Fequency Offset Estimation for OFDM Systems in Non-Gaussian Noise Channels

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***Abstract*—In this paper, the frequency offset estimation schemes robust to the non-Gaussian noise for orthogonal frequency division multiplexing (OFDM) systems are addressed. First, a maximum-likelihood (ML) estimation scheme in non-Gaussian noise is proposed, and then a simpler estimation scheme based on the ML estimation scheme is presented. Numerical results show that the proposed schemes offer robustness and a substantial performance improvement over the conventional estimation scheme in non-Gaussian noise channels.**

Keywords-frequency offset estimation; maximum-likelihood; non-Gaussian noise; OFDM; training symbol

I. INTRODUCTION

Due to its immunity to multipath fading and high spectral efficiency, orthogonal frequency division multiplexing (OFDM) has been adopted as a modulation format in a wide variety of wireless systems such as digital video broadcasting-terrestrial (DVB-T), wireless local area network (WLAN), and worldwide interoperability for microwave access (WiMAX) [1]-[4]. However, the OFDM is very sensitive to the frequency offset (FO) caused by Doppler shift or oscillator instabilities, and thus, the frequency offset estimation is one of the most important technical issues in OFDM systems [1], [5]. Specifically, we are concerned about the FO estimation based on training symbols, which provides a better performance than that based on the blind approach [5].

Conventionally, the FO estimation schemes have been proposed under the assumption that the ambient noise is a Gaussian process [6]-[8], which is generally justified with the central limit theorem. However, it has been observed that the ambient noise often exhibits non-Gaussian nature in wireless channels, mostly due to the impulsive nature originated from various sources such as car ignitions, moving obstacles, lightning in the atmosphere, and reflections from sea waves [9], [10]. The conventional estimation schemes developed under the Gaussian assumption on the ambient noise could suffer from severe performance degradation under such non-Gaussian noise channels.

In this paper, we propose robust FO estimation schemes in non-Gaussian noise channels. First, we derive a maximum-likelihood (ML) FO estimation scheme in non-Gaussian noise modeled as a complex isotropic Cauchy noise, and then, derive a simpler estimation scheme with a lower complexity. From numerical results, the proposed schemes are confirmed to offer a substantial performance improvement over the conventional scheme in non-Gaussian noise channels.

The rest of this paper is organized as follows. Section II introduces the related works on the FO estimation, and the signal model is described in Section III. In Section IV, two FO estimation schemes are proposed for OFDM systems in non-Gaussian noise environments. Section V demonstrates the numerical results. Section VI concludes this paper.

II. RELATED WORKS

Several schemes [6]-[8] have been proposed to estimate the FO of OFDM signals assuming the Gaussian noise environments. The FO estimation scheme in [6] uses a training symbol with two identical halves to estimate the FO within the sub-carrier spacing. Then, using the other training symbol containing a pseudonoise (PN) sequence, the scheme corrects the remaining FO that is a multiple of the sub-carrier spacing. The scheme in [7] uses the best linear unbiased estimation (BLUE) principle requiring only one training symbol with more than two identical parts. Moreover, its estimation performance is quite close to the Cramer-Rao lower bound (CRLB). In [8], joint ML FO estimation scheme was derived when the training symbol is repeated multiple times. Specifically, the scheme in [8] exploits the correlation of any pair of repetition patterns providing optimized performance in the OFDM systems.

III. SIGNAL MODEL

The kth OFDM sample $x(k)$ is generated by the inverse fast Fourier transform (IFFT), and can be expressed as

$$x(k)=\frac{1}{\sqrt{N}}\sum_{m=0}^{N-1}X_m e^{j2\pi km/N}, \tag{1}$$

for $k=0,1,\cdots,N-1$, where X_m is a phase shift keying (PSK) or quadrature amplitude modulation (QAM) symbol in the mth subcarrier and N is the size of the IFFT. Then, the cyclic prefix (CP) of the OFDM symbol is inserted, whose length is generally designed to be longer than the channel impulse response, to avoid the intersymbol

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interference (ISI). Assuming that the timing synchronization is perfect, we can express the kth received OFDM sample $r(k)$ after removing the CP as

$$r(k) = \sum_{l=0}^{L-1} h(l)x(k-l)e^{j2\pi k\varepsilon/N} + n(k) \tag{2}$$

for $k = 0,1,\cdots,N-1$, where $h(l)$ is the lth channel coefficient of a multipath channel with length L, ε is the FO normalized to the subcarrier spacing $1/N$, and $n(k)$ is the kth sample of additive noise.

