Dual-Lag Correlation-Based Feature Detection of OFDM Signals with Cyclic Phase Compensation

Vesa Turunen, Marko Kosunen, Visa Koivunen* and Jussi Ryynänen
Aalto University School of Electrical Engineering
Department of Micro- and Nanosciences / *Department of Signal Processing and Acoustics,
SMARAD, P.0 Box 13000, 00076 AALTO, Finland
Email: {Vesa.Turunen, Marko.Kosunen, Visa.Koivunen, Jussi.Ryynanen}@aalto.fi

***Abstract*—This paper presents cyclostationarity-based spectrum sensing algorithm implementation for detection of OFDM signals. The detector utilizes two distinct autocorrelation delays and introduces compensation of the phase difference between the two cyclic autocorrelation functions. This improves detection sensitivity (or alternatively reduces detection time) compared to other similar algorithms while maintaining the constraint on false alarm rate. The phase compensation can be performed without requiring any new information about the signal properties. Furthermore, incorporation of the phase compensation reduces overall computational complexity of the algorithm and therefore leads to simpler implementation that uses fewer logic gates and consumes less power.**

***Keywords*-Autocorrelation, cognitive radio, detection algorithms, OFDM, spectrum sensing.**

I. Introduction

The objective of spectrum sensing is to identify free spectrum or detect the presence of communication signals in certain frequency band quickly and reliably. In general, detection performance is characterized by the probability of signal detection and the probability of false alarm. The first determines the detection sensitivity, i.e. the received signal power level (or SNR) where the signal can still be detected with desired probability in given detection time. The false alarm rate, on the other hand, has to be kept sufficiently low such that the spectrum sensor is able to find the free spectrum.

In most spectrum sensing schemes, increasing the detection time (i.e. the number of received samples) improves the sensitivity. However, short detection time is desirable for many reasons and more powerful algorithms are sought to improve the detection sensitivity without increasing the detection time. Usually, using more complex algorithm leads to increased computational complexity, which translates into higher number of logic gates and increased power consumption in the actual implementation.

Cyclostationarity-based spectrum sensing algorithms (CBSSA) [1] [2] [3] are a strong candidate for future spectrum sensing implementations due to their superior detection sensitivity and inherent ability to distinguish among different type of communication signals. They are especially suitable for detection of orthogonal frequency division multiplex (OFDM) signals that exhibit strong periodic correlation due to insertion of the cyclic prefix (CP) in front of each OFDM symbol (Fig. 1). Well-known tests exist that utilize multiple lags simultaneously to increase the detection sensitivity while keeping the detection time constant [3]. Detector implementations that are based on signal's cyclostationary features have been reported in [4] [5].

This work introduces a new dual-lag CBSSA implementation for detection of OFDM signals that is based on spatial sign cyclic correlation estimator (SSCCE) presented by Lunden et al. in [3]. We show that the test statistics can be written in a simpler form by deducing and compensating the phase difference of the two SSCCE, which are calculated using distinct lag values. The new test statistics achieves better detection sensitivity and also leads to a reduced computational complexity while maintaining the desired false alarm rate.

This paper is organized as follows: Section II is a short review on spectrum sensing algorithms that can be used to detect OFDM signals utilizing the cyclostationary properties. The proposed algorithm is presented in Section III and an example implementation is given. Section IV presents simulation results and a conclusion is given in Section V.

II. Review of Cyclostationarity-Based Spectrum Sensing Algorithms

A. Statistical Test for Presence of Cyclostationarity

The conventional statistical tests for presence of cyclostationarity [1] estimate the (conjugate) cyclic autocorrelation funtion (CAF)

$$\hat{R}_{xx(*)}(\alpha,\tau) = \frac{1}{N}\sum_{n=0}^{N-1} x[n]x^*[n-\tau]e^{-j2\pi\alpha n}, \qquad (1)$$

where $x[n] = x_i[n] + ix_q[n]$ is a complex input signal, α is the cyclic frequency, and τ is the lag parameter in the autocorrelation. N denotes the number of received samples that are used for signal detection and therefore, together with the signal sampling rate, determines the detection time.

