A New Blind Equalization Algorithm for M-PSK Constellations

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***Abstract*—In this paper, we propose an equalization algorithm for *M-PSK* constellations that greatly improves the convergence features and reduces steady-state error rate of the conventional Constant Modulus Algorithm (CMA). The proposed algorithm introduces a buffer and multiple step-size decision layers to the existing Variable Step Size Modified Constant Modulus Algorithm (VSS-MCMA) equalizer. The buffer is employed to combat the convergence issue while the additional layers have been introduced to improve sensitivity of the step size in both the convergence state and the steady-state. Computer simulations reveal that the proposed algorithm has better convergence rate than the VSS-MCMA and the CMA.**

Keywords—Blind equalization; constant modulus algorithm; step size

I. INTRODUCTION

In wireless communications, one of the most important transmission problems is the channel distortion. Channel distortion leads to InterSymbol Interference (ISI) between transmitted symbols. There have been many blind equalization techniques to combat this effect. Constant modulus algorithm, originated by Godard [1] and Treichler and Agee [2], is the most popular equalization technique among all blind equalization methods. As all blind equalizers, constant modulus algorithm works in absence of the training sequence. In the algorithm, step size is a crucial parameter to determine the convergence speed and steady-state error rate. A small chosen step size will result in a low steady-state error rate. However, convergence will be slow. Conversely, a large value of step size will result in a faster convergence yet a higher steady-state error rate. Therefore, the Constant Modulus Algorithm (CMA) has a trade-off between these two criteria. Another drawback of the algorithm is that it is unable to correct phase rotations induced by the channel [3].

Oh and Chin propose the Modified Constant Modulus Algorithm (MCMA), which resolves phase rotation problem of the CMA. This is achieved by minimizing two cost functions, which are separately computed for real and imaginary parts of input signal. The algorithm applies both equalization and phase correction.

Variable Step Size Modified Constant Modulus Algorithm (VSS – MCMA) [4] is an algorithm that applies phase correction and makes the step size more sensitive. In this algorithm, a circular area is defined around each likely-transmitted symbol in the constellation and the step size is changed between two values according to whether equalizer output symbol is located inside an area or not.

Many other algorithms [5]-[17] have been proposed to combat the problems encountered in CMA. Zarzoso and Comon [5] suggested an algorithm that finds optimal step size in each update operation. Tugcu et al. [6] employ cross correlation between channel output and the error signal to overcome the convergence problem. Song et al. [7] employ a signal steering vector and its oblique projection for updating filter coefficients in order to avoid signal steering vector mismatches. Lin and Lee [8] introduce an algorithm that finds a gain factor by the least-mean-squares method when updating filter coefficients to avoid gradient noise amplification problem. Demir and Ozen [9] proposed a new algorithm in which autocorrelation of error signal is used for updating filter coefficients. Gao and Qiu [10] employ a momentum term and autocorrelation of error signal for updating filter coefficients. The additional momentum term improves the convergence rate. Li et al. [11] utilize singular value decomposition of input signal to obtain a new step size in order to improve the convergence rate. A new update equation is proposed by Abrar and Nandi [12] to improve the convergence rate. A nonlinear estimate of error signal is used for updating the equalizer coefficients, and a novel deterministic optimization criterion is given. Ikhlef et al. [13] employ the prewhitening technique and the complex Givens rotations to improve signal to noise and interference ratio performance. Yan et al. [14] employ nonlinear transformation of error signal to suppress the alpha-stable noise. Nassar and Nahal [15] recently proposed Exponentially weighted step-size recursive Least Squares Constant Modulus Algorithm (EXP-RLS-CMA), which can be considered as the combination of the conventional CMA and the exponentially weighted step-size recursive least squares algorithm. The EXP-RLS-CMA provides higher convergence rate than the conventional CMA at minimum mean-squared-error. Liyi et al. [16] introduce a new variable step size algorithm in which a nonlinear function of error signal is employed to calculate the step size in each symbol period. Baofeng et al. [17] employ cross-correlation between the input signal and the error signal to control the step size for a better convergence rate.

In this work, we have generalized the VSS-MCMA to multi-layered case in order to make the step size more sensitive. Moreover, we have added a buffer to the equalizer to overcome the convergence problem.

This paper is organized as follows. We have introduced the general transmission model and system models of the CMA, MCMA and VSS-MCMA equalizers in Section II. In Section III, we have presented our Buffered Multi-layered Modified Constant Modulus Algorithm (BML-MCMA). We have examined the simulation results of the proposed algorithm and the other two algorithms in Section IV. Finally, we have drawn the conclusions in Section V.

