A Proposal for a New OFDM Wireless System using a CAZAC Equalization Scheme

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***Abstract*—It is well known that one of the most serious drawbacks of the orthogonal frequency division multiplexing (OFDM) scheme in wireless applications is its high peak-to-average power ratio (PAPR), which decreases the efficiency of power amplifiers (PAs) and increases transmitter power consumption. We propose a constant amplitude zero auto-correlation (CAZAC) equalization scheme, which is a robust way of overcoming the PAPR problems with the OFDM scheme. The CAZAC equalization scheme makes the PAPR of multilevel quadrature amplitude modulation (M-QAM) OFDM signals into the PAPR of M-QAM single-carrier signals. This paper proposes a new wireless system that introduces the CAZAC equalization scheme. CAZAC improves the estimated power-added efficiency of the PAs for a 16-QAM OFDM system with 52 subcarriers from 10 to 30% because it reduces the PAPR of 5 dB while the system imposes no penalties on the bit error rate (BER). The paper also provides theoretical analysis of CAZAC equalization and information on spectral control and the efficiency of BER under fading environments.**

***Keywords*–OFDM; CAZAC sequence; Zadoff-Chu sequence; PAPR reduction.**

I. INTRODUCTION

Orthogonal frequency division multiplex (OFDM) systems that attain high speeds and high capacity have recently been attracting attention in wireless applications, e.g., wireless local area networks (WLANs), third generation partnership project long-term evolution (3GPP LTE), and the digital video broadcasting-terrestrial (DVB-T) standard [1] [2].

However, the main drawback of OFDM is its high Peak-to-Average Power Ratio (PAPR), which decreases the efficiency of the power amplifiers (PAs) and increases transmitter power consumption [3] [4]. Therefore, a number of techniques have been proposed to reduce the PAPR [3]. Well-known techniques are clipping-and-filtering, partial transmit sequence (PTS), and selected mapping (SLM). Clipping-and-filtering limits the peak amplitude of the transmission signal. However, non-linear distortion causes BER to degrade. PTS partitions input data into disjoint sub-blocks. Moreover, each sub-block are weighted by a phase factor.

This technique chooses the phase factor to minimize the PAPR of combined signals. SLM generates multiple candidate data blocks. All data blocks represent the same information. Although PTS and SLM can be expected to create a certain reduction in PAPR, both techniques need side information in the receiver, which decreases spectral efficiency. The most practical solution to improving PAPR is to introduce single carrier frequency division multiplexing access (SC-FDMA). The 3GPP LTE system adopts SC-FDMA for uplink multiple access systems [2] [5]. However, SC-FDMA has not been considered to be suitable for next-generation high-speed communications.

A new PAPR reduction technique with constant amplitude zero auto-correlation (CAZAC) equalization was recently proposed [6] [7]. This technique multiplies frequency domain OFDM symbols and CAZAC sequences to reduce the PAPR of OFDM signals. However, this technique needs to use all subcarriers in the frequency domain, which violates the spectrum mask defined by the IEEE 802.11 a/g standard. Since this is not an easy problem to solve, CAZAC equalization was used for visible light communications that did not need to take spectral management into consideration.

This paper proposes a new wireless communication system with the Zadoff-Chu sequence scheme, which is one of the most well known CAZAC schemes. The M-QAM OFDM signal acts as if it were an M-QAM single-carrier signal in the system by introducing the Zadoff-Chu sequence scheme as the CAZAC scheme. Therefore, CAZAC improves the estimated power-added efficiency of PAs from 10 to 30% for a 16-QAM OFDM system with 52 subcarriers because it reduces the PAPR of 5 dB while the system imposes no penalties on BER. This paper also provides a theoretical analysis of Zadoff-Chu sequence equalization and information on spectral control and BER under fading environments.

This paper is organized as follows. Section II analyze the CAZAC-OFDM system and consider applying CAZAC-OFDM to wireless communication. Section III presents the effect of CAZAC-OFDM in wireless communication. Finally, conclusions are drawn in Section IV.

