Optimization of Flows in Level-Constrained Multiple Trees for P2P Multicast System

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***Abstract*—Peer-to-Peer multicast defined for overlay networks, has been taking an advantage over IP Multicast during recent years. It derives from the fact that the overlay architecture for P2P streaming provides potential scalability and easy deployment of new protocols independent of the network layer solutions at relatively low costs. In this paper, we focus on modelling and optimization of multiple trees for flow assignment in P2P multicast systems. The optimization covers multicast flow arrangement on multiple paths in order to minimize the overall streaming cost. Due to quality of service requirements in such kind of systems, we apply a basic hop-constrained spanning tree and capacitated spanning tree problems and we define level-constrained multiple trees problem with bandwidth capacity constraints for multicast flow assignment in overlay system. We propose and compare two Mixed Integer Programming formulations for the problem. In addition, we examine multicast flows in relation to various fragmentation of the content.**

***Keywords*-P2P; Multicast; Flows; Network Optimization**

I. Introduction

Multicast network technique delivers information to a group of destinations simultaneously. Over the years, a lot of research, development, and testing efforts have been devoted to multicast support [1][3][12][17][18][21][22][23]. Multicasting implemented by network-aware approaches using Internet Protocol (IP-Multicast) is afflicted with problems derived form scalability, addressing scheme management, flow or congestion control but in contrast, features of overlay architecture provides potential scalability or ease of deployment new, network-layer-independent protocols at relatively low costs. Overlay network strategy expands end-system multicast [4][11][16] and using overlay based systems has become an increasingly popular approach for multicast and streaming, where participating peers actively contribute their upload bandwidth capacities to serve other peers in the same streaming session by forwarding their available content. Overlay multicast flows are realized as multiple unicast flows at level of a network layer.

In this paper, we consider an overlay P2P system and flow assignment problem for multicast application based on multiple delivery trees. We assume constant bit rate of the multicast stream, which can be divided into separate fractional flows. Due to the quality of service requirements for mutlicast trees structure we employ the hop constrained spanning tree problem [5] and we formulate level-constrained multiple trees problem. The main goal of the problem is to minimize the total cost of delivery trees without considering issues related to dynamics of P2P systems and individual algorithm's tree creation. Cost can be treated as a distance between pair of overlay nodes and can refer to delay, physical flow delivery expenses and network maintenance or cross-ISP's payments and depends on bit rate of transferred stream. To solve the problem in optimal way, we formulate two different mixed integer models. First one, based on directed multicommodity flow and second, derived from level-based formulation.

Although the model based on directed multicommodity flows (DMFM) [7][14] has been applied in past works on hop constrained trees for single tree creation, the last one is in general novel. Level-based formulation (LTM) for creating trees was used in our previous works [19][20] but in this paper we improve its first version and apply it for the level-constrained multiple trees flow problem. These formulations provide with offline optimization, which can be used to find lower bound of other solution approaches or for constructing initial topology. Moreover some P2P multicasting systems are generally static (e.g., data distribution in CDN), and results of offline optimization can be applied in such systems to improve the system performance.

Furthermore we compare and contrast these models in quantitative and time-consumption meaning. To compare the performance and effectiveness of the formulations in terms of the execution time we apply the models in Gurobi Optimizer [10]. Our results show that the models can be more efficient and can enable solving instances in shorter time for different cases. We also employ the models for examining in various ISP (Internet Service Provider) topologies the effect of the tree depth (hop limit) and multicast stream fragmentation on the optimal delivery cost.

The remainder of this paper is organized as follows. In Section II we define Level-Constrained Multiple Trees Problem and in Section III we formulate it as a integer optimization problem. In Section IV, we compare and contrast basic features of the models and present time-consuming evaluation of building and solving models with Gurobi Optimizer [10]. Investigations on the impact of stream fragmentation on the total cost are covered in Section V. Section VI concludes the paper and presents ideas for future work.

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II. Level-Constrained Multiple Trees Problem

In this paper we consider an overlay multicast session with single source and multiple participating receivers. We assume that, in the overlay system all nodes except of the root node are receivers. The general objective is to stream the content as multicast session, which is divided into T delivery spanning trees representing fractional flows.

