Iterative Decision Feedback Equalization for Filter Bank Multicarrier Systems

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***Abstract*—Filter Bank Multicarrier (FBMC) systems are a class of multicarrier modulation schemes for high speed wireless communication. These systems are known for their low adjacent channel leakage. In this paper we focus on the problem of channel equalization for FBMC systems. Most solutions in the literature use a per subcarrier equalization suffering from an error floor at high signal-to-noise ratios (SNR) caused by the residual inter-symbol-interference (ISI). We investigate a simple per subcarrier channel equalization method with ISI minimization and an averaging based ISI cancelation technique. We also introduce an iterative decision feedback scheme which outperforms the other known equalization methods. The presented methods are validated using simulation. The results are compared to the performance of Orthogonal Frequency Division Multiplexing (OFDM) systems with cyclic prefix (CP).**

***Keywords*-FBMC; iterative; channel equalization; decision feedback.**

I. Introduction

The introduction of Cognitive Radio (CR) triggered a new interest for researching alternatives of OFDM multicarrier systems [1]. In this paper, we focus on the FBMC systems as a major candidate competing with OFDM in CR scenarios. OFDM is a widely adopted modulation scheme due to its simple modulation/demodulation using IFFT/FFT block and a channel equalization with low complexity. Despite its many advantages it has some significant drawbacks which must be taken into consideration during the design.

These disadvantages include sensitivity to nonlinear distortions – due to the fluctuations in the instantaneous amplitude of the transmitted signal – as well as sensitivity to frequency offsets caused by local oscillator mismatch. Another important aspect is its spectral properties, especially the out of band radiation which is considered moderate in case of OFDM, but in this respect FBMC has a much better performance. The transmission data rate will be higher due the fact that FBMC does not apply a CP. On the other hand, the channel equalization is more complex compared to OFDM due to the ISI caused by the multipath channel. In this paper, we focus on channel equalization for FBMC and OFDM systems. The basic problem of equalization for FBMC systems is presented in [2] and [3].

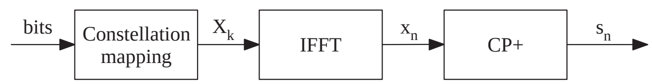

Figure 1. Block diagram of the OFDM transmitter

The paper is structured as follows. In Section II the OFDM and FBMC modulation schemes are described. In Section III we present the baseband signal model. In Section IV we introduce the channel equalization schemes that we intend to analyze in four subsections: first the basic per subchannel Zero Forcing (ZF) and Minimum Mean Square Error (MMSE), then the modified MMSE which minimizes the ISI and the averaged MMSE technique. In the last subsection, we present a new decision feedback equalization technique. In Section V we verify the channel equalization techniques for FBMC systems via bit error rate simulations and we assess their performance by comparing them to OFDM employing MMSE equalization. Finally, the conclusion is drawn.

II. OFDM and FBMC modulation scheme

A. OFDM

In this section, we give only a short description of the OFDM modulation scheme. A general block diagram of an OFDM system can be seen in Fig. 1. First the bits are mapped to constellation symbols X. The time domain samples of an OFDM symbol are generated using IFFT as

$$x_n = \sum_{k=0}^{N-1} X_k e^{\frac{2\pi}{N}nk}, \quad n = 0 \ldots N-1, \tag{1}$$

where X_k is the complex modulation value for the k^{th} subcarrier. The CP is added to the OFDM symbol to form the transmitted signal s_n.

B. FBMC

FBMC systems are derived from the orthogonal lapped transforms [4] and filter bank theory [5]. The block diagram of one possible implementation of an FBMC transmitter can be seen in Fig. 2. Similar to OFDM the bits are first mapped to symbols X drawn from a complex constellation. Then the

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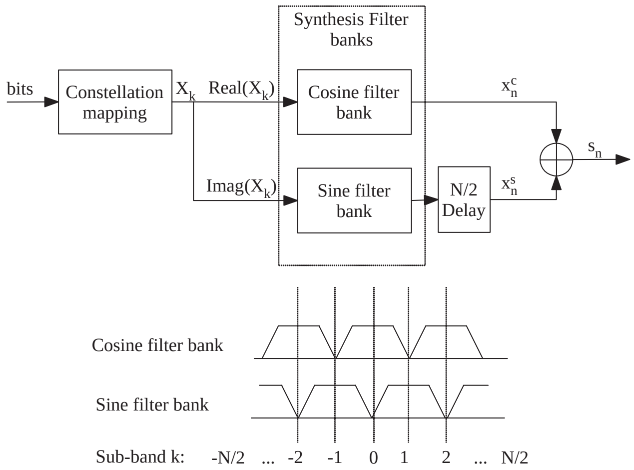