II. SYSTEM MODELS

Consider a baseband transmission model where the received signal can be written as

$$r(n) = \sum_{k=0}^{L-1} h(k)s(n-k) + u(n) \tag{1}$$

where $h(n)$ is the channel's impulse response of length L, $s(n)$ is the transmitted complex baseband symbol at time n and $u(n)$ is additive white noise. If the received signal is fed to the equalizer, the output is

$$z(n) = R^H W(n)\,. \tag{2}$$

In the above equation, $W(n)$ is the equalizer tap vector of length-N, which is defined as $W(n) = [w_0(n), w_1(n), \ldots, w_{N-1}(n)]^T$. $R(n)$ is the tapped delay line vector of received signal $r(n)$ and it is defined as $R(n) = [r(n), r(n\text{-}1), \ldots, r(n\text{-}N\text{+}1)]^T$.

A. Constant Modulus Algorithm

The CMA developed by Godard [1] and Treichler [2] is a stochastic gradient-based algorithm. The cost function is given by

$$J(n) = E\left[\left(\gamma - |z(n)|^2\right)^2\right] \tag{3}$$

where parameter γ is given by $\gamma \triangleq E\{|s(n)|^4/|s(n)|^2\}$. Here, $E\{.\}$ denotes expectation. In this algorithm, equalizer coefficients are updated by minimizing the cost function in (3). The update rule is

$$W(n+1) = W(n) + \mu\vec{g} \tag{4}$$

where μ is the step size parameter, $\vec{g}$ is the gradient vector defined as $\vec{g} = \nabla J(n) = e(n)R^*(n)$. The error signal $e(n)$ is given by $e(n) = z(n)\left(\gamma - |z(n)|^2\right)^2$. Expanding right-hand side of the expression in (4) using the above definitions yields

$$W(n+1) = W(n) + \mu z(n)\left(\gamma - |z(n)|^2\right)^2 R^*(n)\,. \tag{5}$$

B. Modified Constant Modulus Algorithm

MCMA blind equalization algorithm in [3] applies modifications to CMA in calculation of cost function. Computations are performed using real and the imaginary parts of equalizer output separately. The purpose of separating the real and imaginary parts is to correct phase rotations encountered in CMA. The cost function for MCMA has the form

$$J(n) = J_R(n) + J_I(n)\,. \tag{6}$$

In (6), $J_R(n)$ and $J_I(n)$ are cost functions of real and imaginary parts of equalizer output, respectively, and they are defined as

$$J_R(n) = \left(|z_R(n)|^2 - \gamma_R\right)^2 \tag{7}$$

$$J_I(n) = \left(|z_I(n)|^2 - \gamma_I\right)^2. \tag{8}$$

Here, γ_R and γ_I are two parameters for real and imaginary parts of transmitted symbol respectively and they are defined as $\gamma_R = E\{|s_R(n)|^4/|s_R(n)|^2\}$ and $\gamma_R = E\{|s_R(n)|^4/|s_R(n)|^2\}$ where $s(n) = s_R(n) + js_I(n)$.

C. Variable Step Size Modified Constant Modulus Algorithm

The VSS – MCMA algorithm proposed in [4] is an improvement on MCMA. The algorithm employs step-size adaptation to boost the performance in both the convergence state and the steady state. The change of step size depends on the regions defined in the signal space.

Let the signal space contain M regions, namely, a circular area with radius l is placed around each of total M likely transmitted symbols. One of two step size parameters is selected according to whether the equalizer output is located inside the circular area around the nearest symbol or not. In other words

$$\mu = \begin{cases} \mu_{VSS,0}, & z(n) \notin B_i\,,\ i = 1,2,\ldots,M \\ \mu_{VSS,1}, & z(n) \in B_i\,,\ i = 1,2,\ldots,M \end{cases} \qquad \mu_{VSS,0} > \mu_{VSS,1}. \tag{9}$$

The above equation indicates that if equalizer output is not located inside the area B_i, the algorithm selects the larger step size, $\mu_{VSS,0}$. If not, the algorithm selects the smaller one, $\mu_{VSS,1}$.

