuclear fusions exist. In this model the ratio of ρ_0 between the two sides reaches 1.0×10^5. The extremely strong discontinuity with density and pressure calls out another baptism for meshfree methods. For this model, RKPM is not able to keep the computation going on as the acute numerical oscillation makes the density and energy less than zero. The numerical results of RKPM in Figure 4 were obtained by the help of artificial controls in time steps and smoothing lengths. Despite the artificial measures used by RKPM, the computation was still unstable and many particles flew off the computational domain, and that the particles left in the domain presented a disordered distribution which implied very large deviations with exact solutions. Simulations of this model are very difficult to keep stable and accurate even for the numerical methods with meshes, but PECM still exported results according well with the exact solutions. Numerical tests of this model indicate that the kernel function in (16) ensures PECM to be stable and accurate, and the second pressure factor in (20) is absolutely necessary to eliminate the nonphysics wall heating especially when very large density ratio exists.

In Model 5, the discontinuity of the initial velocity makes the materials of the two districts rapidly moving apart from each other. As the velocities are high enough, the materials are expanded entirely and vacuum occurs between the two districts. Owing to instability, RKPM gives the results as in Figure 5 by constraining the evolution of smoothing length within 3 times the initial value, from which we still see the large deviations with exact solutions, whereas PECM can exactly predict the interface separation that exists in reality. As the modularizing factors for pressure and heat flux do not work in this situation, the reform of discrete equations, in addition with the newly introduced kernel and velocity factor may be the most important contributors for PECM to keep excellent stability and accuracy.

Through a large number of numerical tests with various models, we have got adequate evidences indicating that PECM is of super stability and accuracy even with series kinds of strong discontinuities and, the accuracy order is the second for continuous models or lower than the first for most discontinuous problems.

VI. CONCLUSIONS

The deficiencies of SPH and its derivative methods may be easily exposed when they are applied in the models with strong detonations and shock waves. As there are no concrete geometries of the discrete bodies in meshfree methods, it is very difficult to construct mathematic schemes which can sufficiently reach the physical mechanisms hided between the discrete bodies disorderly scattered in space. So it is very important to unearth these potential mechanisms and reflect them in the meshfree numerical schemes.

The practice to add density into kernel function for momentum equation may contribute the most for PECM to make clouds moving harmoniously, and this is very important in developing meshfree schemes, for which no vertexes of micro-bodies could be described to prevent the undefined boundaries from splitting and overlapping.

The modularizing factors and the forms of discrete equations in PECM also have great contributions for computing stability and accuracy, which are designed according to the physical mechanisms apparently or recessively existed in the arbitrarily spread clouds which have no perfect geometries.

The properties such as consistency, stability and accuracy are usually difficult to be theoretically proved but, by posteriori assessments in this paper, PECM has achieved enough evidences for its excellent properties.

ACKNOWLEDGMENT

The author thanks CAEP Foundation (CX2019026) to partially support the works in this paper.

TABLE I. PARAMETERS FOR INITIAL STATES IN ENTITY MODELS

District	Model	Density/kg.m^{-3}	Energy/J.kg^{-1}	Pressure/Pa	Velocity/m.s^{-1}
Left	1	2.500×10^3	6.496×10^6	3.500×10^{10}	1.500×10^3
	2	2.000×10^3	5.000×10^6	2.000×10^{10}	2.000×10^3
	3	2.500×10^3	7.000×10^6	3.500×10^{10}	0.000
	4	2.000×10^{-1}	1.000×10^7	1.000×10^6	0.000
	5	2.500×10^3	6.496×10^6	3.500×10^{10}	-7.000×10^3
Right	1	2.500×10^3	6.496×10^6	3.500×10^{10}	-1.500×10^3
	2	2.000×10^4	2.500×10^5	2.000×10^{10}	2.000×10^3
	3	2.000×10^4	0.000	0.000	0.000
	4	2.000×10^4	2.500×10^5	2.000×10^{10}	-1.500×10^3
	5	2.000×10^3	2.400×10^6	2.500×10^{10}	7.000×10^3

 ISBN: 978-1-61208-737-5

TABLE II. PARAMETERS OF PHYSICS MODELS

District	Model	EOS Form	c_0^2/m^2.s^{-2}	ρ_0/kg.m^{-3}	γ
Left	1	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	3.600×10^6	1.800×10^3	3.000
	2	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	2.250×10^6	2.000×10^3	3.000
	3	$p = (\gamma - 1)\rho e$	0.000	0.000	3.000
	4	$p = (\gamma - 1)\rho e$	0.000	0.000	1.500
	5	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	3.600×10^6	1.800×10^3	3.000
Right	1	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	3.600×10^6	1.800×10^3	3.000
	2	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	2.500×10^7	2.000×10^4	5.000
	3	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	1.500×10^7	2.000×10^4	5.000
	4	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	2.500×10^7	2.000×10^4	5.000
	5	$p = c_0^2(\rho - \rho_0) + (\gamma - 1)\rho e$	2.500×10^6	1.600×10^3	6.000

