A Two-Stage Pilot-Assisted CFO Estimation Scheme for OFDM Signals

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***Abstract*— In this paper, a pilot-assisted two-stage carrier frequency offset (CFO) estimation algorithm is proposed for OFDM signals. The proposed scheme performs a coarse search over a window of possible carrier offsets to obtain an initial estimate of the CFO. Then, a fine search is carried over a much smaller search window to obtain better accuracy of the carrier frequency offset. The pilot signal used in this algorithm results in a slowly varying correlation function with peaks at the correct offset. This is exploited by the proposed algorithm to reduce the search window for the second stage. Simulation results presented in this paper show that a significant improvement in the mean-square error performance is achieved by using this two-stage approach without increasing the complexity of implementation compared to single-stage conventional algorithms. It is also demonstrated that the proposed scheme outperforms the conventional cyclic-prefix based CFO estimation algorithm under both flat and frequency-selective Rayleigh fading channel conditions.**

Keywords-Carrier Frequency Offset Estimation, OFDM Synchronization; Multi-stage Search.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been proposed to support high data rate applications in future mobile radio systems. This is due to its improved spectral efficiency and efficient implementation structure. Furthermore, OFDM has inherent features in terms of mitigating multipath and intersymbol interference (ISI) that are dominant in high data rate scenarios [1]–[3]. OFDM avoids the ISI problem by transmitting the data over a large number of narrow band channels (subcarriers) and by using a cyclic prefix at the start of every OFDM symbol.

A successful deployment of OFDM-based systems, however, needs to overcome several challenges. One of the main problems is the high peak-to-average power ratio of the transmitted signal resulting in a loss of power efficiency. Another major issue, which is the focus of this paper, is the need for accurate subcarrier synchronization, also called carrier frequency offset (CFO) estimation, to maintain orthogonality between the subcarriers and, consequently, prevent inter-carrier interference (ICI). Frequency offsets occur in OFDM systems mainly due to frequency mismatches between the transmitter and receiver oscillators and partly due to Doppler shifts owing to user mobility. Since all the subcarriers should be orthogonal to one another to ensure successful data recovery, frequency offsets pose a major issue that must be accurately resolved for a successful OFDM system implementation [4][5].

OFDM frequency offset estimators can be classified into non pilot-based and pilot-based techniques. Non pilot-based schemes rely on the OFDM symbol structure features such as the cyclic prefix (CP) [6]. These schemes are more bandwidth efficient than pilot-based schemes but they tend to suffer under frequency-selective fading channel conditions [7]. However, pilot-based schemes tend to be more accurate in estimating the CFO. Furthermore, the use of pilot-based schemes is justified since the pilot data can be utilized for other purposes such as channel estimation.

Pilot-based CFO estimation schemes use known training symbols sent by the transmitter over specific subcarriers and time slots. The pilot symbols may be transmitted as a preamble over all subcarriers during the first OFDM symbol and then regular data transmission is started. Different maximum likelihood (ML) CFO estimators were developed in [8]-[13] for both flat and frequency-selective fading channels. Although ML schemes provide good performance in terms of the mean square estimation error but they tend to require high computational complexity.

In this paper, we introduce a two-stage pilot-based CFO synchronization technique that provides accurate carrier offset estimation. The proposed scheme uses different search steps to find the offset resulting is a reduced complexity compared to single-stage searching strategy. The performance of the proposed scheme is presented under both flat and frequency selective fading channel conditions.

The rest of the paper is organized as follows: Section II describes the system model. The proposed scheme is presented in Section III. Simulation results and conclusions are given in Sections IV and V, respectively

II. SYSTEM MODEL

The block diagram of the OFDM system under consideration in this work is shown in Figure 1. An OFDM signal is formed by modulating several orthogonal subcarriers with different data symbols. The input binary data is first applied to the modulation block that maps the input data to corresponding modulation symbols. Then, serial to parallel (S/P) conversion is performed followed by inverse Fast Fourier Transform (IFFT) operation to distribute the symbols to the different subcarriers. A cyclic prefix is added to form an OFDM symbol and then converted from parallel

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to serial (P/S) for transmission after digital to analog conversion (DAC).