For live media applications, a minimized delays or end-to-end latency at each receiver are significant to guarantee the high liveness and quality of service of the streaming media. On the other hand multicast system features should include reliability, availability or even ease of creation of delivery trees. One of the concept for designing of network-based systems with overall quality of service constraints is the hop-constrained minimum spanning tree problem (HMST) [5][6], which is defined as follows: Given a graph $G = (N, E)$ with node set N and edge set E as well as a cost c_e associated with each edge e of E and a natural number H, we wish to find a spanning tree $t \in G$ with minimum total cost and such that the unique path from a specified root node, node r, to any other node has no more than H edges (hops). Service quality can refer to availability and reliability, redundancy at network layer, quality or ease of creation and managing of the tree.

In accordance to flow problems more useful is arc-based (directed edges) formulation of the HMST. The directed formulation replaces every edge $e = \{i, j\}$ in the graph by the two arcs (i, j) and (j, i) and associates to each of these two arcs the cost of the original edge. Analytically, costs can be defined in asymmetric way, that is, cost of arc c_{ij} can vary from c_{ji}. Let A denote set of directed arcs in the directed model. Notice also that - according to overlay network structure - we practically consider a complete directed graph. We assume that there is a bi-directed edge between any pair of overlay nodes except of the root node. In case of the root node r we "remove" directed arcs to the root from any other node, edge $e = \{i, r\}$ is only replaced by one single arc (r, i).

Second, every peer is connected to the overlay network with link, which has a limited upload and download capacity, hence peer's number of all children in all fractional trees is constrained. Speaking in more precise way, the limitation of child nodes for peer depends on number of fractional trees and their streaming rates. The assumptions stated for single tree can be expressed as the capacitated minimum spanning tree (CMST) [15]. In this paper we extend this capacity constraint to multicast system with multiple trees.

The Level-Constrained Multiple Trees Problem (LCMT)

The main aim of the problem is to construct an overlay multicast topology and assign flows on multiple trees, which are limited with maximum hops. We consider stream S of constant bit rate and define constant number of fractional flows as T with fractional streaming rates s_t for each tree t where $S = \sum_t s_t$. This concept can be easily deployed in real systems with existing codecs, i.e., we can steer or manage the number of packets or frames as a bit rate in each tree t. Upload capacity (u_i) and download capacity (d_i) refer to peer's i available upstream and downstream bandwidth, respectively.

Note that, the LCMT contains as particular cases (the case with $T = 1$, $H = N - 1$) a degree constrained spanning tree or capacitated minimum spanning tree, which are NP-Complete problems [15] or (the case with $T = 1$, $SN <= u_i$) a NP-Hard version of the hop-constrained minimum spanning tree [6].

III. Mixed Integer Programming Formulations

The general formulation derives from the well discussed, based on the multicommodity flow model [7][14], which was successfully implemented for the HMST problem. Several previously known and developed formulations for the HMST (single tree) were mainly derived from the multicommodity flow model and were used for problems with limited sizes of either hops or arcs [5][6] or for optimization at network-aware level [2][8][13].

A. Directed Multicommodity Flow Model (DMFM)

The general multicommodity flow model, denoted DMFM or MCF, uses two sets of binary variables. Variables x_{ijt} indicate whether the spanning tree for fractional flow t contains the arc (i, j) and additional set of directed flow variables f_{ijkt} specify if the unique path from the root node r to node k traverses the arc (i, j) in tree t.

In the following IP formulation we consider an overlay system, which can be represented as a complete directed sub-graph, which consists of $N\backslash\{r\}$ nodes and in addition node r with arcs to any i. Hence $(i, j) \in A \equiv (i, j) : i \in N; j \in N\backslash\{i, r\}$. Further notation replaces set N by indexing scheme $1, ..., N$.

indices

$i, j, k = 1, 2, ..., N$ vertices (peers, application layer nodes)
$t = 1, 2, ..., T$ trees (fractional flows)

constants

r root node ($r \in 1, 2, ..., N$)
H maximum hops
d_i download capacity limit in kbps
u_i upload capacity limit in kbps
s_t streaming rate of tree t in kbps
c_{ij} defines cost of 1 kbps transferred from i to j

variables

x_{ijt} = 1 if the spanning tree t contains arc (i, j); 0 otherwise (binary variable)
f_{ijkt} = 1 if path from root r to k contains arc (i, j) in tree t; 0 otherwise (binary, auxiliary variable)