II. PROPOSED SYSTEM

A. OFDM system

The frequency domain symbol, $\boldsymbol{X} = [X_0, X_1, ..., X_{N-1}]^T$ in OFDM systems is modulated by N-size Inverse Fast Fourier Transform (IFFT), which converts the frequency domain to the time domain. The discrete-time OFDM signal with N subcarriers is represented as

$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \tag{1}$$

where $j = \sqrt{-1}$ and n are the discrete time indices. However, receiver acquires frequency domain symbol Y by applying

FFT to received signal y.

$$Y_k = \frac{1}{N}\sum_{n=0}^{N-1} y_n e^{-j2\pi kn/N} = \frac{1}{N}\sum_{n=0}^{N-1}\{x_n+\sigma^2\}e^{-j2\pi kn/N}, \quad (2)$$

where σ^2 is noise power.

The PAPR of the OFDM signal (1) can be expressed as:

$$\mathrm{PAPR} = \frac{\max|x_n|^2}{\mathrm{E}[|x_n|^2]}, \quad (3)$$

where $\mathrm{E}[\cdot]$ is the expectation operator. PAPR is an index that represents the amplitude fluctuation of the OFDM signal for each frame. We can see from Eq. (2) that the OFDM signal is composed of a plurality of subcarrier signals, which causes an increase in amplitude fluctuations. A high PAPR signal requires the power amplifier to operate at a large input-back-off (IBO) due to the corresponding value of the PAPR to amplify the transmission signal without distortion. Increasing the IBO generally greatly decreases the efficiency of PA.

B. CAZAC-OFDM

CAZAC sequences involve constant amplitude and provide excellent cross-correlation properties. Therefore, CAZAC sequences are used in wireless communication systems such as channel estimation and time synchronization. The Zadoff-Chu sequence is one of these CAZAC sequences. The Zadoff-Chu sequence, $\{c_k\}$, is represented as:

$$c_k = \begin{cases} e^{j\pi k^2/L} & (L \text{ is even}) \\ e^{j\pi k(k+1)/L} & (L \text{ is odd}) \end{cases}, \quad (4)$$

where L is the length of the CAZAC sequence and k = 0,1,...,N^2-1 denotes the sequence index. CAZAC sequences are generated by cyclic shift of the original CAZAC sequence. The periodic cross-correlation function, ρ, is defined as:

$$\rho(m) = \sum_{n=1}^{L-1} c_n c^*_{(c-m) \bmod L} = \begin{cases} L & (m=0) \\ 0 & (m \neq 0) \end{cases}, \quad (5)$$

where m represents integer variable. In this paper, we choose $L = N^2$ in this paper, where CAZAC $N \times N$ precoding matrix $\boldsymbol{M}$ is represented as:

$$\boldsymbol{M} = \begin{bmatrix} c_0 & c_1 & \cdots & c_{N-1} \\ c_N & c_{N+1} & \cdots & c_{2N-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(N-1)N} & c_{(N-1)N+1} & \cdots & c_{N^2-1} \end{bmatrix}. \quad (6)$$

Frequency domain symbol $\boldsymbol{X'} = [X'_0, X'_1, ..., X'_{N-1}]$ in CAZAC-OFDM is represented as:

$$\boldsymbol{X'} = \boldsymbol{M}\boldsymbol{X}. \quad (7)$$

Therefore, the CAZAC-OFDM time signal is represented as:

$$x'_n = \sum_{k=0}^{N-1} X'_k e^{j2\pi kn/N}. \quad (8)$$

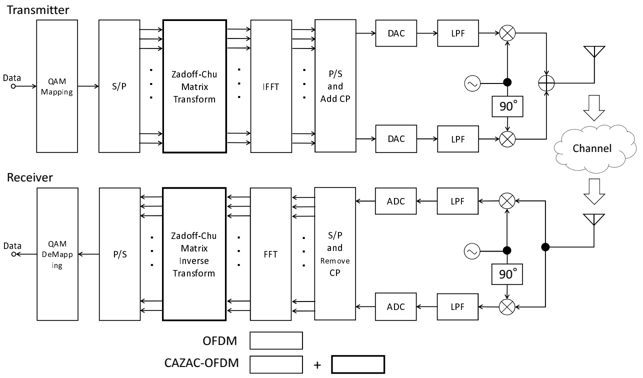


Figure 1. Proposed CAZAC-OFDM system

All sub-carriers in a CAZAC-OFDM system include data symbols. This system cannot include null sub-carriers. Therefore, the spectrum of the proposed system does not satisfy the spectrum mask.