The transmitted OFDM symbol is expressed as

$$x(t) = \sum_{k=0}^{N-1} b_k \, e^{j2\pi kt/T}, \quad (1)$$

where T is the symbol period, N is the number of orthogonal subcarriers, and b_k is the data symbol of the kth subcarrier. A sampled version of the OFDM signal is obtained by taking samples every nT/N seconds, where n is an integer, to get

$$x(n) = \sum_{k=0}^{N-1} b_k \, e^{\frac{j2\pi kn}{N}}. \quad (2)$$

The OFDM signal is transmitted through a wireless channel and experiences fading, noise, and carrier frequency offset to obtain the received signal given by

$$y(t) = e^{j2\pi\Delta f t} \{h(t) * x(t)\} + \omega(t), \quad (3)$$

where $h(t)$ is the impulse response of the channel, $\omega(t)$ is the additive white Gaussian noise (AWGN), and Δf is the unknown frequency offset (in Hz) to be estimated. In discrete form, this reduces to

$$y(n) = e^{\frac{j2\pi\rho n}{N}} \{h(n) * x(n)\} + \omega(n); \quad n = 0,1, \ldots \quad (4)$$

where $\rho = \Delta f T$ is the normalized frequency offset (frequency offset normalized by the symbol rate). For a flat fading channel, the received signal is written as

$$y(n) = \alpha e^{\frac{j2\pi\rho n}{N}} \sum_{k=0}^{N-1} b_k \, e^{\frac{j2\pi k(n-\tau)}{N}} + \omega(n), \quad (5)$$

where α is the complex channel gain modeled with Rayleigh amplitude and uniform phase and τ is the channel delay. In case of slowly varying channels, this represents a sinusoidal signal shifted in frequency by ρ and multiplied by a constant complex number.

To recover the transmitted data, the receiver performs reverse operations on the received signal as shown in Fig. 1. However, a frequency synchronization block is needed to estimate the unknown frequency offset that is provided to the FFT block for correction. The proposed frequency synchronization algorithm is discussed in the following section.

III. PROPOSED SYSTEM

The proposed CFO estimation scheme, shown in Fig. 2, performs a search over a window of possible frequency offsets in order to estimate the unknown offset Δf. The scheme uses a two-stage, also known as double-dwell (DD), search strategy. The first stage performs a coarse search over the possible frequency offset search window with a relatively large frequency separation (called step size) between the test offsets in order to reduce the complexity of the synchronization algorithm. The second stage uses a much smaller step size than the first stage but searches over a much smaller window of possible offsets as specified by the largest correlation peaks from the first stage. This results in a significant improvement in the estimation accuracy while maintaining a low complexity of implementation.

We assume that a known pure sinusoidal preamble pilot data sequence, $\{b_k^P\}$, is transmitted for synchronization purposes. Although using a pilot sequence causes some degradation to the bandwidth efficiency, especially in fast fading channels, these pilots are needed for channel estimation and hence their use for accurate frequency offset estimation is justified. Furthermore, the distribution of pilot symbols over the time-frequency grid can vary depending on channel conditions (time and frequency selectivity). During the pilot sequence transmission, the received signal is reduced to

$$y(n) = \alpha \sum_{k=0}^{N-1} b_k^P e^{\frac{-j2\pi k\tau}{N}} e^{\frac{j2\pi(k+\rho)n}{N}} + \omega(n). \quad (6)$$

The first stage of correlation is performed by correlating the received signal in (6) with a number of complex-conjugate versions of the pilot sequence each with a test normalized frequency offset $\gamma_m = m\epsilon_1$, where $\epsilon_1 = ST$ is the normalized search step size for the first stage and S is the search step size in Hz and m is an integer with values $m = -\frac{1}{2\epsilon_1}, -\frac{1}{2\epsilon_1}+1, \ldots -1,0,1, \ldots, \frac{1}{2\epsilon_1}-1, \frac{1}{2\epsilon_1}$. Without loss of generality, we assume $\frac{1}{\epsilon_1}$ to be an even number so we have m as an integer. The step size for the first stage is typically large, e.g., 10% of the frequency offset $S = \Delta f/10$ in order to reduce the number of correlations and hence reduce the complexity. For the m^{th} offset, correlating the received signal with the known pilot symbols during the first search stage results in

$$R^{(1)}(m) = C \sum_{n=0}^{N-1} e^{\frac{j2\pi(\rho-\gamma_m)n}{N}} + v(m), \quad (7)$$

where $C = \alpha \sum_{k=0}^{N-1} |b_k^P|^2$ is a complex constant and $v(m)$ is due to the noise component.