In order to test for the presence of cyclostationarity a hypothesis test is formulated as follows:

$$H_0: \quad \hat{\mathbf{r}}_{\mathbf{xx}(*)} = \epsilon_{\mathbf{xx}(*)} \qquad (2)$$

$$H_1: \quad \hat{\mathbf{r}}_{\mathbf{xx}(*)} = \mathbf{r}_{\mathbf{xx}(*)} + \epsilon_{\mathbf{xx}(*)}, \qquad (3)$$

where

$$\hat{\mathbf{r}}_{\mathbf{xx}(*)} = [\Re\{\hat{R}_{xx(*)}(\alpha,\tau_1)\},\ldots,\Re\{\hat{R}_{xx(*)}(\alpha,\tau_K)\}, \Im\{\hat{R}_{xx(*)}(\alpha,\tau_1)\},\ldots,\Im\{\hat{R}_{xx(*)}(\alpha,\tau_K)\}] \qquad (4)$$

 ISBN: 978-1-61208-197-7

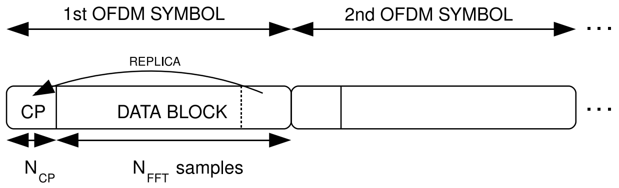

Fig. 1. OFDM symbol consists of a data block and a cyclic prefix that contain N_{FFT} and N_{CP} samples, respectively.

contains the estimates of conjugate cyclic autocorrelation functions for K lags, $\mathbf{r_{xx(*)}}$ is the vector of true nonrandom cyclic autocorrelation functions and $\epsilon_{\mathbf{xx}(*)}$ is the estimation error of $\hat{\mathbf{r}}_{\mathbf{xx}(*)}$. Under the null hypothesis the cyclostationarity does not exist and (4) contains only the estimation error.

Test statistics for the generalized likelihood ratio test is then derived [1] and is given as

$$T = \hat{\mathbf{r}}_{\mathbf{xx}(*)}\hat{\mathbf{\Sigma}}^{-1}\hat{\mathbf{r}}^{\mathbf{T}}_{\mathbf{xx}(*)}, \tag{5}$$

where $\hat{\mathbf{\Sigma}}^{-1}$ is the inverse covariance matrix of $\hat{\mathbf{r}}_{\mathbf{xx}(*)}$ [1].

Under the null hypothesis the test statistics is chi-square distributed with $2K$ degrees of freedom. Consequently, a Neyman-Pearson test can be performed by comparing the test statistics (5) to the threshold that is obtained from the inverse of the chi-square cumulative distribution function (cdf). If the observed test statistics value exceeds the pre-calculated threshold, then it is concluded that signal is present.

To extract the cyclostationary features of that are induced by the basic modulation schemes, such as amplitude modulation, the received signal usually needs to be oversampled with respect to its baseband sample rate. The lag values that are utilized in test are then in the order of the baseband sample period. The algorithm can be used to detect cyclostationary features of the OFDM signal that result from insertion of the cyclic prefix, but then the cyclostationary features occur at the OFDM symbol level. Assuming that the signal is sampled at the baseband sampling rate, the detection can be performed using autocorrelation delay (lag) values $\tau = \pm N_{FFT}$ and cyclic frequencies $\alpha = k/(N_{FFT} + N_{CP}), k = 0, \pm1, \pm2 \ldots$, where N_{FFT} denotes the size of the IFFT that is used to form the data part of the symbol and N_{CP} is the length of the cyclic prefix as presented in Fig. 1. Extension of this algorithm that tests also for multiple cyclic frequencies is presented in [2].