TABLE III. PARAMETERS OF NUMERICAL COMPUTATIONS

ΔX/mm	h_0/mm	n_c	τ	α	β	ε	g_1	g_2	η	δ	θ
1.00	1.5	0.00	0.10	1.00	2.00	0.01	1.00	1.00	0.01	0.10	0.01

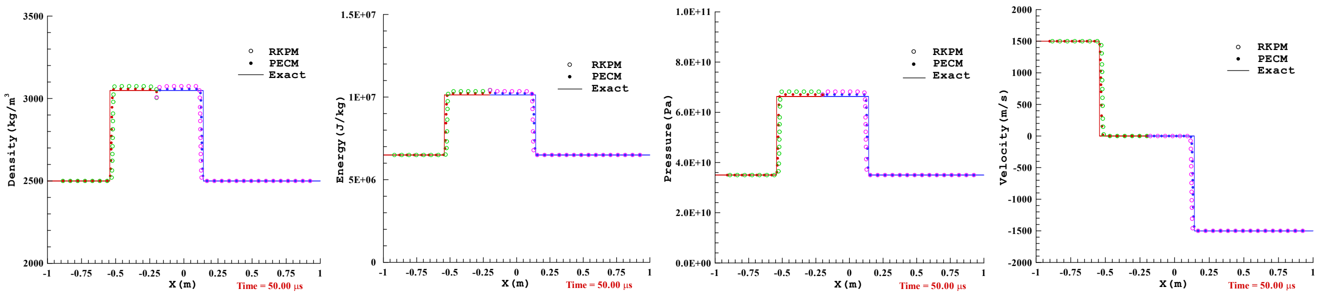

Figure 1. Numerical results about model 1.

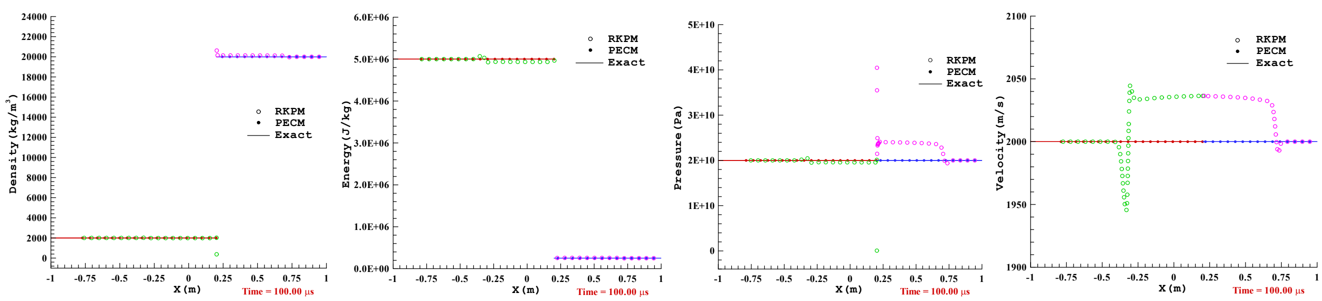

Figure 2. Numerical results about model 2.

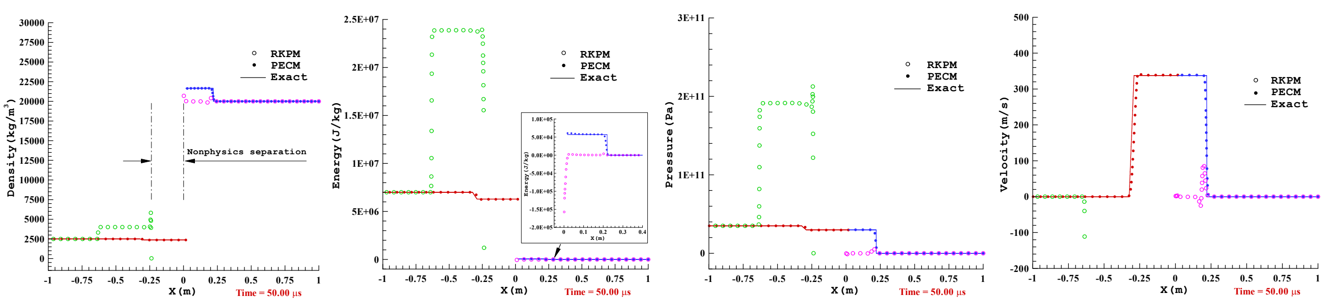

Figure 3. Numerical results about model 3.

