Further Throughput Optimization of IEEE 802.11 Networks based on Successive Interference Cancellation

Ho Chun Leung Sammy Chan

Department of Electronic Engineering
City University of Hong Kong
Hong Kong SAR
P.R. China
Email: {hcleung35-c@my. eeschan@}cityu.edu.hk

Masaki Bandai

Department of Information and Communication Sciences
Sophia University
Tokyo
Japan
Email: bandai@sophia.ac.jp

***Abstract*—Successive interference cancellation (SIC) is a physical layer mechanism, which eases packet collisions. It can decode simultaneously transmitted packets from multiple stations with different transmitting power levels, and hence raises the throughput of wireless networks. In an earlier work, the optimal throughput of an IEEE 802.11 network with SIC for a given initial contention window (W) is investigated. In this paper, we suggest that an optimal W can be chosen to further improve the optimal throughput. We re-visit the throughput optimization problem with W being another degree of freedom, and propose an efficient way to obtain the optimal W and the corresponding probability mass function of power levels. Numerical results have verified that the optimal throughput can be further increased by allowing W to be an optimal variable.**

Keywords*—*multiple-packet reception; successive interference cancellation; power randomization; 802.11.

I. Introduction

An IEEE 802.11 wireless local area networks (WLAN) is a shared medium network. When operated in the infrastructure mode, it comprises an access point (AP) and a number of stations. Each station communicates with each other or external networks via the AP. When stations transmit packets to the AP, they need to contend for the channel. How each station accesses the channel is governed by the contention-based distributed coordinated function (DCF) [1]. Because of the shared medium, if more than one station transmits packets at the same time, a collision happens and no packet can get through the channel. It results in packet retransmissions and hence throughput degradation.

To improve such a situation, multiple packet reception (MPR) techniques can be used at the physical layer. They enable an AP to resolve collisions and successfully decode multiple packets. Early MPR techniques are based on single-user-detection approaches [2], which can only achieve low information rate. On the other hand, various multi-user-detection approaches [3] have been proposed, including zero-forcing, maximum likelihood, parallel interference cancellation and successive interference cancellation (SIC). These approaches can support high information rate.

Recently, an in-depth study of using SIC for MPR is reported in [4]. The authors propose that, when transmitting a packet, each station randomly chooses a power level so that the probability of recovering the signals during a collision can be increased. Hereafter, this scheme is referred to as SIC with power randomization (SPR). More importantly, they derive a discrete set of *optimal* power levels which only depends on the target information rate. In other words, the set of optimal power levels is applicable to any shared-medium wireless networks, irrespective of their medium access control (MAC) protocols. On the other hand, the MAC layer throughput depends on the probability mass function of the optimal power levels and the MAC protocols.

In [5], the throughput performance of an IEEE 802.11 WLAN using SPR is evaluated. Analytical expressions relating the probability mass function and throughput are obtained. Furthermore, an optimization problem is formulated to determine the probability mass function which maximizes the throughput. When solving the optimization problem, the authors of [5] assume that the initial contention window, W, is fixed and given. This limits the achievable optimal throughput. In this paper, we relax this assumption and treat W as one of the optimizing variables. As will be shown in the results, this allows an IEEE 802.11 WLAN based on SPR to achieve a higher optimal throughput.

The remainder of the paper is organized as follow. Section II briefly reviews SPR. Section III summarizes the throughput model developed in [5] for DCF with SPR. The formulation of throughput optimization is given in Section IV. Then, Section V presents our solution approach and Section VI provides some numerical results. Finally, conclusions are given in Section VII.

II. Review of SPR

First, let us consider the case of Gaussian channel. Assume that the mean and variance of the channel noise power are 0 and N_0, respectively. Let E_i be a positive real value recursively

 ISBN: 978-1-61208-473-2

defined below,

$$E_i = \begin{cases} 0 & i = 0, \\ (2^R - 1)(E_{i-1} + N_0) & i = 1, 2, 3, \dots \end{cases} \tag{1}$$

A set $\mathcal{E}$ of discrete power levels can be formed as follows,

$$\mathcal{E} = \begin{cases} \{E_1, \cdots, E_Q\} & R < 1, \\ \{E_1, \cdots, E_i, \cdots\} & R \geq 1, \end{cases} \tag{2}$$

where R is the target information rate, and E_Q is the solution of the equation $E_Q = (2^R - 1)(E_Q + N_0)$.

