PAPR Reduction of OFDM Signals using Partial Transmit Sequence and Clipping Hybrid Scheme

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***Abstract*—The Orthogonal Frequency Division Multiplexing is one of the widely used modulation techniques in the broadband wireless technology. One of the main problems is the high peak-to-average power ratio of transmitted signal due to the superposition of many subcarriers. This paper presents a new hybrid peak-to-average power ratio reduction technique, which combines a partial transmit sequence method with the clipping method. The paper highlights the performance and advantages of the mixed technique and compares it with other existing methods. The simulations shown that the proposed technique realizes an increased peak-to-average power ratio with a decreased signal distortion compared to clipping method.**

Keywords-OFDM; PAPR; Partial Transmit Sequence; Clipping

I. INTRODUCTION

The Orthogonal Frequency Division Multiplexing (OFDM) is used in broadband wireless communication systems like Worldwide Interoperability for Microwave Access (WiMAX), Terrestrial Digital Video Broadcast (DVB-T), or wireline systems like Asymmetric Digital Subscriber Line (ADSL). The main problem of the OFDM is the high value of Peak-to-Average Power Ratio (PAPR) of the transmitted signal. Due to the superposition of the many data subcarriers, the OFDM signal exhibits Rayleigh-like characteristics. The large amplitude variations lead to high values for the PAPR. These peaks require the high power amplifiers (HPA) to support wide linear dynamic range.

Higher signal level at the input of HPA causes non-linear distortions at its output, leading to an inefficient operation of HPA. These distortions cause intermodulation products resulting unwanted out-of-band power. In order to reduce the PAPR of OFDM signals, many solutions have been proposed and analyzed. Some of the main characteristics of these methods are non-linearity, computation complexity and size of side information needed to be sent to receiver.

Some of the well known linear methods are selective mapping (SLM) [1], partial transmit sequence (PTS) [2], and tone reservation (TR) [5].

In the SLM method, the vectors from the original frequency domain OFDM signal are rotated based on a set of predefined phase arrays. For each signal variant obtained, its corresponding PAPR is evaluated. The one with the lowest PAPR is chosen for the transmission.

The PTS method uses a similar principle with the difference that same rotation angle is applied to more than one vector. The method considers the N complex values representing OFDM signal vectors as being grouped into K sub-blocks of N/K elements each. The case of blocks with contiguous carriers has the advantage of simplicity and is more suitable for detection systems. The case of non-contiguous carrier blocks offers better peak factor (PF) reduction capability at the cost of extra complexity.

The method generates a set of signal variants by rotating the vectors from each block with one phase from a given set of K phases with values from a given finite set. Then, after calculation of the corresponding PAPR of each signal variant, the one with minimal PAPR is being chosen for the transmission.

The efficiency of these methods increases with the number of phases from the considered set. The efficiency of the PTS method also increases when a higher number of blocks are used. The disadvantage is that a better efficiency requires an increased amount of computation at the transmitter's side and receiver's side. Because the receiver must know those phases' sets and block sizes, another drawback of these methods is the additional information required to be sent to receiver. Optimizations of those methods have been proposed in several papers [3, 4].

The TR method represents another linear technique which instead of altering the existing data subcarriers modifies the vectors from an additional set of non-data subcarriers. The method calculates the signal's PAPR values which correspond to the allocated reserved subcarriers. The signal replica corresponding to minimal PAPR is chosen for the transmission.

The important advantage of this technique is that at receiver's side no additional information and no computation are required. Because not all subcarriers are used to transmit useful information, this method has the disadvantage of a lower data rate.

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In order to reduce the computation complexity and to improve the performance, several PAPR reduction techniques were derived from the original tone reservation method: selective mapping of partial tones (SMOPT) [6], One-Tone One-Peak (OTOP) [7] and one-by-one iteration [8], fast TR described in [5].

The class of non-linear methods is represented by approaches like clipping, partial clipping (PC), signal compression and active constellation extension (ACE).

The clipping method is another very well known non-linear PAPR reduction technique, where the amplitude of the signal is limited to a given threshold.

Taking in consideration the fact that the signal must be interpolated before A/D conversion, a variety of clipping methods has been proposed. Some methods suggest the clipping before interpolation, having the disadvantage of the peaks regrowth. Other methods suggest the clipping after interpolation, having the disadvantage of out-of-band power production. In order to overcome this problem, different filtering techniques have been proposed. Filtering can also cause peak regrowth, but less than the clipping before interpolation [9].