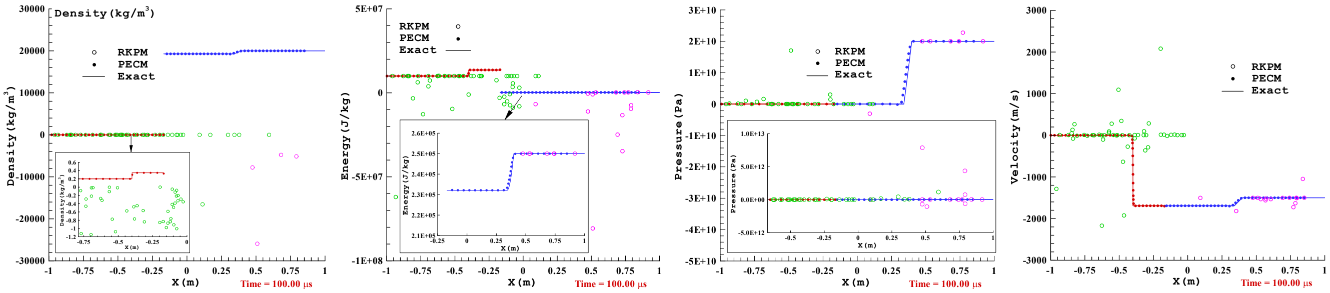

Figure 4. Numerical results about model 4.

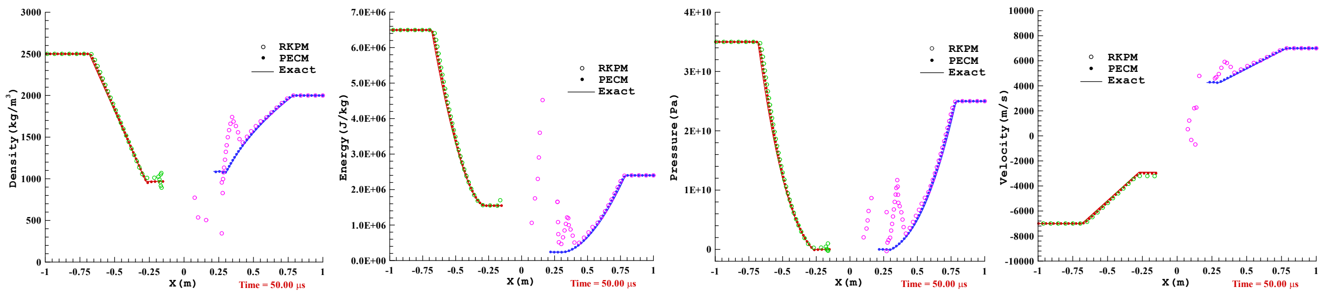

Figure 5. Numerical results about model 5.

REFERENCES

[1] W. L. Oberkampf and C. J. Roy, “Verification and validation in science computing,” Cambridge, UK: Cambridge University Press, 2010.

[2] Z. B. Ma, Y. T. Sun, and J. W. Yin, “QMU Decision based on modeling & simulation,” Chinese Journal of Computational Physics, vol. 33, no. 6, pp. 661-670, 2016.

[3] J. VonNeumann and R. D. Richtmyer, “A method for the numerical calculation of hydrodynamic shocks,” Journal of Applied Physics, vol. 21, no. 3, pp. 232-237, 1950.

[4] S. K. Godunov, “A finite deference method for the computation of discontinuous solutions of the equations of fluid dynamics,” Mathematics of the USSR-Sbornik, vol. 47, pp. 357-393, 1959.

[5] R. A. Gingold and J. J. Monaghan, “Smoothed particle hydrodynamics: Theory and application to non-spherical stars,” Monthly Notices of the Royal Astronomical Society, vol. 181, no. 2, pp. 375-389, 1977.

[6] L. B. Lucy, “A numerical approach to the testing of the fission hypothesis,” Astronomical Journal, vol. 82, no. 12, pp. 1013-1024, 1977.

[7] W. K. Liu, S. Jun, and S. F. Li, “Reproducing kernel particle methods for structural dynamics,” International Journal for Numerical Methods in Engineering, vol. 38, no. 10, pp. 1655-1679, 1995.

[8] J. K. Chen, J. E. Beraun, and C. K. Jih, “An improvement for tensile instability in smoothed particle hydrodynamics,” Computational Mechanics, vol. 23, no. 4, pp. 279-287, 1999.

[9] G. M. Zhang and R. C. Batra, “Modified smoothed particle hydrodynamics method and its application to transient problems,” Computational Mechanics, vol. 34, no. 2, pp. 137-146, 2004.

[10] Z. B. Ma, “Physics evoked cloud method: a versatile systematic method for numerical simulations,” Chinese Journal of Computational Physics, vol. 34, no. 3, pp. 261-272, 2017.

[11] Z. B. Ma and Y. Z. Zhao, “Code verification of mesh free method with computation of derivatives,” Journal of Aerospace Power, vol. 32, no. 8, pp. 1886-1899, 2017.

[12] Z. B. Ma and Y. Z. Zhao, “Physics evoked cloud method,” International Journal of Computational Method, vol. 15, no. 3, pp. 1846006.1-1846006.20, 2018.

[13] Z. B. Ma and Y. Z. Zhao, “Verification and validation of common derivative terms approximation in meshfree numerical scheme,” Journal of Applied and Computational Mechanics, vol. 4, no. 3, pp. 231-244, 2018.

[14] Z. B. Ma, L. Shi, M. Yu, and H. Zheng, “Code verification of physics evoked cloud method for compressible fluid dynamics,” Journal of Aerospace Power, vol. 34, no. 6, pp. 1253-1272, 2019.