In this paper, we adopt the complex isotropic symmetric α stable (CISαS) model for the independent and identically distributed noise samples $\{n(k)\}_{k=0}^{N-1}$ this model has been widely employed due to its strong agreement with experimental data [11], [12]. The probability density function (pdf) of $n(k)$ is then given by [11]

$$f_n(\rho) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\gamma\left(u^2+v^2\right)^{\frac{\alpha}{2}} - j\Re\{\rho(u-jv)\}} dudv, \tag{3}$$

where $\Re\{\cdot\}$ denotes the real part, the dispersion $\gamma > 0$ is related to the spread of the pdf, and the characteristic exponent $\alpha \in (0,2]$ is related to the heaviness of the tails of the pdf: A smaller value of α indicates a higher degree of impulsiveness, whereas a value closer to 2 indicates a more Gaussian behavior.

A closed-form expression of (3) is not known to exist except for the special cases of $\alpha = 1$ (complex isotropic Cauchy) and $\alpha = 2$ (complex isotropic Gaussian). In particular, we have

$$f_n(\rho) = \begin{cases} \dfrac{\gamma}{2\pi}\left(|\rho|^2 + \gamma^2\right)^{-\frac{3}{2}}, & \text{when } \alpha = 1 \\ \dfrac{1}{4\pi\gamma}\exp\left(-\dfrac{|\rho|^2}{4\gamma}\right), & \text{when } \alpha = 2. \end{cases} \tag{4}$$

Due to such a lack of closed-form expressions, we concentrate on the case of $\alpha = 1$: We shall see in Section V that the estimation schemes obtained for $\alpha = 1$ are not only more robust to the variation of α, but they also provide a better performance for most values of α, than the conventional estimation scheme.

IV. Proposed Schemes

A. Maximum-likelihood FO Estimation Scheme

In estimating the FO, we consider a training symbol $\{x(k)\}_{k=0}^{N-1}$ with two identical halves as in [6], i.e., $x(k) = x(k+N/2)$ for $k = 0,1,\cdots,N/2-1$. From (2), we have

$$r(k+N/2) - r(k)e^{j\pi\varepsilon} = n(k+N/2) - n(k)e^{j\pi\varepsilon} \tag{5}$$

for $k = 0,1,\cdots,N/2-1$. Observing that $n(k+N/2) - n(k)e^{j\pi\varepsilon}$ obeys the complex isotropic Cauchy distribution with dispersion 2γ (since the distribution of $-n(k)e^{j\pi\varepsilon}$ is the same as that of $n(k)$, and assumed that the noise samples of CISαS model are independent as in [13]), we obtain the pdf

$$f_{\mathbf{r}}(\mathbf{r} \mid \varepsilon) = \prod_{k=0}^{\frac{N}{2}-1} \frac{\gamma}{\pi\left(\left|r(k+N/2) - r(k)e^{j\pi\varepsilon}\right|^2 + 4\gamma^2\right)^{\frac{3}{2}}} \tag{6}$$

of $\mathbf{r} = \{r(k+N/2) - r(k)e^{j\pi\varepsilon}\}_{k=0}^{N/2-1}$ conditioned on ε. The ML estimation is then to choose $\hat{\varepsilon}$ such that

$$\begin{aligned} \hat{\varepsilon} &= \arg\max_{\tilde{\varepsilon}}[\log f_{\mathbf{r}}(\mathbf{r} \mid \tilde{\varepsilon})] \\ &= \arg\min_{\tilde{\varepsilon}}\Lambda(\tilde{\varepsilon}), \end{aligned} \tag{7}$$

where $\tilde{\varepsilon}$ denotes the candidate value of ε and the log-likelihood function $\Lambda(\tilde{\varepsilon}) = \sum_{k=0}^{N/2-1}\log\left\{\left|r(k+N/2) - r(k)e^{j\pi\tilde{\varepsilon}}\right|^2 + 4\gamma^2\right\}$ is a periodic function of $\tilde{\varepsilon}$ with period 2: The minima of $\Lambda(\tilde{\varepsilon})$ occur at a distance of 2 from each other, causing an ambiguity in estimation. Assuming that ε is distributed equally over positive and negative sides around zero, the valid estimation range of the ML estimation scheme can be set to $-1 < \varepsilon \le 1$, as in [6]. The estimation scheme (7) will be called the Cauchy ML estimation (CMLE) scheme.