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objective

$$\min \quad F = \sum_{i} \sum_{j \neq \{i,r\}} \sum_{t} c_{ij} s_t x_{ijt} \tag{1}$$

constraints

$$\sum_{i \neq j} x_{ijt} = 1 \quad \forall j \neq r \quad \forall t \tag{2}$$

$$\sum_{j \neq \{i,r\}} f_{ijkt} - \sum_{j \neq \{i,k\}} f_{jikt} = \begin{cases} 1 & i = r, \forall k \neq \{i,r\}, \forall t \\ -1 & i = k, \forall k \neq r, \forall t \\ 0 & \forall i \neq \{k,r\}, \forall k \neq r, \forall t \end{cases} \tag{3}$$

$$\sum_{i \neq k} \sum_{j \neq \{i,r\}} f_{ijkt} \leq H \quad \forall k \neq r \quad \forall t \tag{4}$$

$$\sum_{i \neq j} \sum_{t} s_t x_{ijt} \leq d_j \quad \forall j \neq r \tag{5}$$

$$\sum_{j \neq \{i,r\}} \sum_{t} s_t x_{ijt} \leq u_i \quad \forall i \tag{6}$$

$$f_{ijkt} \leq x_{ijt} \quad \forall i, \forall j \neq \{i,r\}, \forall k \neq \{i,r\}, \forall t \tag{7}$$

The main goal of the problem (1) is to find T level-constrained spanning trees, which minimizes the total cost of all flows in the system. Formula (2) refers to the completion constraint and assures each node except of the root node has exactly one parent node in each tree t. Constraint (3) derives from the flow conservation concept and guarantees that the solution of fractional flow t is a directed spanning tree rooted at node r. Constraints (4) state that no more than H arcs are in the path from the root node r to any other node k in tree t. Limitation of downloads in the systems is constrained by formula (5). By analogy, we introduce the upload capacity constraint, which must be satisfied with regards to physical outgoing bandwidth and available capacity of any node i. To bound flow variable f and tree variable x, constraints (7) are introduced. These constraints satisfy that every arc (i,j), which transports flow to any node k in tree t $f_{ijkt} = 1$ exists in tree $x_{ijt} = 1$ simultaneously. Equivalently, if arc (i,j) is not belonging to the tree t, $x_{ijt} = 0$, there cannot be any flow $f_{ijkt} = 0$.

B. Level-based Tree Model (LTM)

To model the multicast trees, the LTM exploits a single set of binary variables x_{ijlt}, which equal to 1 if the spanning tree t contains the arc (i,j) and node i is located at level l. We assume that the root r of each tree t is located at the first level ($l = 1$). All children of the root are located at level 2, etc. The proposed notation enables us to set the value of L as a limit on the maximal depth of the tree. The LTM is based on a precedence relation between two adjacent nodes and is defined by analogy to hop-constrained walk [5] or Steiner Tree Problem in a Layered Graph [9].

indices

$i, j = 1, 2, ..., N$ vertices (peers, application layer nodes)
$l = 1, 2, ..., L$ level of the node
$t = 1, 2, ..., T$ trees (fractional flows)

constants

r root node ($r \in 1, 2, ..., N$)
d_i download capacity limit in kbps
u_i upload capacity limit in kbps
s_t streaming rate of tree t in kbps
c_{ij} defines cost of 1 kbps transferred from i to j

variable

x_{ijlt} = 1 if arc (i,j) belongs to spanning tree t and i is located at level l; 0 otherwise (binary variable)

objective

$$\min \quad F = \sum_{i} \sum_{j \neq \{i,r\}} \sum_{l > 1} \sum_{t} c_{ij} s_t x_{ijlt} + \sum_{j \neq r} \sum_{t} c_{rj} s_t x_{rj1t} \tag{8}$$

constraints

$$x_{rj1t} + \sum_{i \neq \{j,r\}} \sum_{l > 1} x_{ijlt} = 1 \quad \forall j \neq r, \forall t \tag{9}$$