Signal compression is another group of non-linear methods which improves the PAPR reduction. For this purpose some papers proposed μ-law/A-law companding functions [13], exponential companding function [12], piecewise-scales [11] or polynomial ratio functions [14] after the clipping. One of the drawbacks of these methods is the increased noise level generated by the corresponding signal decompression.

The partial clipping is another nonlinear PAPR reduction method which performs additional signal processing in frequency-domain to reduce the distortions of the signal's spectrum. This method supposes that only subcarriers having the highest phase difference between the original signal and its clipped variant will be changed [10].

The ACE method relays on the idea that the outer points from the original constellation may be moved toward outside in order to reduce the PAPR level of the transmitted signal. The domain for allowed alternative points is chosen so that the signal processing does not reduce the constellation's minimum-distance but lowers the PAPR level [15-17].

The rest of the paper is organized as follows. The second section describes the OFDM signal, some of its properties, and some aspects of the high power amplifier. The third section describes the proposed hybrid PAPR reduction scheme. Next, the numerical results highlighted by the computer simulation are presented and discussed. Based on the results obtained, some conclusions are presented.

II. THE OFDM SIGNAL

An OFDM-based communication system uses a complex multicarrier modulation. Each of the N subcarriers, having the frequencies $\{f_n, n=0,1,\ldots,N-1\}$, are modulated with a data sample from a block of symbols $\{X_n, n=0,1,\ldots,N-1\}$.

In order to reduce the bandwidth of the required frequency spectrum, the subcarriers are chosen to be orthogonal, that is $f_n=n\Delta f$, where $\Delta f=1/T$, and T is the OFDM symbol period.

Considering these aspects, the resulting signal can be written as:

$$x(t)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}X_n e^{j2\pi f_n t}\ . \tag{1}$$

In the many cases, the OFDM-based system communicates through multipath channels, which generate intersymbol interference (ISI). In order to avoid this problem, a fraction from the current signal period, corresponding to a guard interval, is added to the signal. After Digital-to-Analogue (D/A) conversion, the signal is modulated and applied to a HPA. The amplified signal is applied to the antenna.

At the receiver, after demodulation, the guard interval will be removed, the symbols being evaluated for time intervals of length T.

Due to statistical independence of subcarriers, the low-pass time-domain OFDM signal in the complex domain presents a Gaussian distribution. Therefore, sporadically, the signal presents peaks, causing the PAPR problem. The expression of the PAPR for a given OFDM signal block is given by:

$$PAPR(x)=\frac{\max\left(|x(t)|^2\right)}{E\left[|x(t)|^2\right]}, \tag{2}$$

where E[.] denotes the expectation operator.
The PAPR is usually evaluated using the complementary cumulative distribution function (CCDF) of the PAPR:

$$\begin{aligned} CCDF(Y) &= \Pr(PAPR > Y) = \\ &= 1-\Pr(PAPR < Y). \end{aligned} \tag{3}$$

The non-linearity of the transmitted signal, produced by the HPA, can be evaluated with another quality measure. This is the Signal-to-Distortion Ratio (SDR) defined as:

$$SDR=\frac{\|x\|^2}{\|x-g(x)\|^2}, \tag{4}$$

where g(.) is the memoryless nonlinearity representing the effects of the HPA.

These effects are described by various models. One of the well known models is the Saleh Model, which is described by the following input-output equations:

$$A_{HPA}(u)=\frac{\alpha\cdot u}{1+\beta\cdot u^2}, \tag{5}$$

$$P_{HPA}(u)=\frac{\alpha\cdot u^2}{1+\beta\cdot u^2}, \tag{6}$$

where $A(u)$ is the amplitude transfer function, and $P(u)$ is the phase transfer function and α and β are two parameters.
An example for the parameter values α=2 and β=1, is presented in Figure 1.

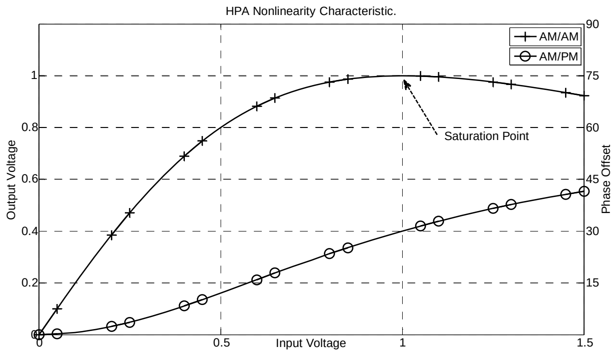

Figure 1. Example of a HPA nonlinear characteristic. Amplitude and phase transfer functions are presented.