III. BUFFERED MULTI-LAYERED MODIFIED CONSTANT MODULUS ALGORITHM

In the proposed system, existing VSS – MCMA algorithm is generalized to a multi-layered system and parallel buffer-delay elements are added before the equalizer. The purpose of the changes is to overcome the convergence problem and lower the steady state error rate. Block diagram of the proposed system is shown in Fig. 1. In the system, initially, first S baud segment of input signal $r(n)$ is used for updating the equalizer coefficients only. At the end of S baud periods, namely, at time ST_S, thanks to the control element, the system turns off the counter and switches to delay output. This time, input signal is fed into the equalizer through the delay element. Although a delay of S baud periods is introduced to the signal, because the equalizer coefficients are updated previously, the convergence problem is removed on a large scale.

To begin analysis of the system, let the signal space contain D regions around every symbol, namely, around every symbol, D circular layers are constructed. This is depicted in Fig. 2, where D has the value of 2. Here, $l_{i,j}$ (i = 1,2, … , D, j = 1,2,…, M) denotes the radius of ith layer around jth symbol. Let us denote radius of the outmost layer by l and the distance between two nearest symbols by p. Then, the restriction to the size of outmost layer is $l<0.5p$. The step size is selected according to whether the equalizer output is located in a layer around nearest symbol or not. The general selection rule can be expressed as

$$\mu = \begin{cases} \mu_0 , & l_{D,j} < \left\| z(n) - s_j \right\| < \dfrac{p}{2} \\ \mu_1 , & l_{(D-1),j} < \left\| z(n) - s_j \right\| < l_{D,j} \\ \cdot & \\ \cdot & \\ \cdot & \\ \mu_{D-1} , & 0 < \left\| z(n) - s_j \right\| < l_{1,j} . \end{cases} \tag{10}$$

In (10), s_j is the jth transmitted symbol in the constellation and $z(n)$ is the output signal of the equalizer at time instant n. Relationship between the step size parameters is $\mu_0 > \mu_1 > \ldots > \mu_{D-1}$.

IV. NUMERICAL RESULTS

In this section, we explore the convergence rate performance of the proposed algorithm through computer simulations. We also compare performance of the proposed algorithm with the conventional CMA and the VSS – MCMA.

In the simulations, we have used the 2-tap complex channel $h(n) = [1 + 0.5\delta(n-1)] + j[1 + 0.4\delta(n-1)]$ [13] and the 3-tap Proakis B-channel $h(n) = 0.407 + 0.815\delta(n-1) + 0.407\delta(n-2)$ [18], where $\delta(n)$ is the Kronecker delta function. The channel is normalized such that the total power is unit watts. Equalizer tap number is selected as 11. For the 2-tap complex channel, all of the taps are initialized to zero except the first tap is initialized to one. For Proakis-B channel, all of the taps are initialized to zero except the center tap is initialized to one. Additive noise is assumed to be zero-mean complex white Gaussian noise. Transmitted signal length N is chosen as 10^6 symbols, which is assumed to be long enough to examine the performance of the three systems. Number of buffer samples S is chosen as 10^4 for the 2-tap complex channel case and 5×10^4 for the Proakis B-channel case.

We present the performance of the three equalizers through constellations of equalizer outputs. Constellations of outputs of the three equalizers at QPSK, 8-PSK, 16-PSK and 32-PSK under the 2-tap complex channel, are given in Fig. 3, Fig. 4, Fig. 5 and Figs. 6 and 7, respectively. SNR values at which the

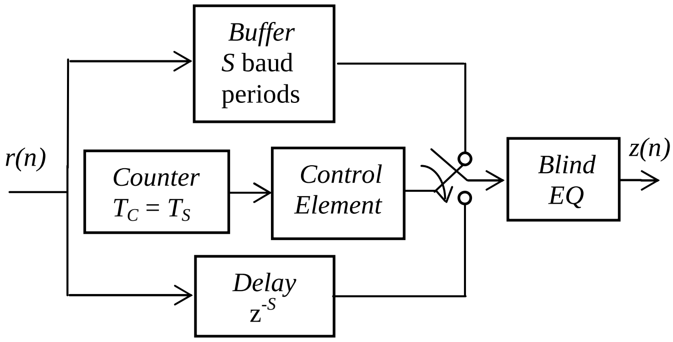

Fig. 1. Block diagram of the proposed BML – MCMA system

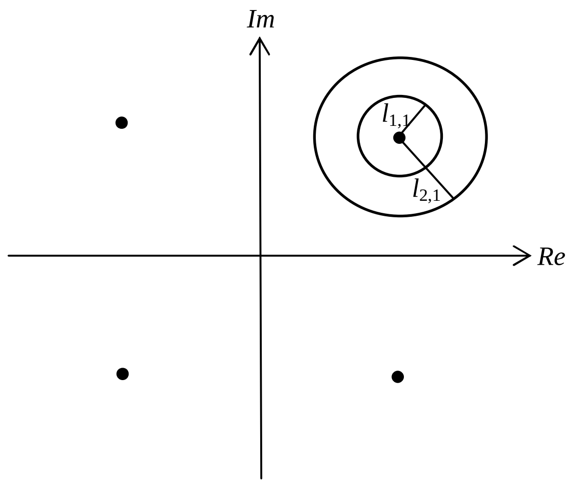

Fig.2. Step-size decision layers (D = 2, QPSK).