The Zadoff-Chu sequences in Eq. (5) have periodic cross-correlation performance. Therefore the original frequency domain symbol, X, can be demodulated by using complex conjugate $\boldsymbol{M}^H$.

$$\boldsymbol{M}\boldsymbol{M}^H\boldsymbol{X} = N\boldsymbol{X}. \quad (9)$$

Figure 1 shows the configuration for a transmitter and receiver in the CAZAC-OFDM system, which applies CAZAC precoding matrix $\boldsymbol{M}$ to the mapping data after serial-parallel conversion.

C. Analysis of CAZAC OFDM

We clarify why the PAPR of CAZAC-OFDM was the same as the PAPR of mapped data signals such as 16 QAM. Frequency domain symbol $\boldsymbol{X'}$ is represented as:

$$\boldsymbol{X'} = \boldsymbol{M^T X} = \begin{bmatrix} c_0X_0 + c_1X_1 + \cdots + c_{N-1}X_{N-1} \\ c_NX_0 + c_{N+1}X_1 + \cdots + c_{2N-1}X_{N-1} \\ \vdots \\ c_{(N-1)N}X_0 + \cdots + c_{N^2-1}X_{N-1} \end{bmatrix} \quad (10)$$

We assumed that $L = N^2$ was even in the following since the OFDM system uses FFT, which rapidly computes the discrete Fourier transform (DFT) with input data having a length with a power of two. Therefore, we propose that baseband OFDM signal x_n can be represented as:

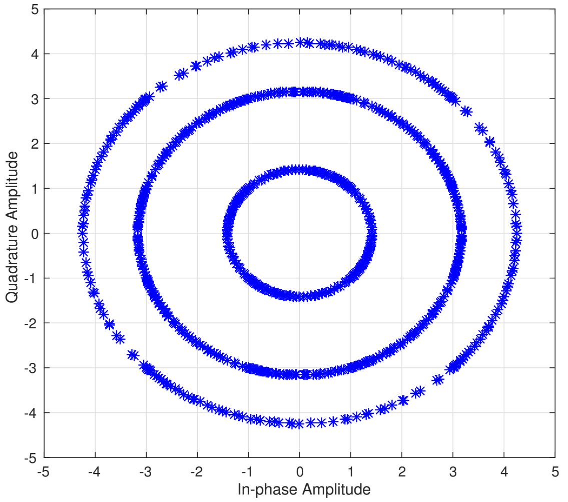


Figure 2. Constellation diagram in time signal which is equalized by CAZAC sequence

$$\begin{aligned} x_n &= \sum_{k=0}^{N-1} \left\{ \sum_{l=0}^{N-1} c_{l+kN} X_l \right\} e^{j2\pi kn/N} \\ &= \sum_{k=0}^{N-1} \left\{ \sum_{l=0}^{N-1} e^{j\pi(l+kN)^2/L} X_l \right\} e^{j2\pi kn/N} \\ &= \sum_{l=0}^{N-1} e^{j\pi l^2/N^2} \left\{ \sum_{k=0}^{N-1} e^{j2\pi k(l+n)/N} e^{j2\pi k^2} \right\} X_l \\ &= \sum_{l=0}^{N-1} e^{j\pi l^2/N^2} \left\{ \sum_{k=0}^{N-1} \left\{ -e^{j2\pi(l+n)/N} \right\}^k \right\} X_l . \quad (11) \end{aligned}$$

Equation (11) can be transformed as:

$$\sum_{k=0}^{N-1} \left\{ -e^{j2\pi(l+n)/N} \right\}^k = \begin{cases} N & (-e^{j2\pi(l+n)/N} = 1) \\ 0 & (-e^{j2\pi(l+n)/N} \neq 1) \end{cases} . \quad (12)$$

In this case, if $-e^{j2\pi(l+n)/N} = 1$, then $2(l+n)/N$ is odd. From $0 \leq l \leq N-1$ and $0 \leq n \leq N-1$, l and n correspond one to one. Therefore, when Eq. (12) $= N$, l is represented as:

$$l = (N/2 - n) \bmod N. \quad (13)$$

Therefore, x_n is represented as:

$$x_n = N \cdot c_{(N/2-n) \bmod N} \cdot X_{(N/2-n) \bmod N} . \quad (14)$$

The signal at time n in Eq. (11) is obtained by rotating the symbol of X_m. Therefore, the time signal of the proposed system looks like a single-carrier signal. Figure 2 plots the baseband signal on the complex plane. We applied 16 QAM as constellation mapping to the frequency domain symbol in this case. The PAPR of a single-carrier signal is generally lower than that of a multi-carrier signal.