Figure 2. Block diagram of the FBMC transmitter, with the spectral structure of the cosine and sine filterbanks

real parts are modulated by a cosine filter bank where only the even-index subbands are used and the imaginary parts are modulated by a sine filter bank where only the odd-index subbands are modulated. An offset of half of the symbol overlapping $N/2$ duration is applied to the output of the sine filter bank – similarly to the offset quadrature amplitude modulation technique. The basic structure of the filter banks can be also seen in Fig. 3. First the frequency domain data is spread over M subcarriers forming a subband, then it is filtered by a prototype filter of the k^{th} subband $F_k(z)$ which is designed so that it fulfils the Nyquist criterion. In FBMC applications these filter bank structures are implemented in a computationally efficient manner using an N-IDFT and a polyphase network [5]. The filter bank yields symbols that span $N * M$ samples each. In order not to lose data rate they will overlap bya factor M – due to the Nyquist criteria, the symbols can be separated in the receiver and a perfect reconstruction is possible –. For example if $M = 4$ then 4 FBMC symbols overlap. This can be seen in Fig. 4 where the signal structure of FBMC is compared to the signal structure of the OFDM signal. The FBMC signal is given for an overlapping/oversampling factor of $M = 4$. The resulting transmitted signal is the sum of overlapping FBMC symbols generated by the filter banks.

III. Baseband transceiver chain

The applied baseband model for the transceiver chain can be seen in Fig. 5. The discrete received signal r_n can be expressed as

$$r_n = x_n * h_n + w_n, \tag{2}$$

where x_n, h_n and w_n are the samples of the transmitted signal, channel impulse response and AWGN noise respectively. We will use this model when dealing with the equalization algorithms, where the samples of the channel

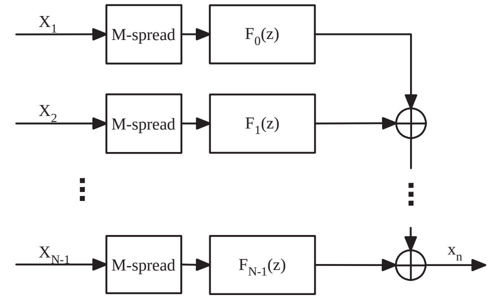

Figure 3. Basic structure of the filter bank with an oversampling ratio of M.

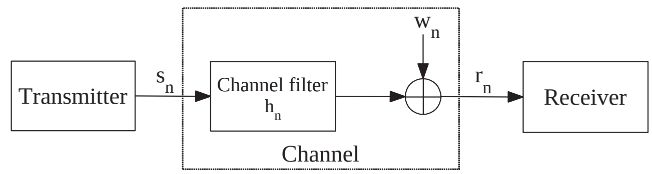

Figure 5. Model of the baseband transceiver chain.

impulse response are Rayleigh distributed, and the following expression is valid

$$\sum_{n=0}^{L-1} |h_n|^2 = 1. \tag{3}$$

where L is the length of the channel impulse response. In case of OFDM systems, if the CP is longer than the channel impulse response (2), after removing the cyclic prefix we can write for an OFDM symbol

$$Y_k = X_k H_k + W_k, \qquad k = 0 \ldots N-1, \tag{4}$$

where Y_k is the N-FFT of r_n, belonging to one OFDM symbol. X_k, H_k and W_k are also an N-FFT of the signal x_n, h_n and w_n respectively. For FBMC systems the frequency domain description is more complicated due to ISI from the neighboring symbols. One of the implications of this ISI is that FBMC systems will require different equalization strategies.