B. Spatial Sign Cyclic Correlation Estimator

Recently, it was shown in [3] that the amplitude of the received signal samples can be normalized while preserving the cyclostationary features. Normalization is performed in order to improve the robustness of the employed detector in the face of impulsive noise and interference. This is achieved at the cost of minimal performance loss in AWGN channel. This also leads to a simpler implementation since the noise statistics are known a priori and therefore do not need to be estimated from the received signal samples. The input sample normalization is denoted in [3] as a spatial sign function

$$S(x[n]) = \begin{cases} \frac{x[n]}{|x[n]|} & \text{if } x[n] \neq 0 \\ 0 & \text{if } x[n] = 0. \end{cases} \tag{6}$$

The spatial sign cyclic correlation estimator (SSCCE) is then defined as [3]

$$\hat{R}_S(\alpha, \tau) = \frac{1}{N} \sum_{n=0}^{N-1} S(x[n]) S(x^*[n-\tau]) e^{-j2\pi\alpha n}. \tag{7}$$

Constant alarm rate test similar to what was described in Sec. II-A is then derived in [3]. The test statistics is

$$T_{S,K} = N||\mathbf{r}_{S,K}||^2, \tag{8}$$

where

$$\mathbf{r}_{S,K} = [\hat{R}_S(\alpha, \tau_1), \hat{R}_S(\alpha, \tau_2), \ldots, \hat{R}_S(\alpha, \tau_K)]. \tag{9}$$

For AWGN, the test statistics $T_{S,K}$ is shown to be gamma distributed with shape factor K and scale factor 1 [3] and, therefore, the threshold for the test is obtained from inverse of gamma cdf. By comparing Eq. (5) and (8) we see that the application of the spatial sign function simplifies the test statistics considerably. Although it does introduce the need for calculating the spatial sign function, the overall complexity cost of that is much less than that of calculating the inverse covariance matrix in (5).

III. Cyclic Phase Compensation

Let us start by rewriting (8) for the dual-lag case ($K = 2$), which is of special interest for detection of the OFDM signals using the cyclic frequency of $1/(N_{FFT} + N_{CP})$. Equations (7)-(9) can be combined to yield

$$T_{S,2} = N \left| \underbrace{\hat{R}_S(\alpha, \tau_1)}_{C_1} \right|^2 + N \left| \underbrace{\hat{R}_S(\alpha, \tau_2)}_{C_2} \right|^2. \tag{10}$$

where τ_1 and τ_2 are set to $+N_{FFT}$ and $-N_{FFT}$, respectively.

The two SSCCE in (10), namely C_1 and C_2, present two complex values that have some magnitude and phase in complex plane. The stronger the correlation, the larger are the magnitudes. The difference in their phase can be written as

$$\phi = arg(C_1) - arg(C_2). \tag{11}$$

Fig. 2 presents the time-domain autocorrelation sequences of C_1 and C_2 of the OFDM signal, showing the periodically alternating correlating and non-correlating subsequences. It follows from the structure of the OFDM symbol stream that when detecting an OFDM signal with using the two lags ($\pm N_{FFT}$), then for sequential OFDM symbols the phase difference ϕ is constant and can be expressed as function of τ and α as

$$\phi = 2\pi\tau_1\alpha. \tag{12}$$

Now if we compensate the phase difference, the two SSCCE can be combined. We define a new test statistics as

$$T^c_{S,2} = 2N \left| \hat{R}_S(\alpha, \tau_1) + \hat{R}^{\phi}_S(\alpha, \tau_2) \right|^2, \tag{13}$$

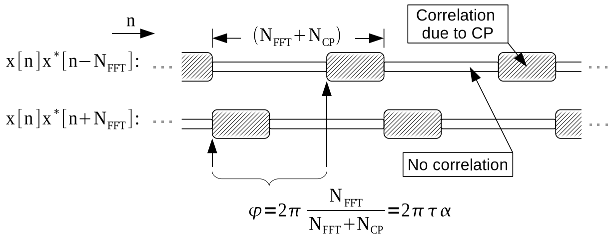

Fig. 2. The time-domain autocorrelation sequences of the OFDM signal for lags $\pm N_{FFT}$. Cyclic phase offset is deducted from the structure of the OFDM symbol stream.

where

$$\hat{R}_S^{\phi}(\alpha,\tau) = \frac{1}{N}\sum_{n=0}^{N-1} S(x[n])S(x^*[n-\tau])e^{-j2\pi\alpha n+\phi} \quad (14)$$

is the SSCCE with cyclic phase compensation.