The proposed system firstly applies FFT to input signal $\boldsymbol{Y}$ on the receiver side to obtain for get symbol $\boldsymbol{Y}$ in the frequency domain. The proposed system then multiplies the received signal $\boldsymbol{Y}$ and the inverse matrix, $\left\{\boldsymbol{M}^T\right\}^H$.

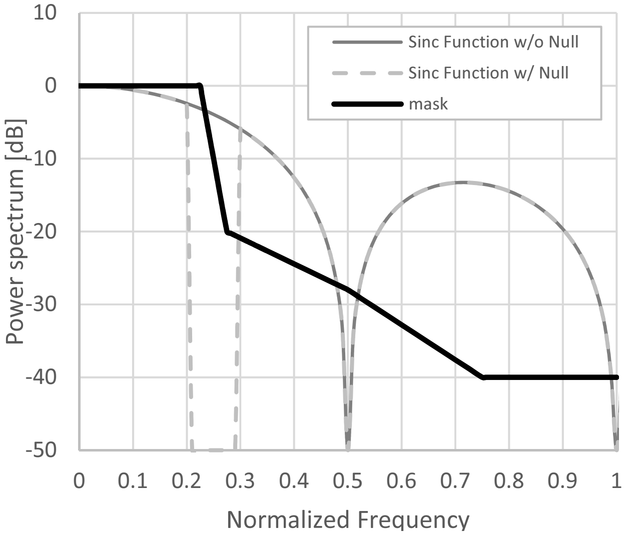


Figure 3. Power spectrum of sinc function with and without Null sub-carriers

$$\begin{aligned} \boldsymbol{Y'} &= \left\{\boldsymbol{M^T}\right\}^{\boldsymbol{H}} \boldsymbol{Y} \\ &= \left\{\boldsymbol{M^T}\right\}^{\boldsymbol{H}} \left\{\boldsymbol{M^T X} + \boldsymbol{G}\right\} \\ &= N \cdot \boldsymbol{X} + \overline{\boldsymbol{M}} \boldsymbol{G}, \quad (15) \end{aligned}$$

where $\boldsymbol{G}$ is the noise added to each subcarrier and $\overline{\boldsymbol{M}}$ is the conjugate of the matrix $\boldsymbol{M}$. All elements of the matrix $\boldsymbol{M}$ are complex number on the unit circle. In addition, all elements of the matrix $\overline{\boldsymbol{M}}$ are also complex number on the unit circle. Therefore, noise is dispersed for all sub-carriers. As a result, the proposed system is robust against noise including only specific frequency components such as frequency selective fading.

D. Proposed system

The frequency domain symbol of CAZAC-OFDM in Eq. (10) includes data sub-carriers in all sub-carriers. Figure 3 plots the difference between the spectrum with and without null sub-carriers. If the IEEE 802.11 specifications are taken into consideration, the normalized frequency can be multiplied by 40 MHz. The filtering normalized frequency in Fig. 3, which is smaller than 0.25, degrades BER. It is necessary, on the other hand, to reduce the power spectrum by 20 dB between the normalized frequencies of 0.225 and 0.275 according to the specifications of the spectrum mask. Therefore, it is not possible for filtering to satisfy the spectrum mask.

We solved this problem in this research by decreasing the symbol rate. Moreover, data sub-carriers were allocated to all sub-carriers without using null sub-carriers. Therefore, the proposed system did not decrease the data rate despite the decreasing symbol rate.

III. PERFORMANCE EVALUATION

We conducted simulation experiments with the matrix laboratory MATLAB/Simulink to evaluate the performance

TABLE I. SIMULATION SPECIFICATION.