Consider that two stations are transmitting packets simultaneously, with randomly chosen power E_i and E_j from $\mathcal{E}$, respectively. When receiving the combined signal due to the two packets, the AP can first decode the stronger signal while treating the weaker signal as noise. Subsequently, the AP can subtract the stronger signal from the combined signal, and then decode the weaker signal. In other words, as long as $E_i \neq E_j$, both packets can always be decoded successfully. This is because, for any E_i and E_j where $E_i \neq E_j$, (1) guarantees that the following conditions for reliable communication [6] are always satisfied:

Condition for first decoding step:

$$\log_2 \left(1 + \frac{E_i}{E_j + N_0}\right) \geq R. \tag{3}$$

Condition for second decoding step:

$$\log_2 \left(1 + \frac{E_j}{N_0}\right) \geq R. \tag{4}$$

It has been proved in [4] that (2) gives an optimal set of power levels in that sense that the achieved throughput is not worse than any other power profiles while less average power is consumed.

For the case of fading channel, each station only needs to ensure that the power levels received by the AP fall into $\mathcal{E}$. Assuming that the instantaneous channel gain g is known and that the channel is reciprocal. Then, the optimal power levels for each station are $\{E_1/g,\ E_2/g,\ \dots\ \ E_i/g,\ \dots,\}$.

III. Throughput of DCF with SPR

Consider an IEEE 802.11 WLAN with N stations deploying SPR with M available power levels. When transmitting a packet, each station chooses power level E_i, $i = 1, \dots, M$, with probability p_i. Assuming that, for each station, packets arrive at the MAC layer from the upper layer with rate λ (packet/second), and that each station has an infinite buffer. Let τ be the attempt rate per slot of each station. In [5], a fixed point equation relating τ to λ, W, $\{p_i, i = 1, \dots, M\}$ is derived, and is denoted in here as

$$\tau = \mathcal{F}(\tau, \lambda, W, \{p_i\}) \tag{5}$$

In other words, τ is determined by a given λ, W and $\{p_i\}$.

Packets can be successfully received by the AP if no more than two stations are transmitting simultaneously. Therefore, the average throughput T is given by

$$T = \frac{LP_1 + 2LP_2}{T_v}, \tag{6}$$

where L is the payload size of a packet, P_1 is the probability that only one station transmits, P_2 is the probability that two stations are transmitting simultaneously, and T_v is the mean slot duration after taking into account the deferment process in DCF.

Clearly, we have

$$P_1 = \binom{N}{1} \tau (1 - \tau)^{N-1}, \tag{7}$$

and

$$P_2 = \binom{N}{2} \tau^2 (1 - \tau)^{N-2} (1 - \sum_{i=1}^{M} p_i^2), \tag{8}$$

where $1 - \sum_{i=1}^{M} p_i^2$ is the probability that the power levels of the two simultaneously transmitted packets are different.

From [1], T_v is given by

$$T_v = (1 - P_b)\sigma + P_b P_s (T_s + \sigma) + P_b (1 - P_s)(T_s + \sigma), \tag{9}$$

where $P_b = 1 - (1 - \tau)^N$, $P_s = \frac{P_1 + P_2}{P_b}$, and both T_s and σ are system parameters. Since T_v is effectively a function of τ, it is thus denoted as $T_v(\tau)$. Then, overall, T is given by

$$T = LN \frac{\tau(1 - \tau)^{N-1} + (N - 1)\tau^2 (1 - \tau)^{N-2} (1 - \sum_{i=1}^{M} p_i^2)}{T_v(\tau)}. \tag{10}$$

This analytical model for throughput of IEEE 802.11 networks operating in DCF mode with SPR has been extensively validated by simulations. Its accuracy is demonstrated by the results reported in [5]. In this paper, this model is used to evaluate the throughput for a given set of network parameters.

IV. Formulation of Throughput Optimization

To optimize the throughput, the following formulations are given in [5].

A. Gaussian Channel

$$\begin{aligned} \max \quad & T \\ \text{subject to} \quad & \sum_{i=1}^{M} p_i = 1 \\ & \sum_{i=1}^{M} p_i \tau E_i \leq E_{av} \\ & 0 \leq p_i \leq 1, \quad i = 1, \dots, M. \end{aligned} \tag{11}$$

where E_{av} is the average power limit.