The objective is to find the value of γ_m that maximizes the magnitude of the correlation function in (7). It can be shown that this is equivalent to maximizing the following function over γ_m

$$U(m) = \left\{ \frac{\left[\left(1-\cos\left(2\pi(\rho-\gamma_m)\right)\right)^2 + \sin^2\left(2\pi(\rho-\gamma_m)\right) \right]}{\left[\left(1-\cos\left(\frac{2\pi(\rho-\gamma_m)}{N}\right)\right)^2 + \sin^2\left(\frac{2\pi(\rho-\gamma_m)}{N}\right) \right]} \right\}, \quad (8)$$

which is maximized when $\gamma_m = \rho$; i.e., when the test offset equals to the actual frequency offset to be estimated. Fig. 3 displays some examples for the above function for a normalized frequency offset ρ of -0.3, 0, and 0.3, demonstrating that the maximum is achieved when the test offset equals to the actual frequency offset.

In practical implementations, only a finite number of test offsets are searched. Thus, it is possible to have an error due to the limited resolution of the search process. This error can be reduced by increasing the number of search steps, i.e., reduce the step size. However, the hardware and computational power could be used more efficiently if the predictability of the correlation function is exploited since the exact offset corresponds to the function's global maximum. Therefore, if the exact offset does not coincide with the search steps, it must then be between the two offsets corresponding to the two highest correlation peaks.

Thus, we use a second search stage with a smaller step size, $\epsilon_2 \ll \epsilon_1$ in order to improve the accuracy of the estimation. The search window is limited to the range between the two frequency offsets with the largest correlation values obtained over the first search stage such that the computational complexity is reduced. Suppose that $|U(m)|^2$ has it is largest or peak value at $m = m_P$, then the second stage search is either performed between γ_{m_P} and γ_{m_P+1} if $|U(m_P+1)|^2 > |U(m_P-1)|^2$ or between γ_{m_P} and γ_{m_P-1} if $|U(m_P+1)|^2 < |U(m_P-1)|^2$. The search is done in steps of ϵ_2 and thus limits the maximum offset estimation error to $\pm\epsilon_2/2$.

The following correlation operation is performed for the second stage

$$R^{(2)}(l) = C \sum_{n=0}^{N-1} e^{\frac{j2\pi(\rho-\vartheta_l)n}{N}} + g(l), \qquad (9)$$

where ϑ_l is the normalized test offset and $g(l)$ is the noise component. The normalized test offset is determined as:

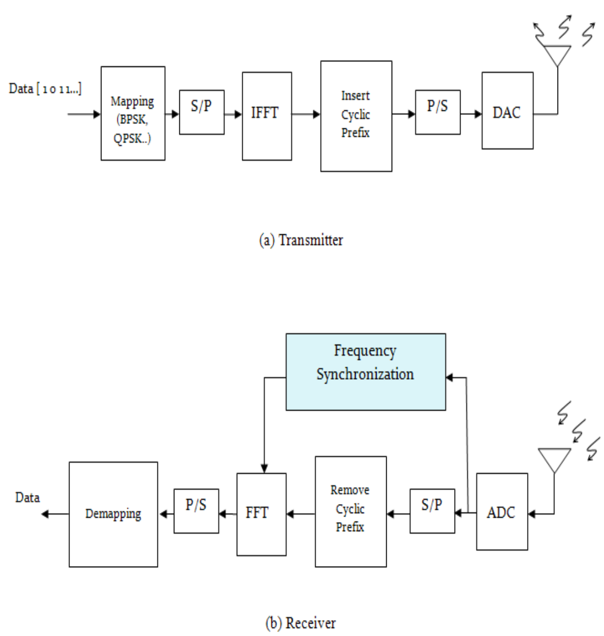

Figure 1. OFDM system block diagram.

