Spectrum Sensing Using Sub-Nyquist Rate Sampling

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***Abstract*—Spectrum sensing in wideband regime requires huge amount of samples. The observed frequency spectrum is usually sparse. Compressed sensing technique provides a viable solution to reconstruct the sparse signals. The observed wideband spectrum can be reconstructed using compressive sensing technique. Inherent constraints of the compressed sensing algorithms hinder the flexible implementation of spectrum sensing process. The structure-based Bayesian sparse recovery algorithm is used in this paper to implement spectrum sensing process. Spectrum sensing performed using the Bayesian estimation approach resulted in better performance compared to the results based on compressed sensing technique. Various cases have been discussed considering the amount of information available for the observed frequency band. Spectrum sensing performed using the Bayesian algorithm showed improvement of more than 5 dB in all cases.**

Keywords; cognitive radio; spectrum sensing; compressive sensing; structure-based bayesian sparse recovery algorithm.

I. INTRODUCTION

The ever-increasing high data rate services and new wireless service providers require more frequency spectrum than available. This appetence of more frequency spectrum has raised a concern of spectrum scarcity. The frequency spectrum is a limited natural resource. Measurements have shown that the current spectrum scarcity is a result of under-utilization rather than the unavailability of spectrum. According to Federal communication comission [1], the spectrum utilization varies from 15% to 85% with high variance in time and space. These statistics puts question on the appropriateness of current regulatory authorities. To overcome this problem, Mitola and Maguire [2] introduced the cognitive radio device in 1999. The cognitive radio (CR) provides an adequate solution to the observed concern of spectrum scarcity. The CR avails opportunistic access to the frequency bands that are not used by the licensed users at a particular instance or space [2].

This paper focuses on performing spectrum sensing in wideband regime. The spectrum sensing process is core of the CR system. It enables the CR to scan range of frequencies and utilize any vacant ones. This process has many challenges associated with it. One key problem relates to the sensing of a wideband signal. Perpetually sampling of signal is done at the Nyquist rate. In the wideband regime, this means acquisition of colossal amount of samples and respectively high sensing time. In this paper solution to the wideband spectrum sensing problem is discussed using the sub-Nyquist rate sampling technique.

Over the years, many algorithms (based on the Nyquist rate sampling criteria) have been developed for the spectrum sensing process. Among them are the energy detection based sensing, wave form based sensing, cyclostationary feature based sensing and match filtering based sensing [4]. Sensing a wideband signal using these techniques require large amount of time. The compressive sensing (CS) technique provides reconstruction of the sparse signals sampled at less than the Nyquist rate [5].

Over the last few years some algorithms have been proposed that perform spectrum sensing using the compressive sensing technique. Some of these algorithms are discussed in this paragraph. In 2007, Tian and Giannakis [6] proposed the idea of performing spectrum sensing using the compressive sensing technique. As the observed signal is sparse in frequency domain, its frequency spectrum was reconstructed using the compressive sensing technique. The estimates of various frequency band locations (within the observed spectrum) were generated using the wavelet edge detection technique. The presence or absence of a primary user within each frequency band was determined by observing the corresponding power spectral density (PSD). In 2009, Polo et al. [7] used the analog to information Converter (AIC) instead of the analog to digital converter (ADC) at the receiver. An AIC can be conceptually viewed as an ADC operating at the Nyquist rate followed by the compressive sampling mechanism. In 2009, Chen et al. [9] improved the work proposed in [6]. A multi-branched spectrum sensing structure was proposed. Each branch repeats the same procedure proposed in [6], i.e., reconstructs the frequency spectrum of received signal and calculates the PSD within each band. The results from all branches were combined to generate a final estimate. In 2010, Nassab et al. [8] assumed a fixed number of frequency bands in the observed spectrum. The wideband filters were used to acquire energies from some frequency bands. As the complete energy vector of the observed spectrum is sparse in nature, it was recovered using the compressive sensing technique. In 2010, Sundman et al. [10] modified the

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proposed work of [7]. The autocorrelation vector achieved in [7] deals with the wide-sense stationary (WSS) signals only. However, the signal at the output of AIC is non-WSS. The autocorrelation vector was modified in order to deal with the non-WSS signals. They also proposed memory based spectral detection, which resulted in the overall reduction of computational complexity. In 2010, Liu and Wan [11] used the *a priori* knowledge of spectrum distribution and proposed a mixed l_2/l_1 norm de-noising operator. They suggested to attain the primary user frequency band information from the regulatory authorities. The *a priori* knowledge of the band gaps and the block sparsity resulted in better performance when compared to the standard mixed l_2/l_1 norm de-noising operator.