B. Fading Channel

For the case of fading channels, a channel gain g is associated with $\{p_i(g), i = 1, 2, \dots, M\}$, where $p_i(g)$ denotes the probability that a station transmits with power $\frac{E_i}{g}$. In order to optimize the throughput, the optimal $\{p_i(g)\}$ for each channel gain g need to be found. Since, in general, g is continuously distributed, this makes finding the exact optimal solution extremely difficult. To simplify the problem, the continuous distribution is approximated by a discrete distribution as follows. The range of g, $[0, \infty)$, is divided into H intervals according to $H+1$ thresholds $\{g^h | h = 0, \ldots, H\}$, and uniform distribution within each interval is assumed. That is

$$p_i(g) = p_i^h, g \in [g^{h-1}, g^h), h = 1, 2, \ldots, H \quad i = 1, 2, \ldots M. \tag{12}$$

Clearly, $\sum_{i=1}^{M} p_i^h = 1$, $\forall h$. Let $\Psi(g)$ is the probability density function of g. Then,

$$p_i = \sum_{h=1}^{H} p_i^h q^h, \tag{13}$$

where $q^h = \int_{g \in [g^{h-1}, g^h)} \Psi(g) dg$.

When the received power is E_i, and the channel gain is $g \in [g^{h-1}, g^h)$, the corresponding transmitted power is E_i/g with probability density $p_i^h \tau \Psi(g)$. The average transmitted power is thus given by

$$\sum_{h=1}^{H} \sum_{i=1}^{M} \int_{g \in [g^{h-1}, g^h)} (E_i/g) p_i^h \tau \Psi(g) dg = \sum_{h=1}^{H} \sum_{i=1}^{M} p_i^h \tau E_i / \overline{g}^h, \tag{14}$$

where $1/\overline{g}^h = \int_{g \in [g^{h-1}, g^h)} \frac{1}{g} \Psi(g) dg$.

Then, the throughput optimization problem can be formulated as follows,

$$\begin{aligned} \max \quad & T \\ \text{subject to} \quad & 0 \le p_i^h \le 1, \quad i = 1, \ldots, M, h = 1, \ldots, H \\ & p_i = \sum_{h=1}^{H} p_i^h q^h, \quad i = 1, \ldots, M \\ & \sum_{i=1}^{M} p_i^h \tau \le 1, \quad h = 1, \ldots, H \\ & \sum_{i=1}^{M} \sum_{h=1}^{H} p_i^h q^h = 1 \\ & \sum_{i=1}^{M} \sum_{h=1}^{H} p_i^h \tau \frac{E_i}{\overline{g}^h} \le E_{av} \end{aligned} \tag{15}$$

V. Optimal Solutions

Referring to the optimization problems given in (11) and (15), it can be seen that both are non-convex. In [5], these problems are simplified by assuming W is fixed and given. When W is given, from (5), τ is effectively a function of $\{p_i\}$. Thus, the optimal variables in (11) and (15) are $\{p_i\}$ only. As a result, the optimization problems become as follows.

Gaussian Channel:

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} p_i^2 \\ \text{subject to} \quad & \sum_{i=1}^{M} p_i = 1 \\ & \sum_{i=1}^{M} p_i \tau E_i \le E_{av} \\ & 0 \le p_i \le 1, \quad i = 1, \ldots, M. \end{aligned} \tag{16}$$

Fading Channel:

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} p_i^2 \\ \text{subject to} \quad & 0 \le p_i^h \le 1, \quad i = 1, \ldots, M, h = 1, \ldots, H \\ & p_i = \sum_{h=1}^{H} p_i^h q^h, \quad i = 1, \ldots, M \\ & \sum_{i=1}^{M} p_i^h \tau \le 1, \quad h = 1, \ldots, H \\ & \sum_{i=1}^{M} \sum_{h=1}^{H} p_i^h q^h = 1 \\ & \sum_{i=1}^{M} \sum_{h=1}^{H} p_i^h \tau \frac{E_i}{\overline{g}^h} \le E_{av} \end{aligned} \tag{17}$$

The problems specified by (16) and (17) are convex and can be solved readily by standard techniques. However, fixing W limits the search space and thus the achievable optimal throughput. We believe optimizing τ and $\{p_i\}$ concurrently would further enhance the optimal throughput. By allowing τ to be an optimal variable, it just means that W is not fixed anymore. Instead, W is determined by the resulting optimal τ. To this end, we propose to solve (11) and (15) exactly and efficiently in the following two-step approach. In the first step, for a fixed τ, we find $\{p_i^*\}$ and $\{p_i^{h*}\}$, which are the solutions of (16) and (17), respectively. In the second step, the optimal τ, τ^*, is obtained by a full search over the range $(0, 1)$.