The difference between (13) and (10) is the constant phase shift in the second exponent term. Computationally (13) is simpler because it does the summation of the two cyclic autocorrelation functions before calculating the absolute square value. This will halve the number of integrators and multipliers that are needed in the hardware implementation of the test statistics calculation. Test statistics in (13) is also gamma-distributed, but with shape factor 1 as opposed to the shape factor 2 in (10). Both detectors calculate the test statistics from a vector of N received samples.

A. Implementation

Next, an implementation for calculating (13) is proposed. Implementation is done mostly in angular domain to avoid complex multiplications [6]. Angular domain representation suits the algorithm well since the signal magnitude is normalized to one for all non-zero samples (amount of zero samples are assumed negligible). Next we denote the phase of each sample as

$$\varphi_x[n] = arg(x[n]) \quad (15)$$

and rewrite (13) as presented in (16). Because additions are difficult to implement in angular domain, the signal is mapped back to Cartesian coordinates after finishing the calculation of the exponents. The calculation of the argument and the mapping back to the Cartesian coordinates can be effectively implemented with the well-known CORDIC algorithm [7].

Fig. 3 presents the proposed implementation for calculating test statistics in (16). First a CORDIC is used to calculate argument of the input samples. A random access memory (RAM) block is used to implement the two delays. A simple integrator is needed to accumulate the $\varphi_\alpha[n]$ term. Five adders are then used to finish the calculation of the two exponent terms, including the cyclic phase offset ϕ. After the exponential terms are resolved, signal is mapped back to Cartesian coordinates using two CORDICs. Finally, the calculation of the test statistics is finished using two integrators, multipliers for calculation of the absolute square and a final division.

IV. Simulations

Performance of the proposed detector is compared to SSCCE presented by Lunden in [3] by conducting a series of Matlab simulations. The simulations utilize OFDM signal ($N_{FFT} = 52$, $N_{CP} = 12$, subcarrier modulation 16-QAM) and $N = 2048$ samples per detection. Three detectors are compared: 1) SSCCE with single lag, 2) SSCCE with two lags and 3) the proposed SSCCE detector with two lags and the cyclic phase compensation. The single lag detector uses delay $\tau_= N_{FFT}$, whereas the dual-lag detectors use $\tau = \pm N_{FFT}$. All detectors make the detection from the single cyclic frequency $\alpha = 1/(N_{FFT} + N_{CP})$ (relative to the sampling rate) and have the probability of false alarm set to 5%.

Fig. 4 presents probability of detection as a function of signal-to-noise ratio (SNR) for the three detectors in an AWGN channel. The simulation shows that the proposed detector has the best detection sensitivity. The improvement over 2-lag SSCCE is less than 1 dB in SNR and approximately 2 dB when compared to the single lag SSCCE. The single lag SSCCE, which is the simplest to implement, would achieve the same performance than the proposed detector by doubling the number of samples N (and thus doubling the detection time).

Fig. 5 presents average test statistics values from the previous simulation. The difference in test statistics of the dual-lag SSCCE and the proposed detector is due to the early combination of the two SSCCE in (13), which leads to a reduction in the degrees of freedom of the distribution of the test statistics under the null hypothesis. Consequently, this reduction in the degrees of freedom enables more efficient test which can be seen as an improvement in the sensitivity of the detector.

Finally, Fig. 6 shows simulated receiver operating characteristics (ROC) curves for the three detectors. In this simulation the SNR is set to -5 dB and the other simulation parameters are identical to the previous simulation. The introduction of the cyclic phase compensation in the proposed detector provides a distinctive improvement over the prior work.