Though compressive sensing algorithms reconstruct the sparse signals with good probability, they do suffer from some deficiencies. They are computationally complex, do not use the structure of the sensing matrix and do not use the *a priori* statistical information about signal support and noise. These algorithms are bottlenecked by the number of observations. Increasing the number of observations leads to the better performance and vice versa. In order to overcome these shortcomings the structure-based Bayesian sparse recovery algorithm (SBBSR) is proposed in [12].

This paper focuses on utilizing the SBBSR algorithm and performing spectrum sensing at the sub-Nyquist rate. The SBBSR algorithm allows flexible implementation in contrast to the compressed sensing based algorithms. The rest of paper is organized as follows. Section II describes the spectrum sensing process performed using the SBBSR algorithm. Section III exploits the flexible implementation of SBBSR algorithm to improve performance. Simulations results are shown and discussed in section IV. Section V provides conclusion to this paper.

II. Spectrum Sensing Using the SBBSR Algorithm

The SBBSR algorithm provides reconstruction of sparse signals using the Bayesian estimation approach. While reconstructing the signal it uses the *a priori* statistical and sparsity information and the sensing matrix structure. Assume the sensing timing window is defined as $t \in [0, NT_0]$ (where T_0 represents the Nyquist sampling rate). According to the Nyquist theorem, N samples are required to reconstruct the original signal without aliasing. The sampling process at a digital receiver can be expressed as

$$\boldsymbol{y} = \boldsymbol{\Theta x} + \boldsymbol{n} \quad (1)$$

where $\boldsymbol{x}$ represents the $N \times 1$ length sparse vector, $\boldsymbol{\Theta}$ is an $M \times N$ projection matrix (or sensing matrix, which is incoherent with the domain in which $\boldsymbol{x}$ is sparse) and $\boldsymbol{n}$ is the complex additive white Gaussian noise vector $\mathcal{CN}(0, \sigma^2{}_n I)$. The process defined in (1) can be explained as the conversion of a continuous domain signal $\boldsymbol{x} \in \mathcal{C}^N$ into the discrete sequence $\boldsymbol{y} \in \mathcal{C}^M$. In (1) when $M = N$ the Nyquist rate uniform sampling is performed whereas setting $M < N$ performs the reduced rate sampling scheme or the sub-Nyquist rate sampling [6].

The sparse signal $\boldsymbol{x}$ can be modeled as

$$\boldsymbol{x} = \boldsymbol{x}_B \odot \boldsymbol{x}_G \quad (2)$$

where $\odot$ represents dot multiplication between the two vectors, $\boldsymbol{x}_B$ is an independent and identically distributed (i.i.d) Bernoulli random variable and the entries $\boldsymbol{x}_G$ can be drawn from any distribution. This model of $\boldsymbol{x}$ provides a sparse signal. The sparsity information is indulged by the Bernoulli random variable and the amplitudes of these observations are drawn from some other distribution [12].

If the support S of $\boldsymbol{x}$ is known it can be written as

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{\Theta x} + \boldsymbol{n} \\ &= \boldsymbol{\Phi \Psi x} + \boldsymbol{n} \\ \boldsymbol{y}|S &= \boldsymbol{\Theta}_S \boldsymbol{x}_S + \boldsymbol{n}_S \end{aligned} \quad (3)$$