Once τ^* is obtained, the corresponding W can be obtained by the following algorithm:

Algorithm 1 Finding W from τ^*

Require: $\tau^*, \lambda, \{p_i\}$
1: Let W=8
2: **repeat**
3: solve $\tau = \mathcal{F}(\tau^*, \lambda, W, \{p_i\})$
4: Set W=$W+1$
5: **until** $(\frac{\tau - \tau^*}{\tau^*} < 0.001)$
6: Obtain optimal W

Initializing $W = 8$ can reduce the computation time of Algorithm 1. Since an extremely small value for W results in many collisions, the resultant throughput would be far from optimal. Thus, $W = 8$ is sufficiently large to initialize the algorithm.

VI. Numerical Results

In this section, we compare the performance of our proposed solution approach with that of [5]. We solve the optimization problems using Matlab with the CVX optimization toolbox for various E_{av}. The fixed system parameters used are listed in Table I.

First, we consider the case of Gaussian channel. The results are shown in Figures 2-3. With $R = 1, M = 5, N = 5$, Figure 2 plots the resulting optimal throughput (normalized) versus E_{av} when the optimal W and $W = 32$ (an arbitrary

 ISBN: 978-1-61208-473-2

TABLE I. SYSTEM PARAMETERS

Slot Time	20 us
SIFS	10 us
DIFS	50 us
Retransmission limit	7
Data rate	11 Mbps
Control bit rate	1 Mbps
Header	576 bits
ACK	272 bits
No. of nodes	5
No. of discrete power levels	5
Packet arrival rate	250
R	2
N_0	1

chosen value) are used, respectively. Obviously, the optimal throughput corresponding to the optimal W is higher than that corresponding to an arbitrary chosen W. Since the optimal W corresponds to the most suitable back-off time, the collisions are resolved in a better manner. This results in a higher optimal throughput. Note that when E_{av} is small, the optimal throughput corresponding to optimal W and $W = 32$ are similar. According to [5], when E_{av} is small, stations are forced to use low power levels with higher probabilities so as to fulfil the constraint of average consumed power. SPR is not effective to resolve collisions under small number of power levels. Therefore, it is reasonable for the low optimal throughput occurring at small E_{av}. Apparently, this phenomenon also happens in the case of optimal W.

Figure 2 shows a similar difference between optimal W and $W = 32$ when R is increased to 2. This demonstrates that SPR is applicable when the network is operated at high information rate. Comparing with Figures 2 and 3, it can be seen that the improvement becomes less. This is due to two reasons. First, it should be recalled that the throughput under $W = 32$ is already sub-optimal; it is obtained by solving the optimization problem given by (11). Second, as explained below, the optimal W increases with N, and $W = 32$ happens to be close to the optimal W. Thus, the improvement that can possibly be made becomes smaller. However, our approach guarantees that the optimal throughput is obtained.

Table II provides more comparison results for various M and N. It can be seen that the optimal throughput is further enhanced by our approach. Note that the optimal W increases with N. As the collision probability increases with N, a larger back-off time is required to reduce the collision probability. This leads to a larger W. Therefore, a network with larger N needs a larger W to achieve the optimal throughput. As a whole, we notice that more discrete power levels provided by SPR is the ultimate key to increase the optimal throughput.

TABLE II. Comparison of optimal throughput under different network configurations for Gaussian Channel and $R = 2$

Config	Throughput (W=32)	Throughput (Variable W)	%	Optimal W
M=5, N=5	0.372	0.403	+8.49	14
M=3, N=5	0.362	0.378	+4.3	14
M=5, N=10	0.3912	0.3967	+1.4	17
M=3, N=10	0.372	0.374	+0.53	24

Then, we consider the case of Rayleigh fading channel with averaged power gain equal to 1. The whole range of

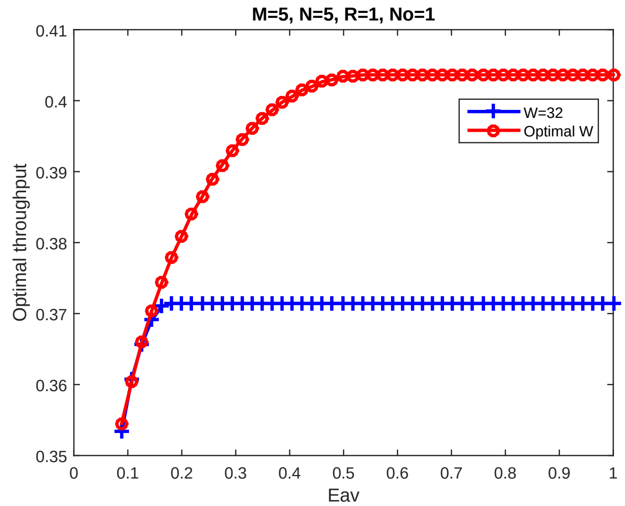

Fig. 1. Optimal throughput comparison in Gaussian channel, $R = 1, M = 5, N = 5$.