The optimal solution for PAPR problem may not be the best solution for the SDR problem and vice versa. Because these two problems are correlated, in practice, a suboptimal solution may be chosen [15].

III. THE HYBRID METHOD

In this section, we present the proposed hybrid PAPR reduction technique which has been obtained by the association of PTS method with clipping method.

The main idea for combining the two methods is relying on the observation that the cumulative signal processing for PAPR reduction significantly improves the overall outcome. Furthermore, the hybrid technique exploits the fact that each of the component methods is based on a different principle.

One performs linear transformation by rotating the vectors from the frequency-domain signal, and the other one performs a non-linear transformation represented by signal limitation. The block diagram of the proposed method is presented in Figure 2.

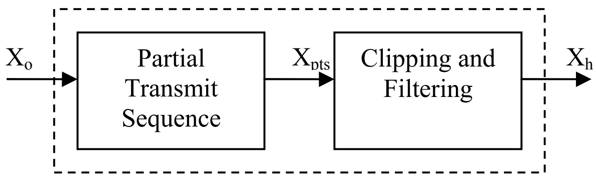

Figure 2. The Hybrid PTS-CLP scheme for PAPR reduction.

The performance of the proposed PAPR reduction technique is analyzed with a MATLAB simulator as presented in Figure 3. Within this simulator, the samples from the generated signal are mapped from binary representation to the M-QAM or M-PSK constellation points. The obtained complex values are grouped in blocks of N elements each, forming the OFDM symbols. The obtained OFDM frames are applied sequentially to PTS block and then to clipping block. For a better evaluation of the proposed method, the results obtained only from clipping [9] are also considered.

The parameters of the resulting signal change according with the signal processing applied by the two PAPR reduction methods.

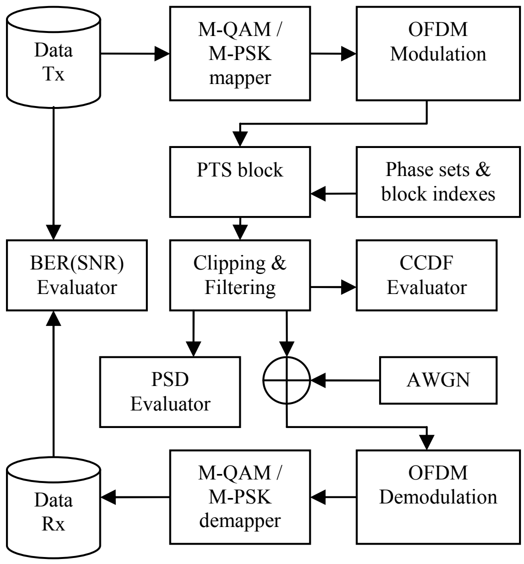

Figure 3. MATLAB model for the analysis of the hybrid PAPR reduction technique.

In order to evaluate the performance and efficiency of o communication system based on the proposed PAPR reduction method, the simulator computes the bit error rate (BER) and power spectral density (PSD) for the original and processed signals.

The PTS method operates on the frequency-domain signal iteratively until the best signal derivate is found. As already mentioned, the main idea of this method is to change the phases of the vectors composing the signal. This method considers the signal's vectors as being grouped in disjoint blocks. The vectors from a block may have a contiguous displacement, or they may be interleaved with the vectors representing another block. The algorithm applies one phase shift for each bloc iteratively until the signal variant having the lowest PAPR is found.

In the present work, the PTS method was implemented considering contiguous blocks of same length each. Additionally, for a better PAPR reduction, the proposed PTS method performs position swap between these blocks. This procedure is depicted in Figure 4.

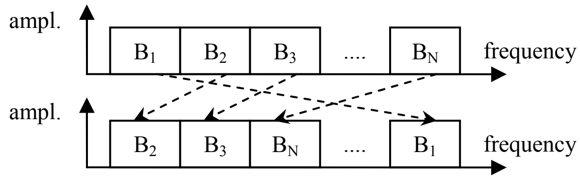

Figure 4. Position swaping between blocks of the OFDM signal.