IV. Channel equalization

A. ZF and MMSE

Zero forcing is known to be the simplest method for channel equalization in the frequency domain. We simply assume that the received noise is zero in equation (4), so the transmitted complex constellation value on the k^{th} subcarrier can be simply calculated as

$$\hat{X}_k^{\text{ZF}} = \frac{Y_k}{H_k}. \tag{5}$$

The MMSE technique gives a better result if we also take the information about the AWNG noise also into account. The problem of ZF occurs if H_k is small, the noise values will be

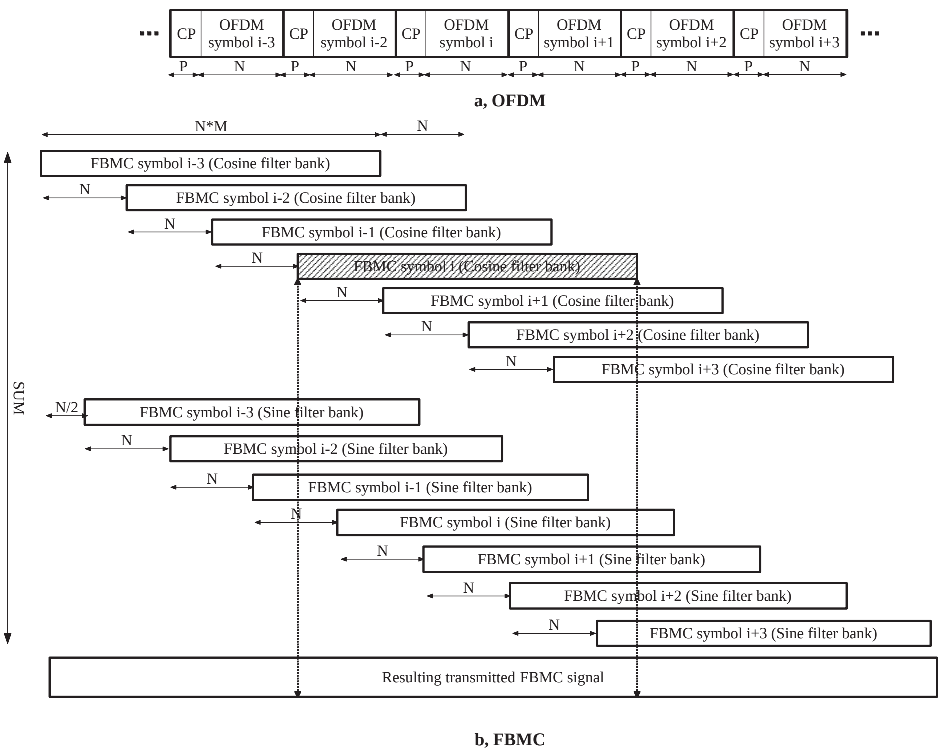

Figure 4. The structure of the transmitted (a) OFDM signal (with a symbol length N and a CP with a length P samples) and the (b) FBMC signal with an overlapping radio of $M = 4$.

also amplified. The equalization coefficient $H_k^{\mathrm{MMSE}} = \frac{1}{C_k}$ for the k^{th} subcarrier is calculated through the minimization the following expression:

$$\min_{C_k} E\left\{\sum_{k=0}^{N-1} |X_k H_k - C_k Y_k|^2\right\}, \tag{6}$$

where $E\{.\}$ donates the expected value of the argument. Using equation (4) the resulting channel compensation value for the k^{th} subcarrier is calculated according to [6] as

$$C_k = \frac{1}{H_k^{\mathrm{MMSE}}} = \frac{H_k^*}{|H_k|^2 + \frac{N_0}{E_s}}, \tag{7}$$

where N_0 is the noise power and E_s is the signal power. It can be seen that with small $\frac{N_0}{E_s}$ values the MMSE solution is equal to the ZF. In case of FMBC and per-subcarrier equalization, (7) has to be modified in order to consider the ISI stemming for adjacent symbols similar to [7][8][9] as

$$\frac{1}{\hat{H}_k^{\mathrm{MMSE}}} = \frac{H_k^*}{|H_k|^2 + \frac{N_0 + I}{E_s - I}} \tag{8}$$

where I is the power of the ISI, for which we present the following equation

$$I = E_s \sum_{n=0}^{L-1} \frac{n}{N} |h_n|^2. \tag{9}$$

Finally, the MMSE estimate results in

$$\hat{X}_k^{\mathrm{MMSE}} = \frac{Y_k}{\hat{H}_k^{\mathrm{MMSE}}}. \tag{10}$$

B. Modified MMSE I.

Observing (9) more closely, we have also concluded that the ISI can be minimized by moving the observation window along all possible positions of the channel impulse response to minimize the following equation

$$\min_{\Delta n}\{I(\Delta n)\} = \min_{\Delta n}\left\{E_s \sum_{n=0}^{L-1} \frac{|n-\Delta n|}{N} |h_{|n-\Delta n|}|^2\right\}, \tag{11}$$
$$\Delta n = 0 \ldots L-1,$$