$$\sum_{j \neq r} \sum_{t} s_t x_{rj1t} \leq u_r \tag{10}$$

$$\sum_{j \neq \{i,r\}} \sum_{l > 1} \sum_{t} s_t x_{ijlt} \leq u_i \quad \forall i \neq r \tag{11}$$

$$\sum_{i \neq \{j,r\}} \sum_{l > 1} \sum_{t} s_t x_{ijlt} + \sum_{t} s_t x_{rj1t} \leq d_i \quad \forall j \neq r \tag{12}$$

$$x_{ij(l+1)t} \leq \begin{cases} x_{ri1t} & \forall i \neq r, \forall j \backslash \{i,r\}, l = 1, \forall t \\ \sum_{k \neq \{i,r\}} x_{kilt} & \forall i \neq r, \forall j \backslash \{i,r\}, \forall l \backslash \{1, L\}, \forall t \end{cases} \tag{13}$$

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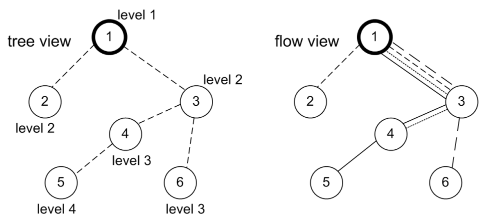

Figure 1. An example of fractional delivery tree for all models (left) and visualization of flow variables in DMFM (right).

The objective function (8) minimizes the spanning trees cost and includes two elements: cost of flows from any $i \neq r$ and arcs, which are starting in the root r. Constraints (9) satisfy that each node is connected to any of tree t, either j's parent is root ($x_{rj1t} = 1$) or any other node i ($\sum_{i \neq \{j,r\}} \sum_{l>1} x_{ijlt} = 1$). Node $j \neq r$ has exactly one and only one parent node situated at exactly one level in each tree t. To clarify upload capacity limitations we introduce (10) and (11) for outgoing streams from root and any other node, respectively. Download capacity constraints include all receiving nodes j (without the root r) and limit the sum of incoming flows in all trees. Due to the fact, that every parent node is situated at exactly one level in each tree t thus to avoid loops, the multiple tree variables x_{ijlt} are created under constraints (13). In this case, every node i can be a parent of any node j at level $l+1$ ($x_{ij(l+1)t} = 1$) if and if only i's parent node k is located at level l ($\sum_{k \neq i} x_{kilt} = 1$).

IV. Models Comparison

Figure 1 presents an example of single, fractional delivery tree t. The flow view refers to the set of flow variables in the DMFM formulation. In Tab. I binary variables x equal to 1 are shown. For the DMFM and the LTM there is only such straightforward representation of the tree from example in Fig. 1

The main advantage of the LTM over directed multi-commodity flow formulation is that we can limit models' sizes (number of variables and constraints) according to the selected hop limit. Table II presents sizes of the models. Variables of the DMFM are x_{ijt}, f_{ijkt}; and the LTM are x_{ijlt}. Note that the maximum usable value for L is $L = N-1$ (in extreme case, the tree is a path bounded with $N-1$ levels for locating parents). In regard to this reason, the number of DMFM's variables exceed variables of the LTM by at least $2N^2 - 2N$ for each fractional tree t. 'Constr.' in Tab. II refers to the number of formulation's constraints in relation to network sizes and represents constraints of (2)-(7) for the DMFM, (9)-(13) for the LTM.

The computational results were obtained on a PC, Intel Core2 Duo, 2.13 GHz, 4GB RAM, Windows 7 Professional. We used the C++ libraries of *Gurobi 2.0.2* with default parameters to obtain the optimal integer solutions of the models tested. We first compare executable build time of the models. It refers to time, which is needed for dynamic creation of variables and merging all constraints of the problem. Fig. 2 presents average time required for building the models. Average building time of LTM with its upper bound of $L = N-1$ is comparable to DMFM but for any value L less than $N-1$ the LTM is expected to be built faster in contrast to the DMFM, which need, in general, the same number of operations irrespective of the hop limit.