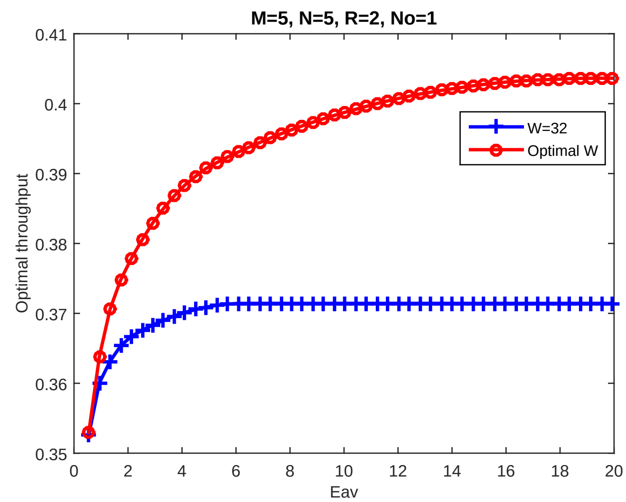

Fig. 2. Optimal throughput comparison in Gaussian channel, $R = 2, M = 5, N = 5$.

g, i.e., $[0, \infty)$, is divided into 20 intervals. Figures 4-6 plot the resulting optimal throughput versus E_{av} for both solution approaches under various system parameters. The observations are similar to that of Figures 2-3. It demonstrates the efficacy of our solution approach for the case of fading channels.

Table III provides more comparison results for various M and N. Again, similar observations as the case of Gaussian channel can be made.

TABLE III. Comparison of optimal throughput under different network configurations in fading channel with R=2

Config	Throughput (W=32)	Throughput (Variable W)	%	Optimal W
M=5, N=5	0.372	0.404	+8.6	14
M=3, N=5	0.362	0.3779	+4.4	14
M=5, N=10	0.3912	0.397	+1.5	17
M=3, N=10	0.372	0.373	+0.26	24

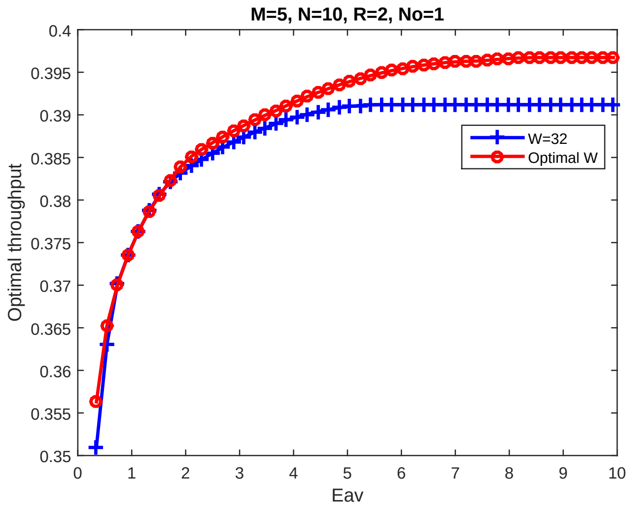

Fig. 3. Optimal throughput comparison in Gaussian channel, $R = 2, M = 5, N = 10$.

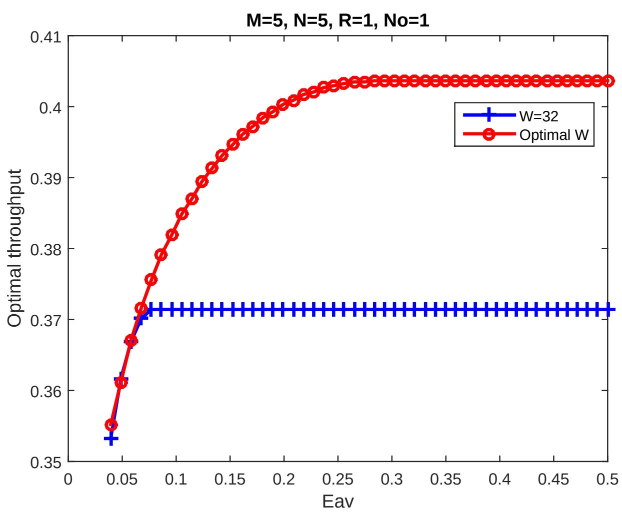

Fig. 4. Optimal throughput comparison in fading channel, $R = 1, M = 5, N = 5$.