V. Conclusion

This paper has introduced an improved dual-lag test for the spatial sign cyclic correlation estimator that can be used for OFDM signal detection in spectrum sensing applications. The key idea in this work has been to deduce and compensate the phase difference between the two SSCCE that are obtained using two distinct lag values. The proposed detection algorithm has been shown to achieve improved probability of detection compared to the prior work while keeping the false alarm rate constant. Moreover, the algorithm has also been shown to result in reductions in the computational complexity, which makes it more suitable for practical implementations.

Acknowledgment

The authors would like to thank Nokia Research Center and Finnish Funding Agency for Technology and Innovation (TEKES) for the financial support in this research.

$$T_{S,2}^{c} = \frac{1}{2N} \left| \sum_{n=0}^{N-1} (e^{j(\varphi_x[n]-\varphi_x[n-\tau_1]-\varphi_\alpha[n])} + e^{j(\varphi_x[n]-\varphi_x[n-\tau_2]-\varphi_\alpha[n]+\phi)}) \right|^2, \quad (16)$$

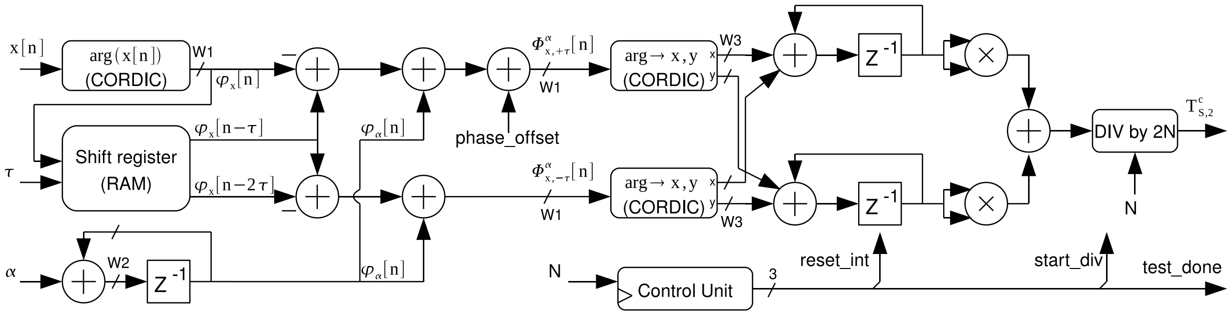

Fig. 3. Implementation of the proposed dual-lag SSCCE algorithm with cyclic phase compensation.

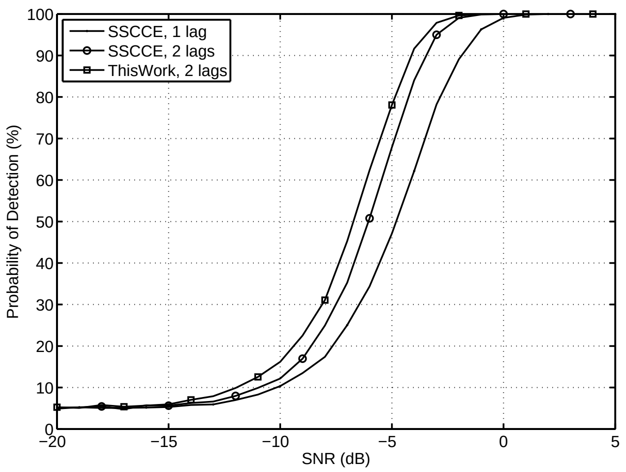

Fig. 4. Probability of detection as a function of SNR. The proposed detector outperforms the prior work in terms of detection sensitivity while maintaining the constant false alarm rate.