simulations are performed, are chosen as 15 dB, 20 dB, 30 dB and 40 dB respectively. Optimum values of μ and l parameters of the three equalizers are obtained heuristically.

In Fig. 3, it can be seen that BML-MCMA has the lowest Mean-squared Error (MSE) by having less scattered symbols than the two other. In the case of VSS-MCMA, a clear QPSK constellation is not visible. Therefore, the VSS-MCMA has the highest MSE and Symbol Error Rate (SER).

From Fig. 4, one can note that the BML-MCMA outperforms the two other. The employment of buffer and multiple step size layers results in a better output constellation. Because there is no buffer used in conventional CMA and VSS-MCMA, number of highly scattered symbols are much greater than those of the BML-MCMA. The conventional CMA has a better constellation than VSS-MCMA.

Fig. 5 clearly indicates that BML-MCMA has the best performance by having the clearest constellation. Here, again, there are large number of highly scattered symbols in constellations of CMA and VSS-MCMA. The BML-MCMA overcomes this problem thanks to its buffer element and its output constellation yields better overall MSE than those of the other two.

Fig. 6 and Fig. 7 show that the BML-MCMA has the best MSE performance and convergence rate. Conventional CMA and the VSS-MCMA have many highly-scattered symbols and they cannot yield a clear constellation. Consequently, these two equalizers have much lower convergence rate.

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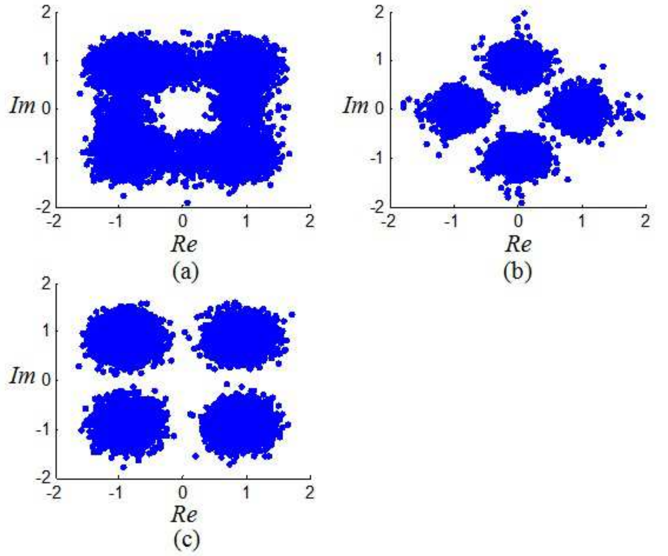

Fig. 3. Constellation diagrams of first 20000 elements of equalizer output at QPSK modulation. (a) VSS-MCMA, (b) CMA, (c) BML-MCMA

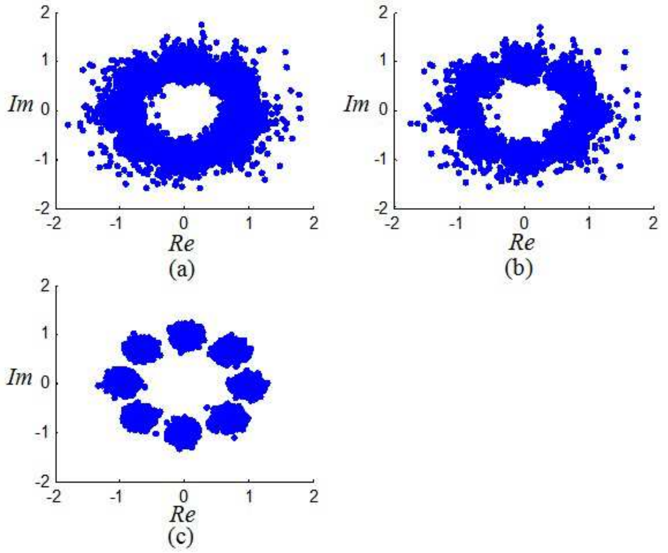

Fig. 4. Constellation diagrams of first 20000 elements of equalizer output at 8-PSK modulation. (a) VSS-MCMA, (b) CMA, (c) BML-MCMA