$\boldsymbol{\Theta}_S$ is the sub-matrix formed from $\boldsymbol{\Theta}$ containing only those columns represented by S. The maximum a posteriori (MAP) estimate of observed signal $\boldsymbol{x}$ is given as [12]

$$\hat{\boldsymbol{x}}_{MAP} = arg\ max_S\, p(\boldsymbol{y}/S)\, p(S) \quad (4)$$

where $p(S)$ is the probability of a given support. Assuming the signal model of (2), the probability of support can be written as [12]

$$p(S) = p^S(1-p)^{N-S} \quad (5)$$

Now, the problem of calculating MAP narrows down to the calculation of $p(\boldsymbol{y}/S)$. In this paper it is assumed that primary user data has Gaussian distribution, $\boldsymbol{x}|S$ is Gaussian, then $\boldsymbol{y}|S$ will also be Gaussian with zero mean and covariance $\boldsymbol{\Sigma}_S$. Corresponding probability is calculated as [12]

$$p(\boldsymbol{y}/S) = \frac{exp(-\frac{1}{\sigma_n{}^2}\boldsymbol{y}^H \boldsymbol{\Sigma}_S{}^{-1}\boldsymbol{y})}{det(\boldsymbol{\Sigma}_S)} \quad (6)$$

where covariance matrix is given as

$$\boldsymbol{\Sigma}_S = \boldsymbol{I} + \frac{\sigma_x{}^2}{\sigma_n{}^2}\boldsymbol{\Theta}_S \boldsymbol{\Theta}_S{}^H \quad (7)$$

To perform the spectrum sensing process using SBBSR algorithm following steps are opted. These steps are also described in Fig. 1.

1- The sub-Nyquist rate sampled signal $\boldsymbol{y}$ is correlated with the sensing matrix $\boldsymbol{\Theta}$.
2- Based on the correlation result P clusters are made.

3- Let P_c denote the maximum possible support size in a cluster. For each cluster find the likelihoods for all support size starting from $l = 1,2, \dots P_c$.
4- Within each cluster the MAP estimates of corresponding likelihoods are calculated as explained in (4).
5- Decision regarding presence or absence of the primary user on certain frequency band is made based upon the MAP estimate. The indexes of maximum valued estimates correspond to the occupied locations by a user.

III. EXPLOITING FLEXIBLE IMPLEMENTATION OF THE SBBSR ALGORITHM

The SBBSR algorithm is used to recover the locations where transmission has been done by primary user. Numerous conditions can be imposed to enhance the sensing ability of a CR. These conditions have been discussed in this section and will be used in the simulation part.

- **Case 1**: Considering only signal sparsity as an assumption for spectrum sensing.
- **Case 2**: In addition to sparsity, assuming the observed spectrum consists of fixed (same) length frequency bands.

Consider a scenario in which *a priori* information about the primary user frequency band is available. As proposed in [11], regulatory authorities assign a certain frequency band to a user following the static spectrum allocation scheme. For instance, the bands 1710-1755 MHz and 1805-1850 MHz are allotted to GSM 1800. This also provides a hint that on a certain frequency band the primary users will appear in the form of clusters. For the observed frequency spectrum these details can be gathered *a priori* from the regulatory authority. Here, it is assumed that on a given spectrum all primary users have been assigned known and fixed length bands. One key advantage is the reduction of computational complexity. Earlier calculation of the estimates for various support sizes $l = 1, 2, \dots P_c$ was required. This resulted in calculation of 2^{P_c} estimates. Now with the length knowledge, the estimates for various support sizes are not required. One estimate is calculated for each cluster.

- **Case 3**: In addition to sparsity, assuming the observed spectrum consists of variable length frequency bands.

Assume that in the observed spectrum, variable length frequency bands are present. The length of these frequency bands is assigned based on some probability distribution function. Assume that this *a priori* length information is also known at the receiver.

Begin
Correlate Observation vector $\boldsymbol{y}$ with sensing matrix $\boldsymbol{\Theta}$
Form P semi-orthogonal clusters of length L each around the positions with high correlation values
Process each cluster independently and in each cluster calculate the likelihoods for support of size $l = 1,2, \dots P_c$
Evaluate MAP estimate
END

Figure 1. Spectrum Sensing Using SBBSR Algorithm

IV. SIMULATIONS

The sensing matrix in case of spectrum sensing is a partial inverse discrete Fourier transform (IDFT) matrix and is given as

$$\boldsymbol{\Theta} = \boldsymbol{S_c}^T \boldsymbol{F_N}^{-1} \quad (8)$$

where $\boldsymbol{S_c}$ is the identity matrix of size $N \times M$ and $\boldsymbol{F_N}^{-1}$ is the IDFT matrix of size $N \times N$. In this case the observation vector can be written as

$$\boldsymbol{y} = \boldsymbol{S_c}^T \boldsymbol{F_N}^{-1} \boldsymbol{x} + \boldsymbol{n} \quad (9)$$