B. Low-complexity FO Estimation Scheme

The CMLE scheme is based on the exhaustive search over the whole estimation range ($|\varepsilon| \le 1$), which requires high computational complexity. Thus, we propose a low-complexity FO estimation scheme with the reduced set of the candidate values.

In order to obtain the reduced set of the candidate values, we exploit the property that $\varepsilon = 1/\pi\angle\{x^*(k)x(k+N/2)\} = 1/\pi\angle\{r^*(k)r(k+N/2)\}$ in the absence of noise. Based on this property, we obtain the set of the candidate values

$$\bar{\varepsilon}(k) = \frac{1}{\pi}\angle\{r^*(k)r(k+\frac{N}{2})\}, \text{ for } k = 0,1,\cdots,\frac{N}{2}-1. \tag{8}$$

Exploiting the set of the candidate values in (8), the FO estimate $\hat{\varepsilon}_L$ can be obtained as follows

$$\hat{\varepsilon}_L = \arg\min_{\bar{\varepsilon}(k)}\Lambda(\bar{\varepsilon}(k)), \quad \text{for} \quad k = 0,1,\cdots,\frac{N}{2}-1. \tag{9}$$

In the following, (9) is denoted as the low-complexity CMLE (L-CMLE) scheme. Using only $N/2$ candidate values, the L-CMLE scheme can offer an almost same performance as the CMLE scheme with the exhaustive search, the performance verified by numerical results in Section V.

V. NUMERICAL RESULTS

In this section, the proposed CMLE and L-CMLE schemes are compared with the Gaussian ML estimation (GMLE) scheme in [6] in terms of the mean squared error (MSE) by computer simulations using Matlab program and computational complexity. We assume the following simulation parameters: The IFFT size $N = 64$, FO $\varepsilon = 0.25$, length 8 samples of CP, the interval of search spacing 0.001 for the CMLE scheme, and a multipath Rayleigh fading channel with length $L = 8$ and an exponential power delay profile of $E[|h(l)|^2] = \exp(-l/L)/\{\sum_{l=0}^{L-1}\exp(-l/L)\}$ for $l = 0,1,\cdots,7$, where $E[\cdot]$ denotes the statistical expectation. Since CISαS noise with $\alpha < 2$ has an infinite variance, the standard signal-to-noise ratio (SNR) becomes meaningless for such a noise. Thus, we employ the geometric SNR (GSNR) defined as $\frac{E[|x(k)|^2]}{4C^{-1+2/\alpha}\gamma^{2/\alpha}}$, where $C = \exp\{\lim_{m\to\infty}(\sum_{i=1}^{m}\frac{1}{i} - \ln m)\} \simeq 1.78$ is the exponential of the Euler constant [14]. The GSNR indicates the relative strength between the information-bearing signal and the CISαS noise with $\alpha < 2$. Clearly, the GSNR becomes the standard SNR when $\alpha = 2$. Since γ can be easily and exactly estimated using only the sample mean and variance of the received samples [15], it may be regarded as a known value: Thus, γ is set to 1 without loss of generality.

Figs. 1-4 show the MSE performances of the CMLE, L-CMLE, and GMLE schemes as a function of the GSNR when $\alpha = 0.5, 1, 1.5,$ and 2, respectively. From the figures, we can clearly observe that the CMLE and L-CMLE schemes have a better estimation performance compared with that of the GMLE scheme for most values of α, except for $\alpha = 2$. Another important observation is that the estimation performance of the L-CMLE scheme is almost same as that of the CMLE scheme. From this observation, it is confirmed that the trial values for the L-CMLE scheme is reasonable. Numerical results show that proposed schemes not only outperform the conventional scheme in non-Gaussian noise environments, but also provide similar performance in Gaussian noise ($\alpha = 2$) environments. This can clearly explain a robustness of proposed schemes to the variation of the channel environments. In short, when the type of the noise is not known, the L-CMLE scheme can be an effective solution with robust performance to the noise.

Table I shows the computational complexity of CMLE, L-CMLE, and GMLE schemes, where S denotes the number of search spacing for the CMLE scheme. The GMLE scheme requires $(3N-2)$ real additions and $(2N+1)$ real multiplications only. On the other hand, the CMLE scheme requires $SN(3N-1)$ real additions and $SN(5N/2)$ real multiplications by choosing the most likely candidate among the SN candidates. Using $N/2$ reliable candidates only, the L-CMLE scheme reduced the number of operations to $N/2(3N-1)+N$ real additions and $(N/2+1)(5N/2)$ real multiplications.