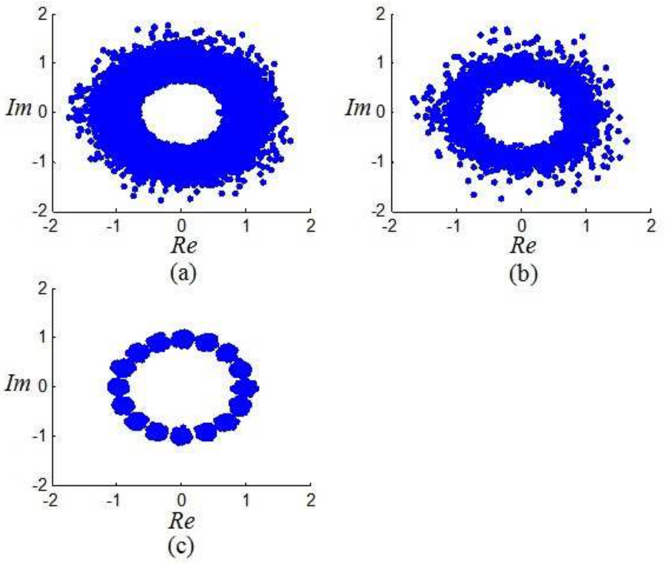

Fig. 5. Constellation diagrams of first 20000 elements of equalizer output at 16-PSK modulation. (a) VSS-MCMA, (b) CMA, (c) BML-MCMA

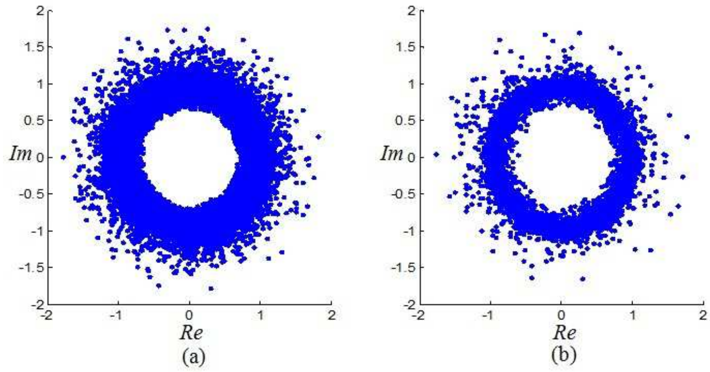

Fig. 6. Constellation diagrams of first 20000 elements of equalizer output at 32-PSK modulation. (a) VSS-MCMA, (b) CMA

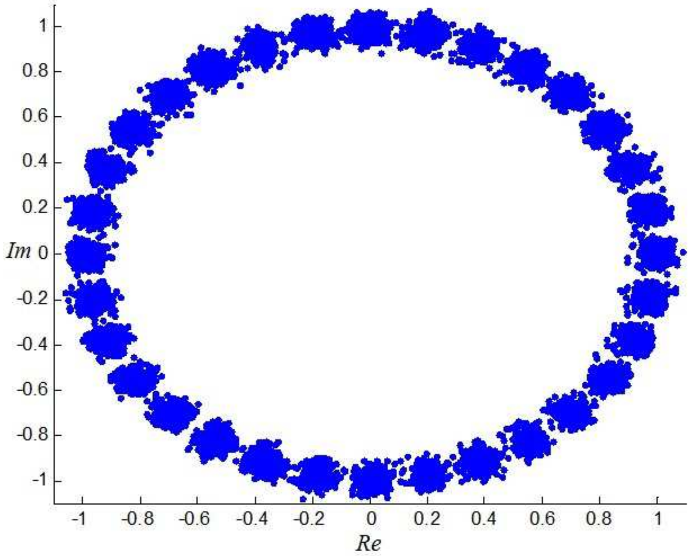

Fig. 7. Constellation diagram of first 20000 elements of BML-MCMA equalizer output at 32-PSK modulation.

Fig. 8 shows the constellation of first 20000 output symbols of equalizer at 8-PSK modulation under Proakis B-channel. Since the Proakis B-channel is a harsh channel, we have increased the buffer size to 50000 symbols. From Fig. 8, it can be observed that BML-MCMA outperforms the two other in convergence rate. Conventional CMA takes the second place and VSS-MCMA again has the lowest convergence rate.