Modulation	OFDM	CAZAC-OFDM
Mapping	16QAM	16QAM
Bandwidth	20 MHz	16 MHz
Symbol time	4 µ sec	5 µ sec
Data rate	48 Mbps	48 Mbps
Carrier frequency	100 MHz	100 MHz
Number of data subcarriers	48	60
FFT size	64	64

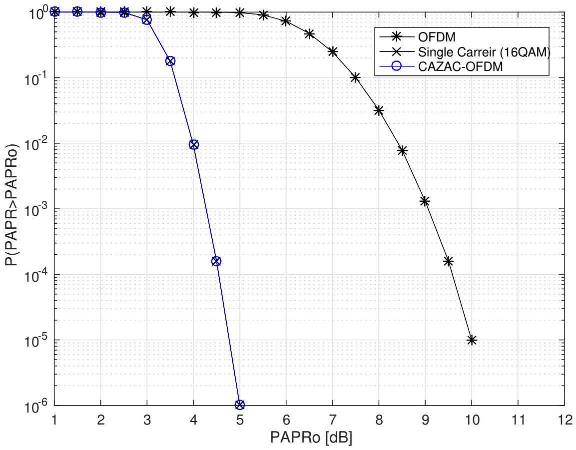


Figure 4. CCDF performance

of the proposed system. Table I summarizes the simulation setting. We will now present the simulation results of OFDM and single-carrier (16 QAM) systems to enable a comparison of performance.

We first considered the complementary cumulative distribution function (CCDF) of PAPR to evaluate the performance of PAPR, which is the probability that PAPR will be higher than a certain PAPR value PAPR_0, i.e., $\Pr(\mathrm{PAPR} > \mathrm{PAPR}_0)$. Figure 4 plots the CCDF of PAPR using the proposed system as well as the OFDM and single-carrier systems. We found that the PAPR of the proposed system was almost equal to that of the single-carrier system. Moreover, the PAPR of the proposed system was improved by 5.8 dB at the CCDF of 10^{-3} compared with the OFDM system. This PAPR reduction resulted from CAZAC equalization. The CAZAC equalization in Fig. 2, converted the amplitude of the OFDM signal to the amplitude of the mapped data such as 16 QAM, which improved PAPR.

We next examined the bit error rate (BER) of the proposed system. We considered three channels: additive white Gaussian noise (AWGN), AWGN & frequency selective fading channels, and Rayleigh fading. A specific frequency (101 MHz) was highly attenuated, as plotted in Fig. 5 in the AWGN & frequency selective fading channels, in addition to AWGN.

Figure 6 plots the BER of the proposed system and the OFDM system under AWGN and the AWGN & frequency selective fading channels. The results indicate that the proposed system does not degrade BER. The signal power of specific sub-carriers is highly attenuated in the OFDM system, and thus BER is large even when the noise power is low. In contrast, the BER of the proposed system is small because the influence of fading is spread to all sub-carriers. This indicates that the proposed system has excellent capabilities to resist frequency selective fading.