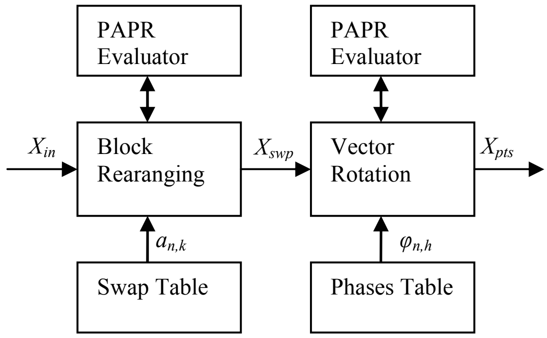

Figure 5. The PTS derivate method for PAPR reduction.

Therefore, the PTS method considered in this paper consists in two stages as the Figure 5 shows.

Based on this block diagram, the PTS algorithm consists in the following steps.

1) Starting with a set of N data symbols X_{in}, representing an OFDM frame:

$$x_{in}(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{in}(n) e^{j2\pi f_n t} , \quad (7)$$

2) Rearrange the values X_{in} using the swap table $a_{n,k}$, where n=1...N is the index of a given vector, and k=1…P is the index of a signal variant from a set of P possibilities. Therefore the output signal can be written as:

$$X_{swp}(n,k) = X_{in}(a(n,k)). \quad (8)$$

3) Apply IFFT to get the corresponding time-domain signal representation $x_{swp}[n,k]$.

$$x_{swp}(t,k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{in}(a(n,k)) e^{j2\pi f_n t} . \quad (9)$$

4) Compute the PAPR for all P variants, and choose the one with lowest PAPR level for the following processing step.

5) Rotate all the constellation vectors from X_{swp} using the phase table $\varphi_{n,k}$, where n=1...N is the index of a vector, and h=1…R is the index of the signal variant from a set of R possibilities. The signal becomes:

$$X_{pts}(n,h) = X_{swp}(n) \cdot e^{j\varphi(n,h)}. \quad (10)$$

6) Apply IFFT to get the corresponding time-domain signal representation $x_{pts}[n,h]$.

$$x_{pts}(t,h) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{pts}(n) e^{j(2\pi f_n t + \varphi(n,h))} . \quad (11)$$

7) Compute the PAPR for all R variants, and choose the one with lowest PAPR level for the transmission.

The signal X_{pts} is derived from X_{in} which has modified values for both amplitude and phase of all vectors. Because the receiver must reconstruct the original signal by applying inverse operations, it has to know the block swapping tables and rotation angle tables. For this purpose, it is necessary either to consider predefined tables or to transmit additional information to receiver.

An important drawback of this method is that an increased size of these tables implies increased computation complexity. Furthermore, if the number of block permutations is too high, the possibility for signal reconstruction decreases very much. In such cases, searching of different block position exchanges is not enough, additional information being required.

In order to overcome this problem, we propose a block marking scheme, as presented in Figure 6. The main idea of this model is to use one data carrier as label for each block.

The PTS method changes the phases of all vectors within all blocks. If vector rotation is avoided for the marking subcarriers, PAPR level may not be efficiently reduced. Therefore, the marking model can not relay on the phase information from the additional subcarriers. Additionally amplitude information may not be enough for block indexing, when amplitude levels are less than the number of blocks within the OFDM frame. More, the amplitude is significantly affected by noise.

In order to overcome these problems, the proposed block marking model, uses differential phase information. For this purpose two subcarriers are considered. One represents the phase reference, and the other one represents the block index.

Since amplitudes of these marking vectors do not carry any information, for an improved demodulation, these amplitudes can be increased.

In order to compensate any degradation of the PAPR reduction due to these marking tones, additional two subcarriers are considered. This technique is similar to the tone reservation PAPR reduction method described in [5].

Their phases should be opposite to the phases of the marking subcarriers. Since a tone compensation method is efficient when frequency difference is minimal, their positions are interleaved as in Figure 6.

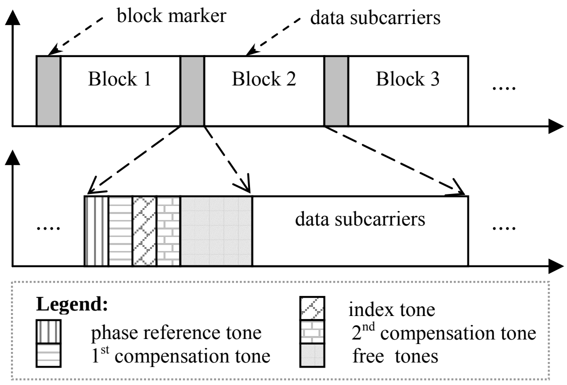

Figure 6. PTS block marking.