In the second part of the simulations, we have investigated the effect of buffer size *S* on the convergence rate of the BML-MCMA equalizer through the constellations obtained for various buffer size values. Fig. 9 shows the constellations of first 20000 elements of equalizer outputs for *S* = 10000, 25000 and 50000 symbols, respectively. The modulation is 8-PSK and we have used the Proakis B-channel. The SNR value is 16 dB. Fig. 9 shows that using only the first 50000 symbols of the input, the BML-MCMA equalizer reduces the convergence problem greatly and results in a clear constellation. For a buffer size of 25000 symbols, the equalizer significantly reduces the convergence problem so it can resolve the 8-PSK symbols. Due to the harsh Proakis B-channel, a buffer size of 10000 symbols is not enough to eliminate the convergence problem.

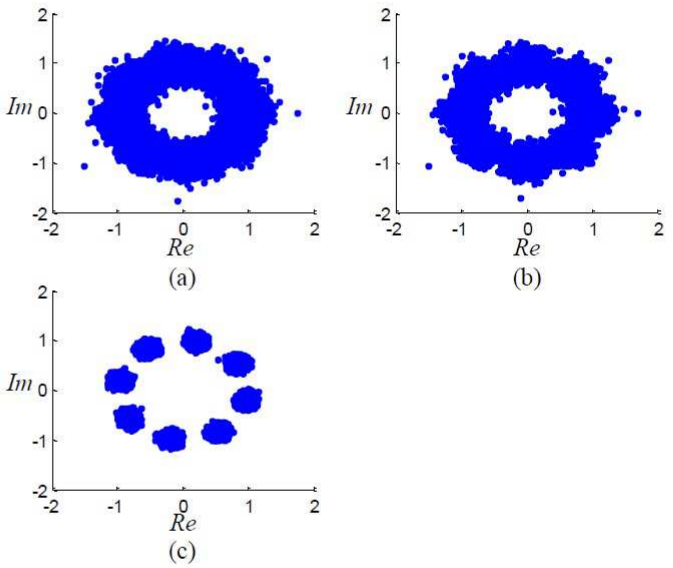

Fig. 8. Constellations of first 20000 elements of equalizer outputs at 8-PSK modulation. (a) VSS-MCMA, (b) CMA, (c) BML-MCMA

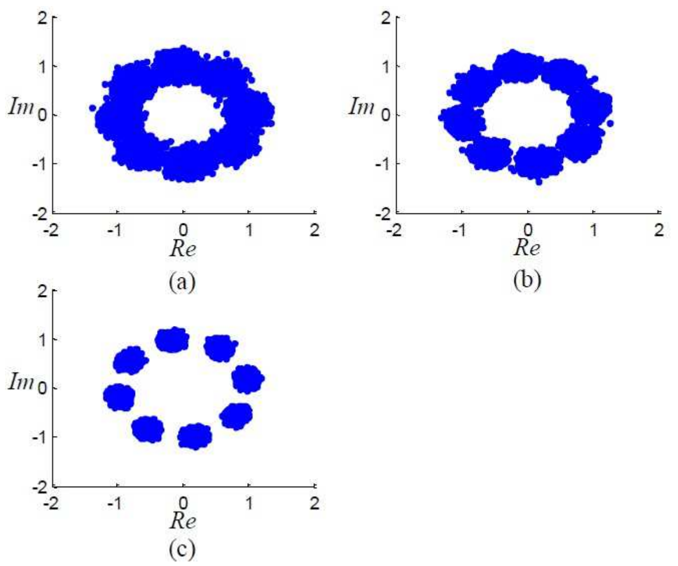

Fig. 9. Constellations of first 20000 elements of equalizer output under different buffer size values. (a) $S = 10000$, (b) $S = 25000$, (c) $S = 50000$ symbols

V. CONCLUSION

Despite there is no need for a training sequence, conventional CMA and VSS-MCMA have convergence problems. In this paper, we have proposed our BML-MCMA algorithm to eliminate the convergence problems and further improve MSE performance of VSS-MCMA. We have presented numerical results which indicate that through using a buffer, the proposed algorithm resolves convergence problem greatly. Furthermore, the proposed algorithm has better MSE performance. Although using a buffer introduces some initial delay to the equalizer, considering the overall performance, BML-MCMA is a good alternative to the existing blind equalization algorithms.

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