Efficiency of Opportunistic Spectrum Sensing by Sequential Change Detection in Cognitive Radio

Karsten Husby, Arne Lie, Jan-Erik Håkegård
Communication Systems
SINTEF ICT
Trondheim, Norway
karsten.husby@sintef.no

Abstract—**A critical point in cognitive radio spectrum sensing is the ability to detect presence and absence of primary users as fast as possible at very low SNR. In this paper, sequential power detection by cumulative sum and recursive generalized likelihood ratio test is used to detect free spectral slots of opportunity. The benefits of these change detection algorithms are the adaptive sensing window, the low processing burden and the optimality in sense of maximum likelihood. A spectrum utilization efficiency metric is proposed that put a cost on late detections as well as on false alarms that might give rise to harmful interference into the primary system. The efficiency metric is then simulated versus the size of the free slot of opportunity and for different SNR. The detectors presented are found useful for cognitive radio.**

Keywords-Cognitive radio; CUSUM; GLRT; Hypothesis testing; Maximum likelihood estimation;

I. INTRODUCTION

Algorithms for quick detection under noisy conditions have been in use since the 1920s. The first control chart originated from Bell Labs, where Dr. Shewhart set up control limits based upon the Bell curve to find out if a process was in control or out of control.

Later, an improved method based on an intuitive cumulative sum was proposed [1]. This method is now named CUSUM, and is closely related to the Neyman-Pearson test that tries to distinguish between two hypotheses [2][3][4] when the probability density function (PDF) for both is known in advance.

Another popular, but often regarded as a high complexity test, is the generalized likelihood ratio test (GLRT), where unknown parameters are part of the PDF of the process. Because this test is based on maximum likelihood estimation (MLE), it is asymptotically optimal for a large number of applications. However, for systems requiring a low false alarm probability, a huge amount of data needs to be stored and processed before an optimal decision can be made [8][9][10][11][12][13]. This is particularly troublesome in high bandwidth systems.

For cognitive radio spectrum sensing, high performance and low processing burden detectors are wanted. Recursion is often an efficient processing method, and in this paper, focus is put on low processing burden recursive change detection algorithms that also offer certain optimality in the maximum likelihood (ML) sense. One useful recursive algorithm is the CUSUM detector, which is optimal for a given signal to noise ratio (SNR). Structures of parallel CUSUM detectors that are optimal for several SNRs may be implemented and lead to GLRT type of detectors. One particularly useful detector is the recursive GLRT detector called R-GLRT [5].

In cognitive radio secondary users need to detect when to transmit or not. This involves both power turn-off and power turn-on detection. All knowledge about the primary user transmit timing and the free window of opportunity W is also important. If this window is unknown it becomes important to estimate W or at least transmit packets short enough not to interfere with the primary user at power turn-on. Detectors both for on and off detection are presented but estimation of W is considered outside the scope of the paper.

This paper contributes in the area of spectrum sensing for opportunistic cognitive radio networks using the well-known CUSUM detector and the novel R-GLRT detector at very low SNR where the processing burden of other GLRT type detectors is significant. Both CUSUM and R-GLRT are very efficient algorithms that do not saturate when the amount of collected data grows. Due to their optimality, these detectors give increased spectral utilization. An efficiency metric is presented to quantify the performance offered by these detectors.

This paper is organized as follows. After a system introduction in Section II, the CUSUM detector and R-GLRT detector are presented in Section III. In Section IV both algorithms are simulated, and in Section V their impact when used in a cognitive radio system is found based on the spectrum efficiency metric.

II. MULTIPLE HYPOTHESES SYSTEM MODEL

The detector operates on an increasing number of n input samples $x(i)$, $i = 1:n$ supplied from any sensor circuitry such as a bandpass filter, a FFT processor, a cyclostationary feature processor, or a Kalman filter matched to the parameter whose change is to be detected. The samples are assumed to be independent and identically distributed although such a limitation is not always required according to [16]. Without loss of generality x(i) is assumed normal distributed with mean value μ and standard deviation σ ; i.e.,

$$p\left(x(i);\theta\right) = N(\theta) = N(\mu,\sigma) \tag{1}$$

Immediately after an unknown sample number m the primary user either turns off or on its transmitter. This is defined as the change point

 ISBN: 978-1-61208-267-7

$$m \in [0:n] \tag{2}$$

Before and at the change point m, we have $x(i) \sim N(\theta_0)$. After the change point for $i > m$, we have $x(i) \sim N(\theta_1)$. It is necessary to handle all possible $n+1$ different change points m as separate hypotheses H_m meaning that the m first samples belong to $N(\theta_0)$ and the rest to $N(\theta_1)$. In [5] it is established that CUSUM and R-GLRT are detectors that solves this multiple hypotheses problem.