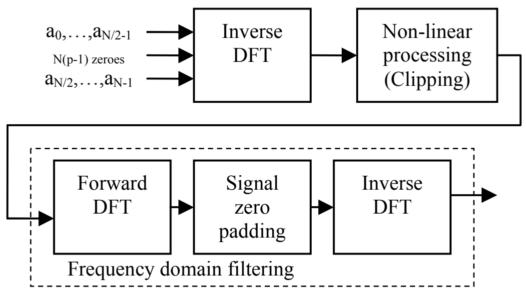

Figure 7. Clipping based PAPR reduction model.

The applied clipping technique [9] is presented in the block diagram from Figure 7.

Here, the input vector $[a_0,\ldots,a_{N-1}]$ is first converted from frequency to time domain using an oversized IFFT. For the oversampling factor p, the input vector is padded with $N(p-1)$ zeroes placed in the middle of the vector. This results in a trigonometric interpolation of the time domain signal, which fits well for the signals with integral frequencies over original FFT window, like is the case of OFDM. The interpolated signal is then clipped by limiting its amplitude.

The clipping ratio is defined as ratio of the clipping level A and the root-mean-square power σ of the unclipped baseband signal,

$$CR = 20 \cdot \log_{10}\left(\frac{A}{\sigma}\right). \qquad (12)$$

The proposed hybrid PAPR reduction technique contains both linear and non-linear signal processing blocks. This fact produces a high flexibility from the signal processing point of view.

When PTS block is configured to use a reduce set of signal variants, its smaller PAPR reduction can be compensated by the clipping block. This approach presents the advantage of a decreased amount of computation with the price of a higher signal distortion. When PTS block is configured to use an extended set of signal variants, the increased PAPR reduction requires a decreased signal distortion in the clipping block. The drawback in this case is represented by the increased amount of computation.

IV. NUMERICAL RESULTS

The MATLAB simulations have been performed for base-band signals with N=128 and N=256 subcarriers using M-QAM and M-PSK modulations, with corresponding constellations having M=16 points. For the clipping block was considered a clipping rate CR set to 12 and the oversampling factor p set to 2.

For the reference clipping-only method, the simulation considers the clipping rate CR having some values in range of 6-16 and the oversampling factor p set to 2 as well.

For the OFDM signal spectrum computation, it was considered that the distance between two adjacent subcarriers is 0.2 MHz.

The results presented in this paper are obtained using a phase rotation array containing 4 sets of phases with values $\varphi_k = k \cdot \pi / 2$, where k=0…3, and 4 sets of phases with values $\varphi_k = k \cdot 2\pi / 5$, where k=0…4. For the block swapping, the proposed method used a set of 8 randomly chosen combinations for the case when the OFDM signal is composed by 8 blocks of same length. The combinations are chosen in order to present a variety of blocks interleaving. Some of them exchange odd blocks with even blocks, other exchange blocks from first half with blocks from the second half. Also, there are cases when only neighborhood blocks are changed.

The numerical results show that the proposed scheme improves the PAPR reduction in comparison with the use of only one of the component methods. For the evaluation of the performance of the proposed technique, two cases are presented.

The first case considers an OFDM signal with N=128 subcarriers with a 16-QAM modulation. The Figure 8 and Figure 9 indicate the corresponding PAPR reduction and signal's frequency spectrum respectively.

In Figure 8, it can be observed that the modified PTS method with markers and the pure clipping method with CR=12 have similar PAPR reduction. The PAPR reduction is significantly better when the PTS method does not add block markers in the signal. The hybrid PTS-clipping method provides better PAPR reduction since it accumulates the effects from the two methods.

In Figure 9, it can be remarked that in the case of the modified PTS method the signal's spectrum is wider and has a different shape. This is caused by the insertion of additional subcarriers for the block marking and reserved tones. In this case each of the eight blocks contains two free reserved tones.

The second case considers an OFDM signal with N=256 subcarriers with a 16-PSK modulation. The clipping rate is set to same value. The resulted PAPR reduction and signal's frequency spectrum are presented in Figure 11 and Figure 12 respectively.

In Figure 11, it can be observed that the difference between the PAPR reduction of the PTS methods and clipping-only method is even smaller. However, the hybrid method provides better PAPR reduction for this case too.