After finding the sample value Δn which minimizes equation (11), the observation window – where we perform the channel equalization – has to be moved by Δn samples and also the channel impulse response has to be circularly shifted, respectively as

$$\hat{X}_k^{\mathrm{MIN}} = \frac{Y_k(\Delta n_{\min})}{\hat{H}_k^{\mathrm{MMSE}}(\Delta n_{\min})}. \tag{12}$$

C. Modified MMSE II.

To further minimize the ISI we introduce the idea of the Averaged MMSE equalizer. The averaged MMSE is driven by the idea that the ISI can be also considered as a noise, which can be eliminated by averaging. So based on the idea of moving the observation window, we perform the demodulation and MMSE equalization for each Δn positions of the possible L observation windows and then we average all complex modulation values belonging to the same subband

$$\hat{X}_k^{\mathrm{AVG}} = \frac{1}{L}\sum_{\Delta n=0}^{L-1}\hat{X}_k^{\mathrm{MMSE}}(\Delta n) = \frac{1}{L}\sum_{\Delta n=0}^{L-1}\frac{Y_k(\Delta n)}{\hat{H}_k^{\mathrm{MMSE}}(\Delta n)} \tag{13}$$

With this calculation we can minimize the ISI.

D. Iterative decision feedback equalization

In this section we will introduce a novel iterative decision feedback scheme where the most reliable decision values are fed back after the decision to minimize the ISI in the received signal. This decision feedback scheme is shown in Fig. 6. The basic idea is to regenerate the transmitted signal, but only the subbands which are reliable, and filter it with the known channel filter. The idea is visualized in Fig. 4: If we want to make a decision for the shaded i^{th} FBMC symbol of the cosine filter bank, then we reconstruct as much as possible from the surrounding symbols $i-3, i-2 \ldots i+3$ which overlap with it (both sine and cosine) based on the selection criteria. Then, during the decision on the i^{th} FBMC symbol the ISI of the known neighboring symbols can be subtracted, reducing the noise stemming from the ISI, leading to better performance. The selection criteria is defined based on the constellation diagram, we take confidence interval around each constellation point. The complex modulation symbols which fall inside this interval are considered as reliable. During the iteration process the interval can be enlarged as the ISI is minimized.

V. Simulation results

To verify the previously described equalization schemes, simulations were performed. The simulation parameters for both OFDM and FBMC system are summarized in Table I. In order to enable a proper comparison of the two different modulation schemes the SNR is defined as

$$\mathrm{SNR_{dB}} = 10\log_{10}\left(\frac{E_s}{N_0}\right) \tag{14}$$

$$= 10\log_{10}\left(\frac{E_b N_c D}{(N+P)N_0}\right), \tag{15}$$

where E_b is the bit energy, N is the number of the subcarriers/subbands available and N_c is the number of subcarriers/subbands used. P is the length of the CP and D is the number of bits transmitted by one subcarrier/subband. During the simulations we have averaged the results of 10 channel realizations.

The simulated bit error rate (BER) for the proposed 3 MMSE equalization schemes can be seen in Fig. 7. For comparison we have also plotted the results for the AWGN channel without multipath propagation. For low SNR values

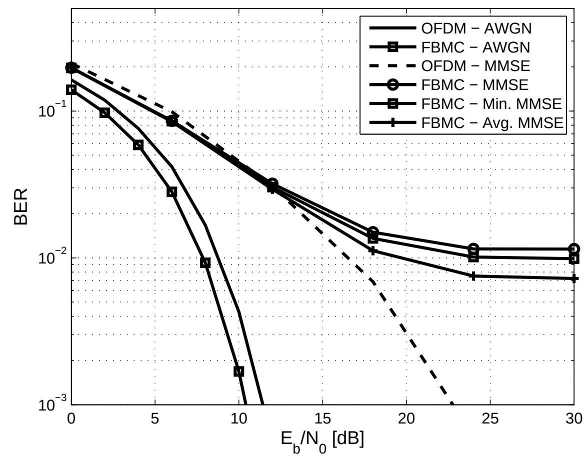

Figure 7. Bit error rate in function of SNR for the three MMSE-FBMC equalization schemes.