$\boldsymbol{n}$ is the complex additive white Gaussian noise vector $\mathcal{CN}(0, \sigma^2{}_n I)$. Here it is assumed that the wideband signal of interest lies in the range of [0,1000]Δ Hz, where Δ is frequency resolution. There are two primary users present in the observed spectrum and are shown in Fig. 2. The observed spectrum is sparse with a sparsity level of 6% and possesses the same structure as described in [6]. This choice of model is helpful in comparing the results of SBBSR algorithm and the approach proposed in [6]. In [6], compressive sensing technique was used for the spectrum sensing process. Frequency spectrum was recovered from the sub-Nyquist rate sampled observations using the l_1 minimization approach. In order to obtain the frequency band edge information the wavelet edge detection technique was applied on the recovered spectrum. The PSD of each frequency band is calculated and decision regarding presence or absence of the primary user is made. In simulation, Gaussian wavelet is used for the edge detection technique.

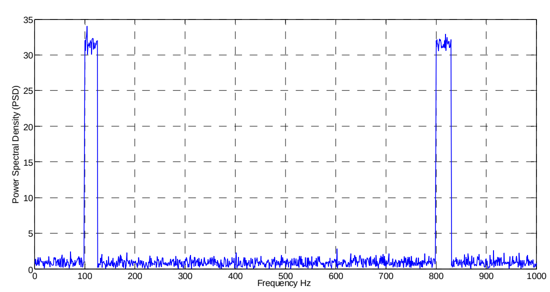

Figure 2. Assumed Wideband Signal-Flat PSD

Aforementioned cases are considered and compared to the compressed sensing approach of [6]. Table I provides the values required by the SBBSR algorithm for these different cases and Table II shows the corresponding working ranges for probability of detection greater than 0.9 for both techniques. Fig. 3 shows the corresponding plots of probability of detection versus signal to noise ratio (SNR).

The spectrum sensing performed using SBBSR algorithm showed better performance than the compressive sensing technique. In all cases, gain of (approximately) more than 5 dB over SNR is observed. The flexible implementation of SBBSR algorithm allowed improvement in the working range of a CR.

V. CONCLUSION

In this paper, the structure-based Bayesian sparse reconstruction algorithm (SBBSR) was used for the spectrum sensing of wideband signals. The SBBSR algorithm provides sub-Nyquist rate sampling solution to the wideband spectrum sensing problem. Spectrum sensing was performed for various cases using both the SBBSR algorithm and the compressed sensing based technique. The results obtained from the SBBSR algorithm showed better performance compared to the other technique. It provided an improvement of more than 5 dB in signal to noise ratio for the observed spectrum. The *a priori* knowledge of the frequency band (whether fixed or variable) helped to achieve better performance. Hence, the SBBSR algorithm improves the performance of spectrum sensing process for the wideband signals and in addition overcomes the shortcomings of compressed sensing technique.

TABLE I. REQUIRED VALUES BY SBBSR ALGORITHM

Cases	Observation Vector Size M	Number of Cluster P	Maximum Support Size P_c	Cluster Size L
1	$\frac{N}{4}$	29	9	9
2	$\frac{N}{4}$	79	1	31
3	$\frac{N}{4}$	79	3	[25 31]

TABLE II. COMPARISON OF CASES FOR PROBABILITY OF DETECTION ≥ 0.9

	Compressed Sensing	Case 1	Case 2	Case 3
SNR ≥	12.95 dB	7.1 dB	7.8 dB	1.8 dB

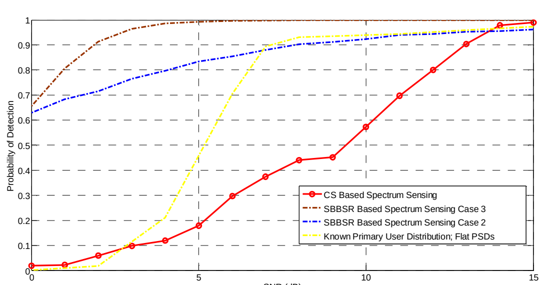

Figure 3. Probability of Detection versus SNR for Case 1

VI. ACKNOWLEDGEMENT

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