VII. Conclusion

In this paper, we have suggested that the initial contention window can be suitably chosen to further optimize the throughput of IEEE 802.11 network based on successive interference cancellation with power randomization. To this end, we have formulated the optimization problem and proposed an efficient way to obtain the optimal initial contention window. We have compared the resultant optimal throughput with the approach of arbitrarily chosen window size for both Gaussian and Rayleigh fading channels. Numerical results have shown that, by allowing the initial window size to be suitably chosen, a higher optimal throughput can be achieved.

References

[1] G. Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function," *IEEE J. Selected Area in Comm.,* vol. 18, no. 3, March 2000, pp. 535-547.

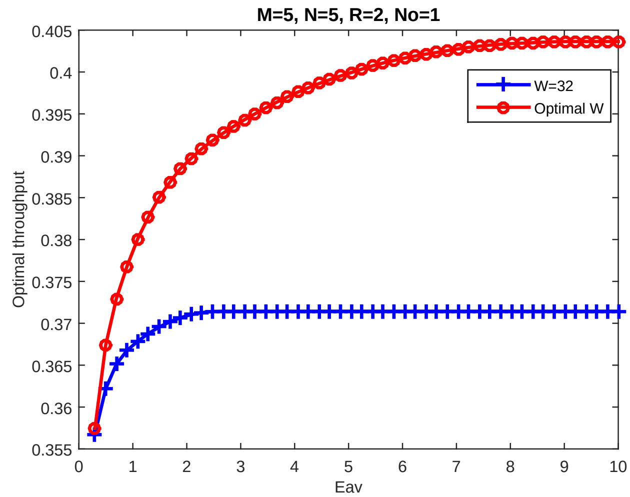

Fig. 5. Optimal throughput comparison in fading channel, $R = 2, M = 5, N = 5$.

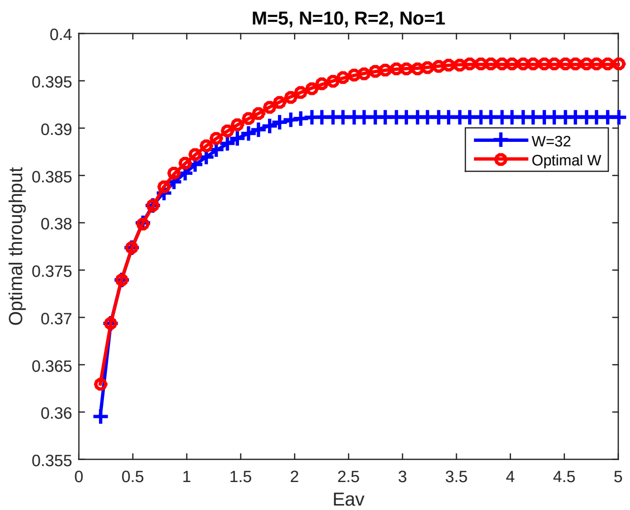

Fig. 6. Optimal throughput comparison in fading channel, $R = 2, M = 5, N = 10$.

[2] J. Luo and A. Ephremides, "Power Levels and Packet Lengths in Random Multiple Access with Multiple-packet Reception Capability," *IEEE Trans. on Inf. Theory*, vol. 52, no. 2, February 2006, pp. 414-420.

[3] S. Verdu, *Multiuser Detection,* Cambridge Univ. Press, 1998.

[4] C. Xu, Li Ping, P. Wang, S. Chan, and X. Lin,"Decentralized Power Control for Random Access with Successive Interference Cancellation," *IEEE J. Selected Areas in Comm.,* vol. 31, no. 11, November 2013, pp. 2387-2396.

[5] M. Zou, S. Chan, H. Vu, and Li Ping, "Throughput Improvement of 802.11 Networks via Randomization of Transmission Power Levels", *To appear in IEEE Trans. on Vehicular Technology*, DOI: 10.1109/TVT.2015.2427845.

[6] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.