Table I
Equivalent solution for various models

	Example of solution (variables equal to 1)				
Model	1	2	3	4	5
DMFM	$x_{1,2,t}$	$x_{1,3,t}$	$x_{3,4,t}$	$x_{3,6,t}$	$x_{4,5,t}$
LTM	$x_{1,2,1,t}$	$x_{1,3,1,t}$	$x_{3,4,2,t}$	$x_{3,6,2,t}$	$x_{4,5,3,t}$

Table II
Size of the models

	Element	Size
DMFM	Variables	$(N^3 - 3N^2 + 4N - 2)T$
	Constr.	$(N^3 - 3N^2 + 7N - 5)T + 2N - 1$
LTM	Variables	$((L-1)N^2 + (4-3L)N + 2L - 3)T$
	Constr.	$((L-1)N^2 + (4-3L)N + 2L - 3)T + 2N - 1$

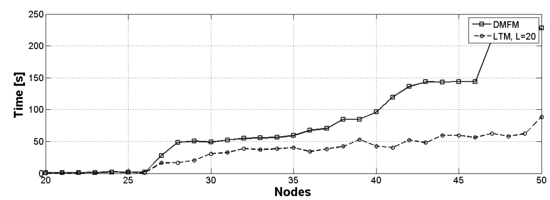

Figure 2. Average time of building models DMFM and LTM

A. Experimentation on Networks

We propose to select the arcs' costs to represent various topologies at level of inter-ISP connections. Fig. 3 shows hypothetical scenarios with different topologies at cross-ISP level. The root node is always in ISP 1 and remaining nodes are distributed among available providers in proportional way. To model these topologies we define cost tables with the following simple algorithm: if any pair of peers belong to the same ISP the cost between them is randomly chosen from 3 to 10. If the shortest path (referred as the number of inter-ISP hops) between peers equals to 1, the cost is set from 20 to 40. Distance of 2 ISP-hops introduces cost in the range of 50-90. We assume symmetric costs ($c_{ij} = c_{ji}$) for all instances but note that costs do not satisfy irregularity of the triangle. In the remaining simulations we assume all peers have the same capacity parameters $512kbps$ for upload and $1024kbps$ for download, streaming rate of the session is $S = 252$ kbps and streaming rate of each fractional tree s_t is derived from the proportional division S into T trees, i.e., if only one tree $T = 1$ is used for the flow allocation its streaming rate $s_1 = 252kbps$, if $T = 3$ then $s_1 = s_2 = s_3 = 84kbps$. We chose $S = 252kbps$ in regards to provide fractional flows with rational values. Note that, for $T = 5$, $s_t = 50.4kbps$, which is still rational number expressed in bps.

Problem's solving times presented in Figs. 4 and 5 were obtained for set of 50 instances with 20 nodes and $T = 1$ for topologies of $S1$ and $S2$ respectively. Results shown there indicate that, the LTM is in general better than DMFM especially in cases of strongly limited number of levels. Average solving time by DMFM and can be approximately treated as constant. Note that time required for solving MIP problems depends not only on the problem size but also on values of input parameters (e.g., costs or root location) and the execution time relation presented in this paper refer to rather overall rough comparison without delving into solver implementation and operating system performance. Moreover, results shown in Fig. 7 presents that, for the much more complicated instances it is worth using DMFM with regards to solve problem in shorter time. This can suggest a general high stability of the DMFM in contrast to the LTM in computational time meaning.

Finally, all of the presented models can be applied for determining the number of required trees for fractional flows, for which total cost of flows assignment can be minimized. Fig. 8 presents optimization costs in relation to number of trees and allowed levels. Note that, the solution is infeasible in cases of hop limit $H = 1$, which means that, the root node is parent of all nodes in all trees and in case $H = 2$ and $T = 1$. The overall conclusion, illustrated in these figures, can be stated as follows: the flow delivery cost for multicast system based on multiple trees can be decreased if more trees with more allowed hops are employed, nevertheless, the most critical impact on delivery cost has limitation of levels.

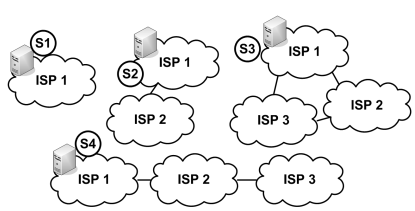

Figure 3. Proposed scenarios of inter ISP connections.