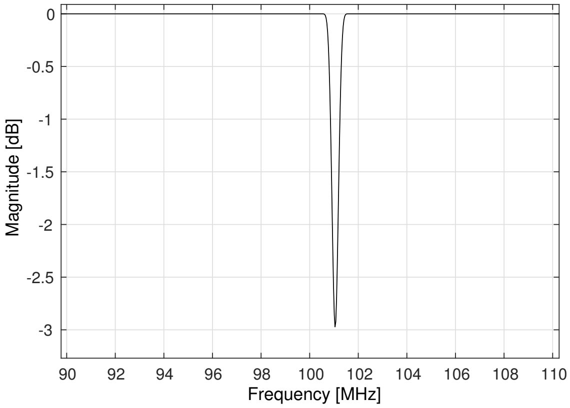


Figure 5. Frequency selective fading model

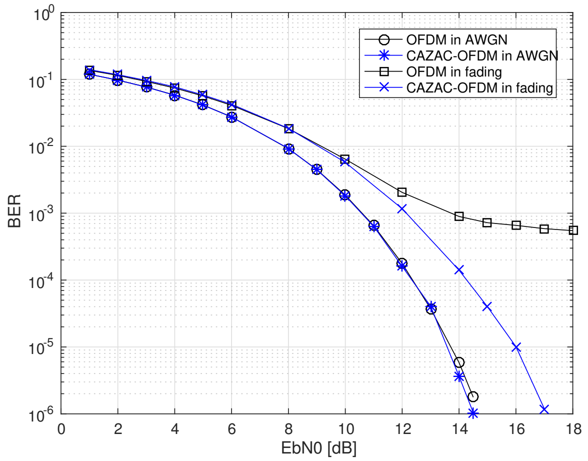


Figure 6. BER versus EbN0 under AWGN and frequency selective fading environments

Figure 7 plots the BER of the proposed and the OFDM systems under AWGN and Rayleigh fading channels. We found that the BER of the proposed system was comparable to that of the OFDM system in both channels. Therefore, CAZAC equalization did not affect the multi-path fading compensation of OFDM.

Finally, we show the spectrum of the proposed system in Fig. 8. By decreasing the symbol rate, the spectrum of the proposed system satisfies the spectrum mask standardized by IEEE 802.11 specification [1]. Therefore, the proposed scheme can be applied to wireless communication systems such as Wi-Fi and LTE.

IV. CONCLUSION

We proposed a new OFDM wireless system using the CAZAC scheme, which made the PAPR of the M-QAM OFDM signal into the PAPR of an M-QAM single-carrier signal. We found the performance of the system was the

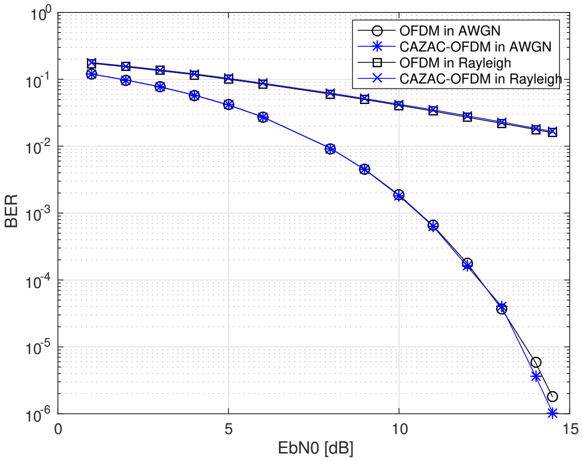
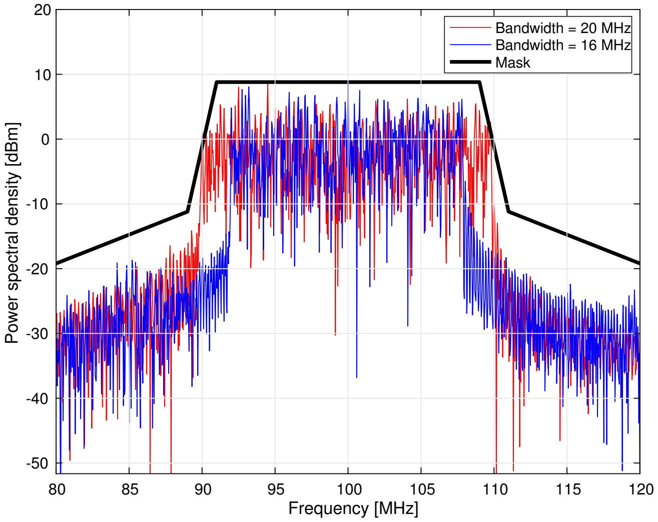


Figure 7. BER versus EbN0 under AWGN and Rayleigh fading environments



RBW=49.97 kHz

Figure 8. Spectrum for proposed system

same as that of single-carrier signals through simulations using MATLAB/Simulink. Therefore, we expected that the CAZAC scheme would approximately improve the power-added efficiency of the PA for a 16-QAM OFDM system with 52 sub-carriers from 10 to 30% because it reduced the PAPR of 5 dB while the system imposed no penalties on BER. The system satisfied the spectrum mask defined by the IEEE 802.11 a/g standards, while maintaining the same data rate, by adjusting the symbol duration of the standards and increasing the number of data sub-carriers. Moreover, the CAZAC scheme had an advantage in reducing BER under frequency selective fading environments and not degrading BER under Rayleigh fading environments.

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