$$\vartheta_l = \begin{cases} \gamma_{m_P+l\epsilon_2} & \text{if } |U(m_P+1)|^2 > |U(m_P-1)|^2 \\ \gamma_{m_P-l\epsilon_2} & \text{if } |U(m_P+1)|^2 < |U(m_P-1)|^2 \end{cases}, \qquad (10)$$

for $l = 0,1,2,\ldots,\lceil 1/\epsilon_2 \rceil$ where $\lceil q \rceil$ is the integer value greater than or equal to q. Finally, the offset that results in the largest correlation in (9) is used as the estimate for the actual normalized frequency offset $\hat{\rho}$. The estimated offset is then provided to the FFT block at the receiver for offset correction prior to the FFT operation.

IV. SIMULATION RESULTS

In this section, the performance of the proposed double-dwell (DD) synchronization technique is presented in terms of the mean-square error (MSE) in estimating the frequency offset. The proposed scheme performance is compared to the performance of the conventional cyclic extension (CE) synchronization technique. The channel is assumed to follow a Rayleigh fading model with either flat fading or frequency selective fading. The OFDM system is assumed to have N = 128, 256, or 512 subcarriers and the cyclic extension is $1/8^{th}$ of the OFDM symbol duration. The double-dwell scheme uses a first stage with a normalized search step size of $\epsilon_1 = 0.1$ and a second stage with a normalized search step size of $\epsilon_2 = 0.01$. These values were used for illustration purposes and other values could be used without loss of generality.

Fig. 4 compares the MSE performance of the double-dwell scheme with that of the cyclic-extension based scheme under flat fading for an OFDM system with 128, 256, or 512 subcarriers. The results show that the double-dwell scheme provides a significant improvement, especially at low signal-to-noise ratio, with a gain of more than one order of magnitude. It is observed that the performance for both schemes improves as the number of subcarriers increases.

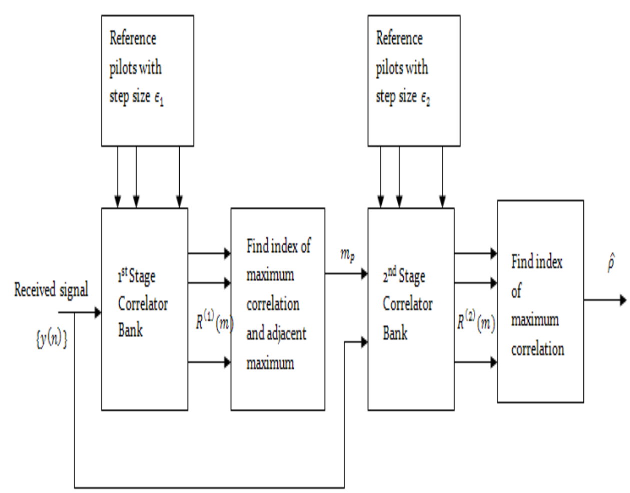

Figure 2. Block diagram of the proposed double-dwell frequency synchronization scheme.

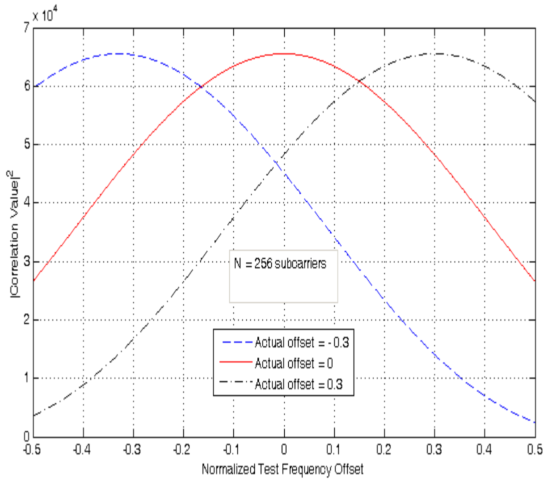

Figure 3. Examples of the correlation function for different frequency offsets.