Table I
Simulation parameters for OFDM and FMBC system

Parameter	OFDM	FBMC
CP	16	-
N	64	64
M	1	4
Modulation (D)	16-QAM (4)	16-QAM (4)
Modulated subcarrier/subbands	48	48
Channel length L	16	16

the FBMC system has a very small gain over OFDM system in the BER results. It can be observed that OFDM outperforms FBMC at higher SNR value (SNR $>$ 12 dB) the FBMC system when only an MMSE equalizer is applied. When introducing the minimized MMSE and the averaged MMSE, a small performance gain becomes apparent for large SNR values in favour of FBMC. This small difference will be crucial for the iterative decision feedback technique. These bit error rates can be considered as the starting values for the iterative algorithm.

The BER results for the iterative decision feedback technique is depicted in Fig. 8. The BER results for the 5. iteration step is plotted together with the initial starting values for the inital iteration. It can be observed that the averaged MMSE performs the best, the minimized MMSE has a similar result and finally the original MMSE has the worst BER.

VI. Conclusion

In this paper, we have investigated channel equalization schemes for FBMC system which were compared to the result of CP-OFDM system using MMSE equalization. First, modifications of the MMSE equalization technique suited for FBMC distorted by ISI were presented and the results were verified via simulation. We have also presented a decision

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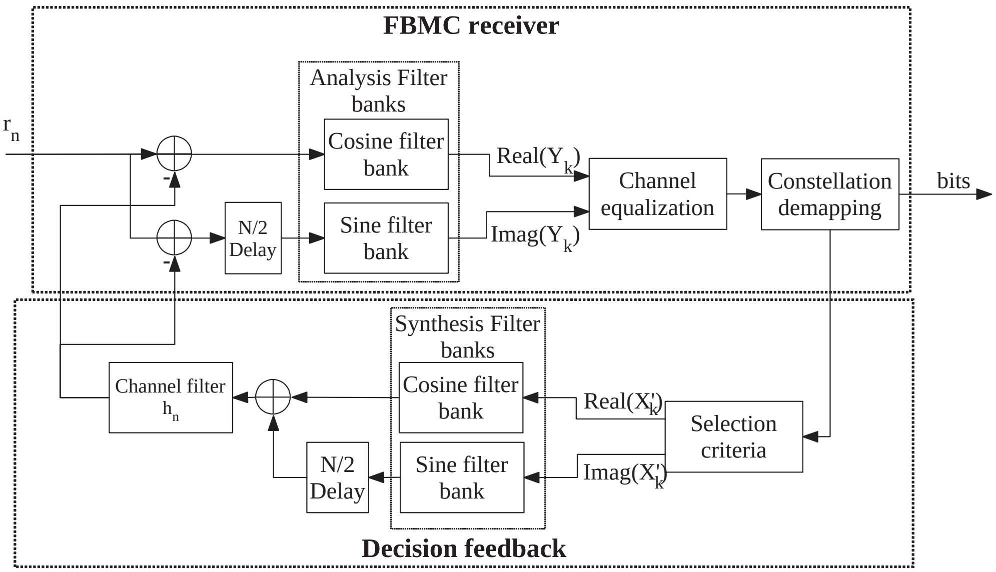

Figure 6. Model of the FBMC receiver with decision feedback loop.

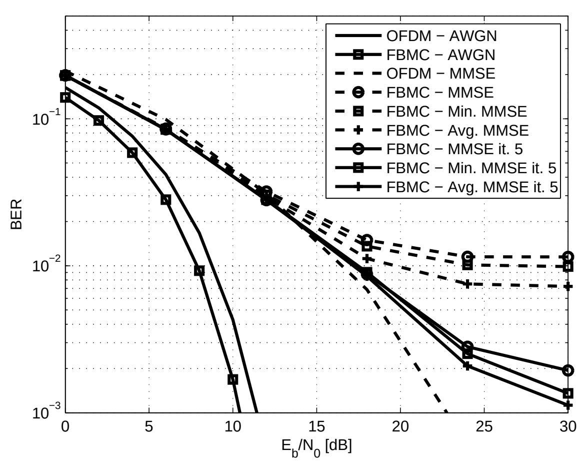

Figure 8. Bit error rate in function of SNR for the three MMSE-FBMC equalization schemes with decision feedback.

feedback scheme, which has a much better performance compared to the simple MMSE methods. The results show that FBMC is a very good candidate for competing with OFDM systems over multipath channels. The criteria for selecting the confidence interval around the constellation points requires further investigation, but the simulation provide promising results.

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