In Figure 12, it can be remarked that in case of the alternative PTS method, the signal's spectrum is slightly wider, but less than the one from the previous case. This is due to the missing of the free reserved tones.

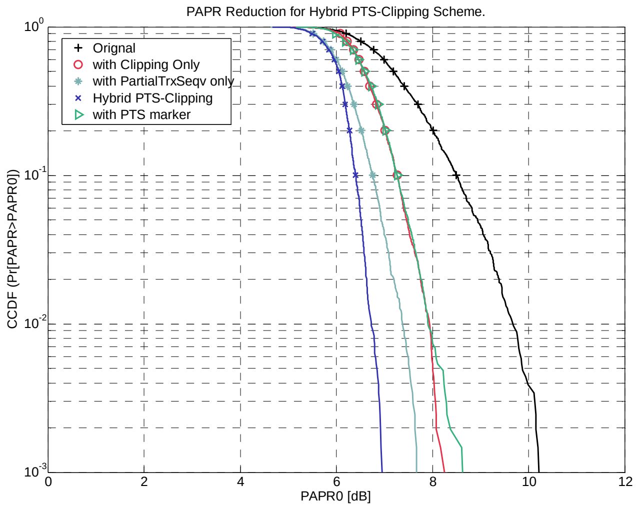

Figure 8. PAPR reduction using hybrid PTS-Clipping method.

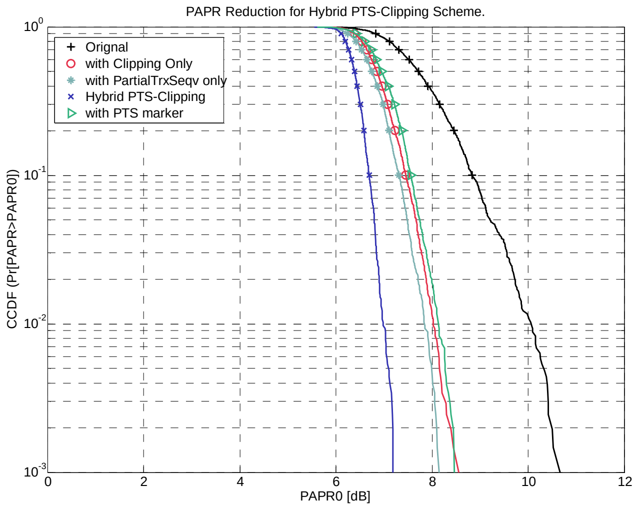

Figure 11. PAPR reduction using hybrid PTS-Clipping method.

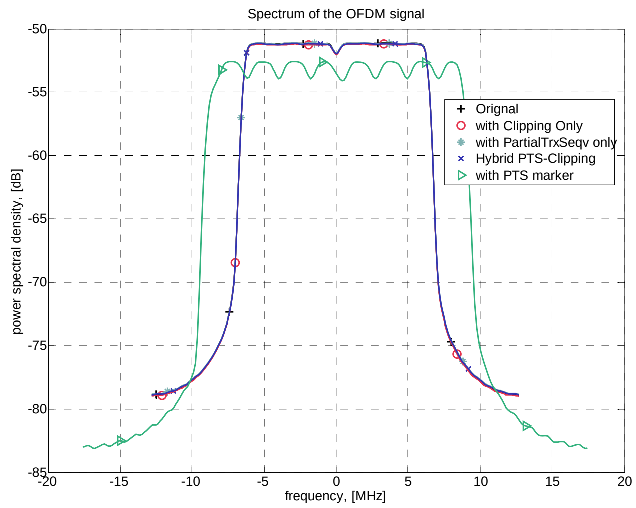

Figure 9. Spectrum of OFDM signal before and after PAPR reduction.

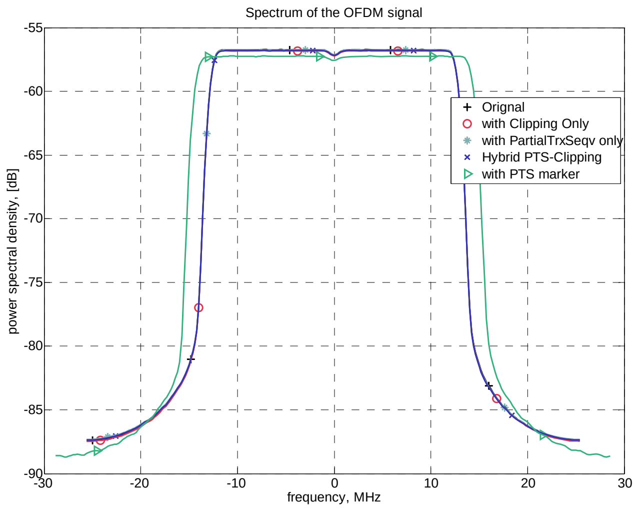

Figure 12. Spectrum of OFDM signal before and after PAPR reduction.

