A Mathematical Model for Predator–Prey Ecosystems Facing Climate Changes

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***Abstract*—We developed a mathematical model for predator–prey ecosystems undergoing climate-related changes. The model introduces the amount of information transferred between the number of individuals of the predator and prey categories, and the regulation performance in a predator–prey ecosystem is measured by a reduction of Shannon entropy, which is achieved by predation events and decay in the ecosystem. We examine the model with a computer simulation for a well-studied bass–crayfish predator–prey ecosystem in a closed lake.**

Keywords-mathematical model; predator–prey ecosystem; climate change.

I. INTRODUCTION

Over the last three decades, environmental changes, such as global warming, desertification, and air pollution have worsened, and their effects on life systems is a serious concern [1]. Previous studies on the environmental responses of life systems have been conducted for specific networks, including genetic regulatory networks and ecological networks [2][3].

Schrödinger suggested that a life system takes orderliness from its environment and sustains itself at a fairly high level of orderliness, or at a fairly low level of thermodynamic entropy [4]. Kauffman investigated how the dynamic behavior of a Boolean network suddenly becomes orderly. He made the analogy that the behavior approximates cell fate which is characterized by expression patterns of multiple genes in an organism [5][6]. Barabási and Albert found that generic mechanisms form an ordered network structure with a scale-free property [7]. However, we could not find a mathematical model that clarifies the varying orderliness of biological systems undergoing environmental changes.

In this study, we quantify the environmental stimuli and orderliness achieved in state variables in life systems with Shannon entropy based on their probability distributions. The state variables represent the state of the system, such as expression levels in a genetic network. We then hypothesize a relationship between environmental changes and orderliness in the life systems. We validate the hypothesis on an ecosystem using numerical experiments on a computational model of differential equations for the ecosystem with the climate-shift model [8]. In the model, a climate-attribute change is modeled as a shift in the probability distribution of the climate attribute. We evaluate control performance by a difference of Shannon entropy as $\Delta H \equiv H(X)-H(X')$, where X and X' represent the state variable X at t_0 and at t_1 (unit time after t_0), respectively [9]. The Shannon entropy $H(X)$ indicates the uncertainty of X [10]. Section II includes our results and discussion, and Section III states our conclusion and future work.

II. RESULTS AND DISCUSSION

We consider a predator–prey ecosystem in a closed lake (Figure 1a). The probability distribution of the number of viable predators, which we call “capacity”, varies according to the climate shift of a climate attribute against a range of climate attributes (survival region) in which the predator is viable. The predator capacity decreases with an increase in climate shift (Figure 1b). We derived (1), which shows that the Shannon entropy ($H(Y)$) of the number of predators decreases with an increase in the climate shift (Figure 1c):

$$H(Y)_{e+\delta e} \le H(Y)_e \,, \tag{1}$$

where e and δe indicate the level of the climate attribute and its increment. Generally, $I(X;Y) \le \min\{H(X),H(Y)\}$, thus

$$I(X;Y)^U_{e+\delta e} \le I(X;Y)^U_e \,, \tag{2}$$

where $I(X;Y)^U (\equiv H(Y))$ denotes an upper bound of the mutual information between X and Y. We merged (2) with an information–theoretic limit for general control systems [9], and thus obtained (3):

$$\Delta H^U_{e+\delta e} \le \Delta H^U_e \,, \tag{3}$$

where ΔH is the Shannon entropy reduction of the state variable X (the number of prey individuals) over the transition $X \rightarrow X'$ between t_0 and t_1 (unit time after t_0). It represents the control performance of the predator–prey ecosystem. Equations (2) and (3) suggest that the mutual information between the number of prey individuals (X) and predators (Y), as well as the control performance of the predator–prey ecosystem, decreases with an increase in climate shift. Furthermore, the control performance of the predator–prey ecosystem appears to degrade from the level of a closed-loop control system to an open-loop control system, based on the information–theoretic limits of control [9].

Numerical experiments on a well-studied bass–crayfish predator–prey ecosystem in a closed lake [11] validate the degradation of the control performance suggested by the model mentioned above.

The derived inequalities, (1), (2) and (3), are independent of the dynamics of the target ecosystem. Thus, our model can be applied to analyses of ecosystems in which the dynamics are unknown. Furthermore, our model and the numerical experiment results suggest that the maintenance of predator numbers is effective for protecting predator–prey ecosystems against climate-related changes.

III. Conclusion and Future Work

We developed an information–theoretic predator–prey ecosystem model that is independent of the dynamics of the ecosystem, and validated the model through numerical experiments.

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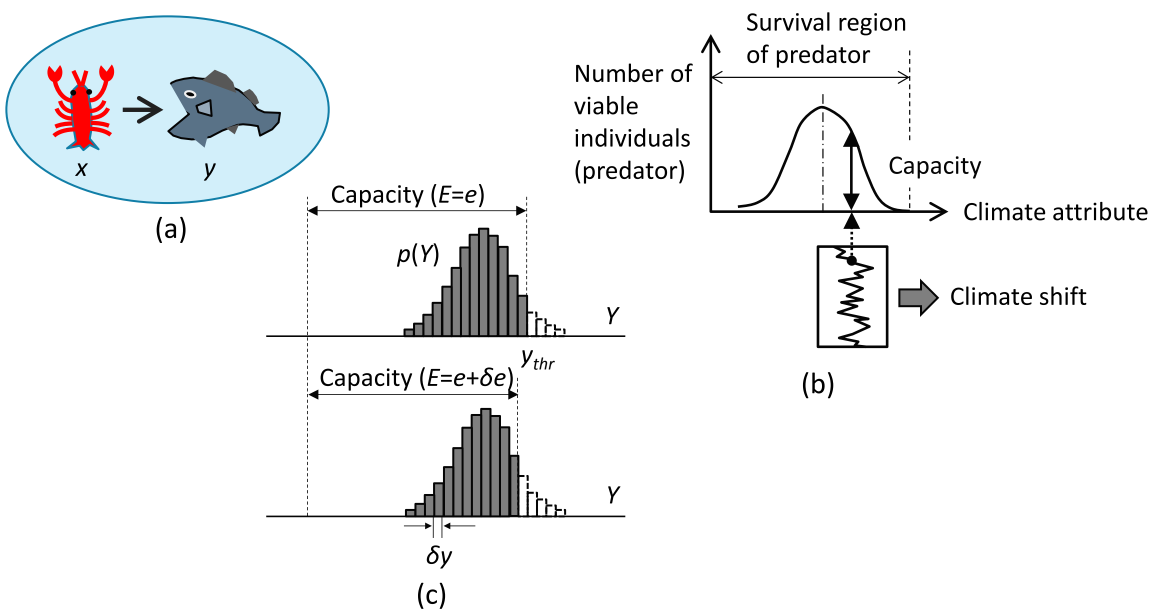

Figure 1. An ecosystem model: (a) Predator–prey ecosystem in a closed lake. The arrow denotes feeding relationship. (b) Number of viable predators and climate shift. (c) Probability distribution of the number of predators before (upper panel) and after (lower panel) an increase in climate shift by δe.

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