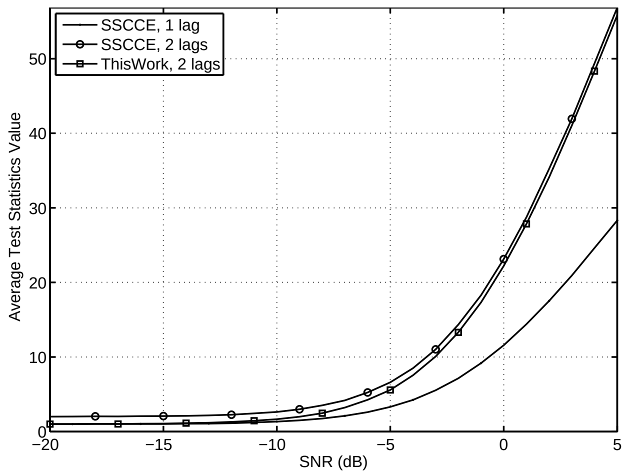

Fig. 5. Average test statistics as a function of SNR. Compared to the 2-lag SSCCE detector, this work achieves reduction in the degrees of freedom in the distribution of the test statistics, which enables the performance improvement.

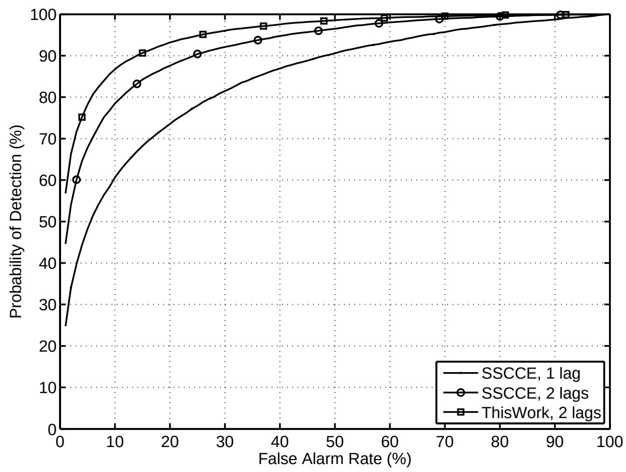

Fig. 6. Receiver operating characteristics (SNR=-5 dB). The introduction of the cyclic phase compensation provides a distinctive improvement over the prior work.

REFERENCES

[1] A. Dandawate and G. Giannakis, “Statistical tests for presence of cyclostationarity,” *IEEE Trans. Signal Processing*, vol. 42, pp. 2355–2369, Sept. 1994.

[2] J. Lunden, V. Koivunen, A. Huttunen, and H. V. Poor, “Collaborative cyclostationary spectrum sensing for cognitive radio systems,” *IEEE Trans. Signal Processing*, vol. 57, pp. 4182–4195, Nov. 2009.

[3] J. Lunden, S. A. Kassam, and V. Koivunen, “Robust nonparametric cyclic correlation-based spectrum sensing for cognitive radio,” *IEEE Trans. Signal Processing*, vol. 58, pp. 38–52, Jan. 2010.

[4] A. Tkachenko, A. Cabric, and R. Brodersen, “Cyclostationary feature detector experiments using reconfigurable BEE2,” in *Proc. IEEE Int. Symp. New Frontiers in Dynamic Spectrum Access Networks*, 2007, pp. 216–219.

[5] V. Turunen, M. Kosunen, A. Huttunen, S. Kallioinen, P. Ikonen, A. Pärssinen, and J. Ryynänen, “Implementation of cyclostationary feature detector for cognitive radios,” in *Proc. Int. Conf. on Cognitive Radio Oriented Wireless Networks and Communications*, 2009, pp. 1–4.

[6] V. Turunen, M. Kosunen, M. Vääräkangas, and J. Ryynänen, “Correlation-based detection of OFDM signals in the angular domain,” *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, 2012.

[7] P. K. Meher, J. Valls, T.-B. Juang, K. Sridharan, and K. Maharatna, “50 years of CORDIC: Algorithms, architectures, and applications,” *IEEE Trans. Circuits Syst. I*, vol. 56, pp. 1893–1907, Sept. 2009.

 ISBN: 978-1-61208-197-7