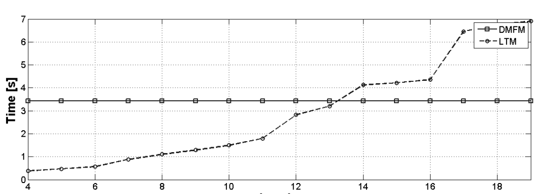

Figure 4. Scenario S1: average time of obtaining optimal result (N=20, T=1).

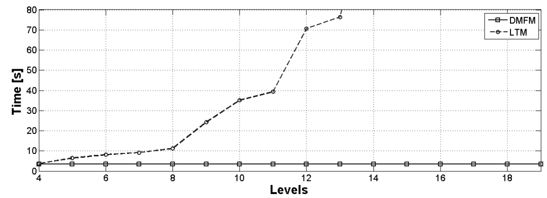

Figure 5. Scenario S2: average time of obtaining optimal result (N=20, T=1).

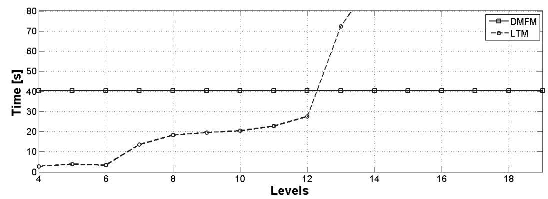

Figure 6. Scenario S1: average time of obtaining optimal result (N=20, T=5).

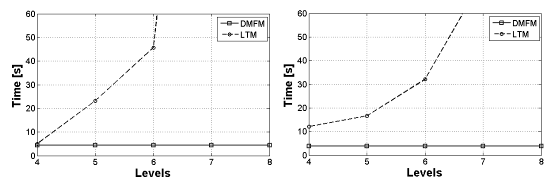

Figure 7. Scenario S3 (left) and S4 (right): average time of obtaining optimal result (N=20, T=1)

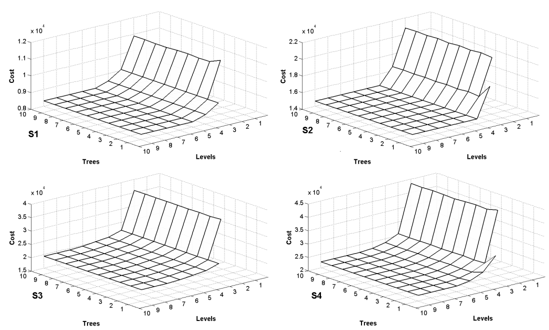

Figure 8. General relation of level and trees number to objective value.

V. Conclusion and Further Work

In this paper, we defined the level-constrained multiple trees problem for multicast streaming purposes. We assumed that the multicast streaming session is divided into separate fractional flows, which spread in the overlay network on multiple spanning trees. These spanning trees are additionally created under the depth (hop, level) constraint. We proposed two mixed integer formulations for the problem and compared as well as contrasted quantitative and time consuming performance of them. The LTM applies a precedence relation between two adjacent nodes, eliminate recording paths to receiving peers and create the spanning tree as short as possible, what requires less number of variables and constraints. Experimentation results with *Gurobi Optimizer* show that the LTM can improve solving the problem in computational time, especially for either limited hops number or small difference between arcs' costs. However, generally most 'deterministic' and predictable results in computational time meaning are provided with DMFM formulation (using *Gurobi*). Moreover, we showed how to manipulate the table cost for simulating various structures of inter-ISP connections and the next step is to analyse more complicated ISP-based topologies with various root locations, different distribution of peers and numerous streaming rates for different tree. Finally, results presented in the paper indicate that, it is worth dividing media stream into separate fractional streams and constructing trees with level limitation greater than 4 provides with relatively short trees without extortionate expenses. Further work includes examination of various overlay networks with much more heterogeneous nodes (i.e., upload and download capacities), more complicated inter-ISP topologies and investigations on different streaming rates of fractional flows. With regards to solve the problem for instances of larger sizes, we plan to design and implement heuristics and metaheuristic algorithms.

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