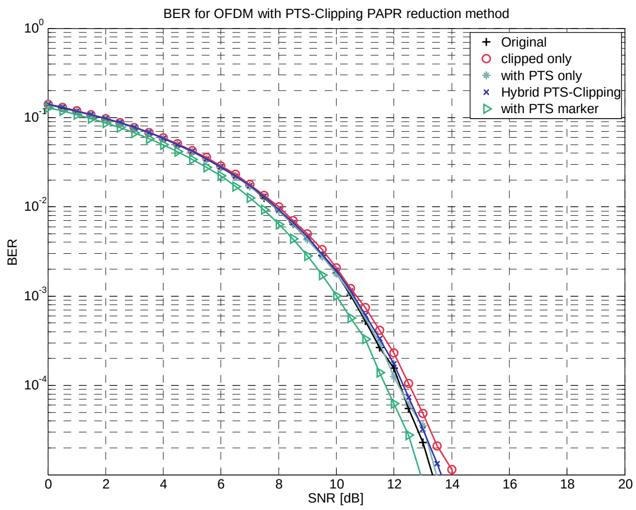

Figure 10. BER of OFDM signal before and after PAPR reduction.

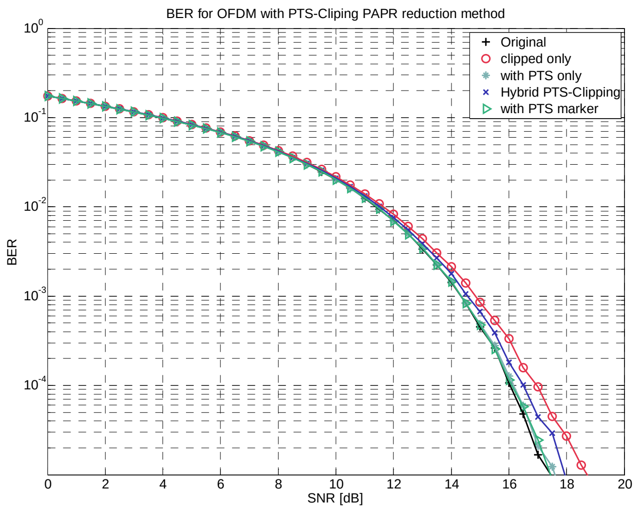

Figure 13. BER of OFDM signal before and after PAPR reduction.

Figure 10 and Figure 13 show that the BER performance is slightly influenced by the proposed PAPR reduction technique, being better than in case of simple clipping.

The PTS method applied in the hybrid scheme does not use the presented block marking technique. The PTS method which includes the block marking technique is separately evaluated.

The Power Spectral Density (PSD) characteristics presented in Figure 9 and Figure 12 highlighted two aspects of the block marking. The first case considers a block marking with two guard subcarriers per block. The second case considers no zero subcarriers within the blocks. The amplitude of the marker subcarriers was set to be equal with the mean value of all data subcarriers.

The required bandwidth for the OFDM signal resulted after the PTS derivate method has increased due of the additional subcarriers. The shape of the frequency spectrum has a slight variation according with the added guard subcarriers.

For the reference clipping method, some PAPR reduction curves together with their corresponding BER characteristics are presented in the Figure 14 and Figure 15, respectively.

The simulations have shown that a smaller value for the clipping ratios increases the PAPR reduction. The case of clipping only with *CR*=6 presents similar results for the PAPR reduction as those obtained with the hybrid method with the previously presented configurations.

The major disadvantage of the clipping only method is that its increased signal distortion leads to a considerable lower BER performance.

Therefore, the presented numerical results shown that, in all cases, the hybrid PTS-clipping method provides better PAPR reduction than in case of use of only one method. Additionally, compared with clipping only method, the combined technique presents insignificantly degradation of the BER performance.