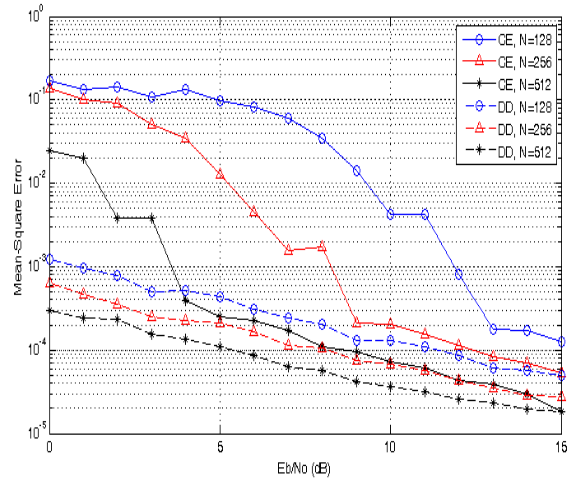

Figure 4. MSE frequency offset estimation performance under flat fading channel conditions: CE – Cyclic Extension; DD – Double-dwell.

The gain achieved by the double-dwell scheme is significantly increased when the channel undergoes frequency-selective fading as shown in Fig. 5. In this case, the channel has five multipath components with equal energy and a maximum delay spread of 75% of the guard time. It is noticed that the cyclic-extension based scheme suffers from the presence of the multipath components while the proposed scheme maintains a good performance.

Finally, we remark that the number of complex multiplications and additions needed by the proposed scheme is $N(1/\epsilon_1 + \epsilon_1/\epsilon_2)$ while a conventional single stage search scheme with the same accuracy as the proposed scheme would require $N(1/\epsilon_2)$ operations. This demonstrates that the proposed scheme reduces the complexity by a factor of $1/(\epsilon_1 + \frac{\epsilon_2}{\epsilon_1})$.

The performance will always be accurate to within $\epsilon_2/2$. Therefore, given a desired maximum error $\epsilon_2/2$, a first stage step size, ϵ_1, that gives the minimum number of search steps, N_s, required for the desired estimation accuracy can be calculated

$$\frac{dN_s}{d\epsilon_1} = \frac{d\left(\frac{1}{\epsilon_1} + \frac{\epsilon_1}{\epsilon_2}\right)}{d\epsilon_1} = 0$$
$$\epsilon_1 = \sqrt{\epsilon_2} \tag{11}$$

V. CONCLUSION AND FUTURE WORKS

In this paper, a two-stage pilot-based CFO estimation algorithm has been proposed. The algorithm uses a large step size to test for the CFO during the first stage to reduce the complexity. Then, a second search stage is used with a small step size to improve the CFO estimation accuracy. Simulation results demonstrate significant improvement in the MSE of the proposed scheme over both flat and frequency-selective Rayleigh fading channels. Future work will focus on optimizing the number of search stages and step sizes to have further performance improvement and complexity reduction.

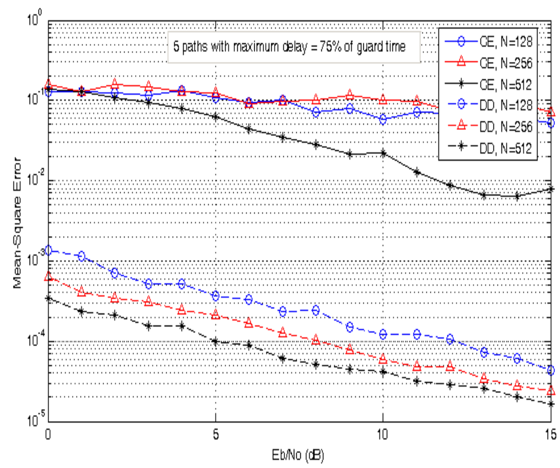

Figure 5. MSE frequency offset estimation performance under frequency-selective fading channel conditions.

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