TABLE I. COMPUTATIONAL COMPLEXITY OF THE FO ESTIMATION SCHEMES

	CMLE	L-CMLE	GMLE
Number of candidates	SN	$N/2$	-
Real additions	$3N-1$ per candidate	$3N-1$ per candidate + N	$3N-2$
Real multiplications	$5N/2$ per candidate	$5N/2$ per candidate $+5N/2$	$2N+1$

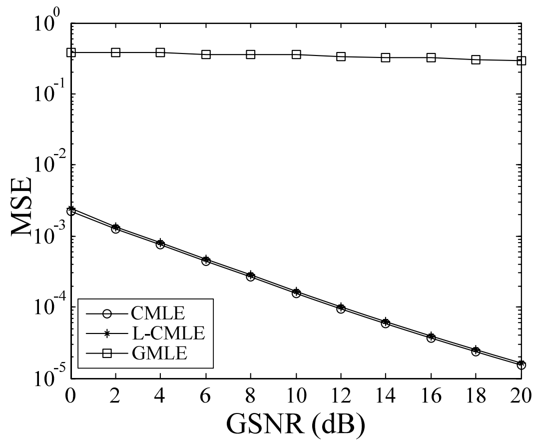

Figure 1. The MSE performances of the CMLE, L-CMLE, and GMLE schemes as a function of the GSNR when $\alpha = 0.5$.

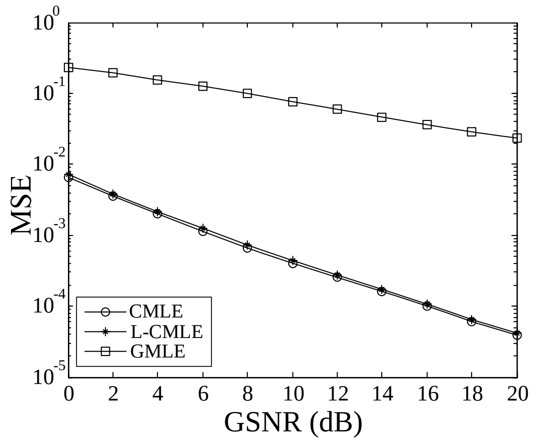

Figure 2. The MSE performances of the CMLE, L-CMLE, and GMLE schemes as a function of the GSNR when $\alpha = 1$.

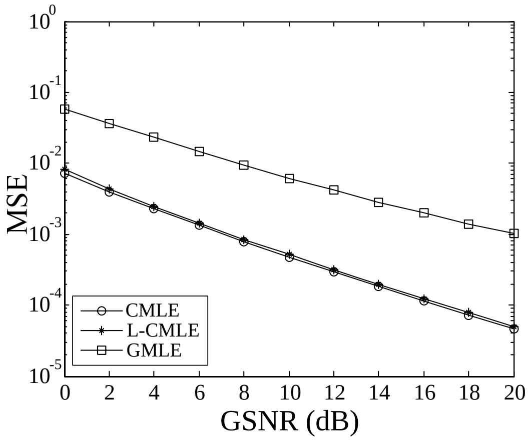

Figure 3. The MSE performances of the CMLE, L-CMLE, and GMLE schemes as a function of the GSNR when $\alpha = 1.5$.

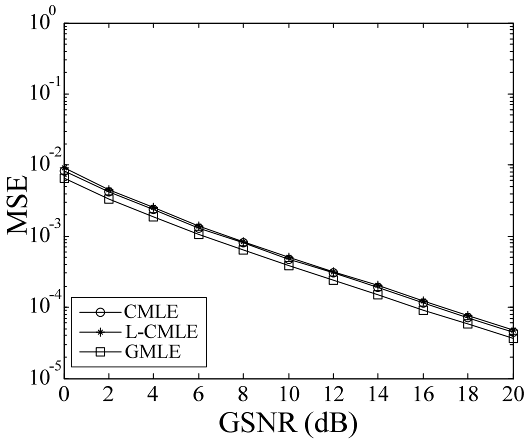

Figure 4. The MSE performances of the CMLE, L-CMLE, and GMLE schemes as a function of the GSNR when $\alpha = 2$.

VI. CONCLUSION

In this paper, we have proposed FO estimation schemes in non-Gaussian noise channels. First, an ML estimation scheme in non-Gaussian noise channel has been proposed, and then a simpler estimation scheme based on the ML estimation scheme has been presented. From the numerical results, it has been confirmed that the proposed schemes offer robustness and a substantial performance improvement over the conventional estimation scheme in non-Gaussian noise channels.

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