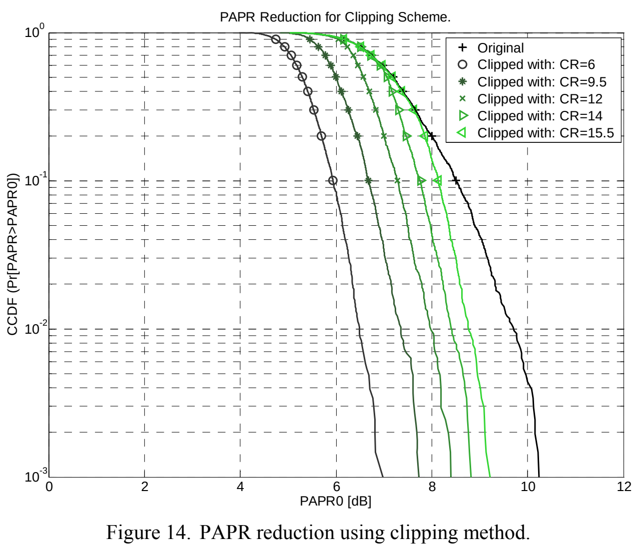

Figure 14. PAPR reduction using clipping method.

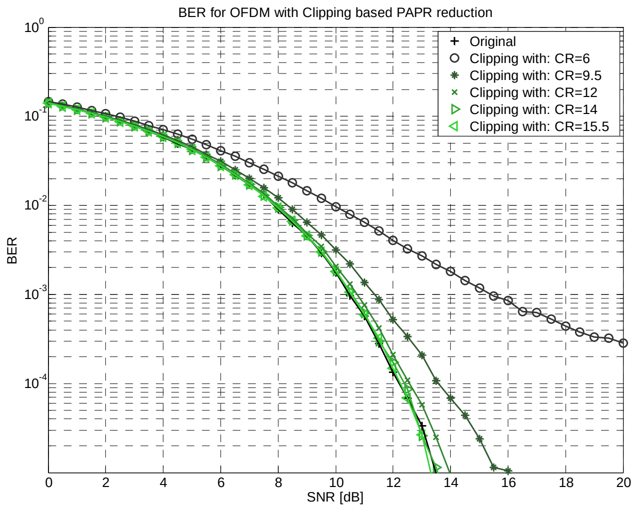

Figure 15. BER of the OFDM signal after clipping.

The computational complexity of the algorithm of the hybrid technique is given by the sum of the computational complexity of the component methods.

The PTS block performs one IFFT, a rearranging of the *N* subcarriers and change of the phase of all *N* subcarriers per iteration. Considering the fact that that the PTS method has to perform *P* block rearranging and *R* phase changes, the amount of operations corresponding to this component method is $O\left(\left(P+R\right)\cdot\left(1+N\cdot\log_2\left(N\right)\right)\right)$.

For the clipping block, the computation complexity is given by the cumulated amount of complexity for effective clipping and frequency-domain filtering.

The effective clipping block requires one IFFT on the frequency-domain signal with zero padding and $N \cdot p$ amplitude limitations operation on the time-domain signal. Therefore, the amount of operations for this sub-block is $O\left(N\cdot p\cdot\left(1+\log_2\left(N\cdot p\right)\right)\right)$.

The frequency domain filtering performs one FFT, one IFFT and $N \cdot (p-1)$ vector resets. Therefore, the amount of operations corresponding to this sub-block is $O\left(N\cdot\left(p-1\right)+2\cdot N\cdot p\cdot\log_2\left(N\cdot p\right)\right)$.

Finally, the computation complexity for the whole hybrid PAPR reduction method is given by the sum of the three computation complexities.

V. CONCLUSION AND FUTURE WORK

In this paper we proposed an alternative PAPR reduction technique based on the combination of a partial transmit sequence method with the clipping method. The PTS and clipping algorithms used within the hybrid technique are explained.

The numerical results show that the hybrid scheme brings higher PAPR reduction than the component methods.

The considered two methods have few variants with different efficiency and performance.

The PTS method may be implemented using different block structures, swapping tables and rotation angle tables. Additionally, the block marking can be implemented in various manners, determining different efficiency for PAPR reduction and different sizes of the search space at the receiver's side.

The clipping method, may consider various filtering techniques, each of them leading to various PAPR reduction efficiency.

Therefore the efficiency of the hybrid method may vary when derivates of the component methods are used.

In future work, we will consider different block configurations and different set of values for the tables within the PTS block. Some clipping derivates also may be considered.

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