III. CUSUM AND R-GLRT CHANGE DETECTION

It has been proven [7] that the CUSUM test only needs to investigate the sequential sum of the log likelihood ratio (LLR) between two probabilities to reach asymptotic optimal performance as long as both θ_0 and θ_1 are known in advance. It has also been proven that CUSUM is optimal for all finite thresholds γ [4] . The LLR is

$$LLR(i) = \ln\left(p(x(i);\theta_0)\right) - \ln\left(p(x(i);\theta_1)\right) \tag{3}$$

which in case of the normal distribution is

$$LLR(i) = \ln\left(\frac{\sigma_1}{\sigma_0}\right) - \frac{\left(x(i)-\mu_0\right)^2}{2\sigma_0^2} + \frac{\left(x(i)-\mu_1\right)^2}{2\sigma_1^2} \tag{4}$$

The sum LLR is very efficiently implemented in recursive form. The cumulative sum C is started from an initial value $C(0)$ that affects the transient response of the CUSUM. It is possible to improve the performance if $C(0)$ is increased to balance out the transient. This is called fast initial response (FIR) CUSUM [14][15]. In this paper, $C(0)$ = 0, even though the R-GLRT algorithm also will benefit from FIR. The CUSUM recursive programming step is

$$C(i) = C(i-1) + LLR(i) \tag{5}$$

for $i = 1:n$. For the traditional single sided CUSUM algorithm, only the final value $C(n)$ is stored for later use. In addition, the maximum value of C at the change point is stored as

$$M = \max\left(C(i)\right) \tag{6}$$

The CUSUM stopping time becomes

$$T = \min\left\{n : M - C(n) > \gamma\right\} \tag{7}$$

The average run length ARL_1 is the average stopping time when all samples are $N(\theta_1)$. ARL_0 is the average stopping time when all samples are $N(\theta_0)$ and is equivalent to the mean duration between false alarms.

The R-GLRT detector simultaneously runs parallel CUSUMs

$$C_{u,s} = L_{0,u} + L_{1,s} \tag{8}$$

indexed by u to adapt to different prior statistics $\theta_{0,u}$ for $i = 1:n$

$$L_{0,u}(i) = L_{0,u}(i-1) + \ln\left(p(x(i);\theta_{0,u})\right) \tag{9}$$

and indexed by s to adapt to different $\theta_{1,s}$ statistics after the change point.

$$L_{1,s}(i) = L_{1,s}(i-1) - \ln\left(p\left(x(i);\theta_{1,s}\right)\right) \tag{10}$$

The ML solution, which is already inherently part of the traditional CUSUM [5] is then solved recursively in multiple dimensions that includes the unknown statistics before and after the change point as well as the MLE of the change point itself $\hat{m}$.

$$[\hat{u},\hat{s},\hat{m}] = \arg\max\left(M_{u,s} - L_{1,s}(n)\right) \tag{11}$$

The stopping time under these constraints becomes

$$T = \min\left\{n : M_{\hat{u},\hat{s}} - L_{0,\hat{u}}(n) - L_{1,\hat{s}}(n) > \gamma_{\hat{u},\hat{s}}\right\} \tag{12}$$

IV. CHANGE DETECTION SIMULATIONS

Simulations are done in MATLAB by a flexible R-GLRT detector that handles any number of pre change parameter sets $\theta_{0,u}$ and any number of post change parameter sets $\theta_{1,s}$. If the number of possible pre change parameter sets and post change parameter sets are only one then the R-GLRT algorithm collapse into the traditional CUSUM algorithm.

The simulations show how well these detectors are able to detect a sudden shift in variance. The signal to noise ratio generating the change is defined as

$$SNR = \frac{(\mu_0 - \mu_1)^2 + \left|\sigma_1^2 - \sigma_0^2\right|}{\min\left(\sigma_0^2, \sigma_1^2\right)} \tag{13}$$

For detection of a change in variance, the mean values are disregarded by setting $\mu_0 = \mu_1 = 0$. To detect primary user power turn-on, $\sigma_0^2 = 1$ and $\sigma_1^2 = \sigma_0^2 + \sigma_p^2$, where the primary user power is represented by σ_p^2. To detect primary user turn-off, $\sigma_1^2 = 1$ and $\sigma_0^2 = \sigma_1^2 + \sigma_p^2$. In both cases the primary user power is swept.

Results from a simulation of ARL for detection of sudden power turn-on and turn-off are given in Fig. 1. ARL_0 should be as high as possible and ARL_1 should be as low as possible. Note that the power turn-on detection is slightly better than the power turn-off detection both with regard to detection speed and false alarm at the optimum SNR which in this example is -3dB. It is important to note that the detection of primary user power turn-on or the detection of primary user power turn-off leads to different ARL although the algorithm is almost the same.

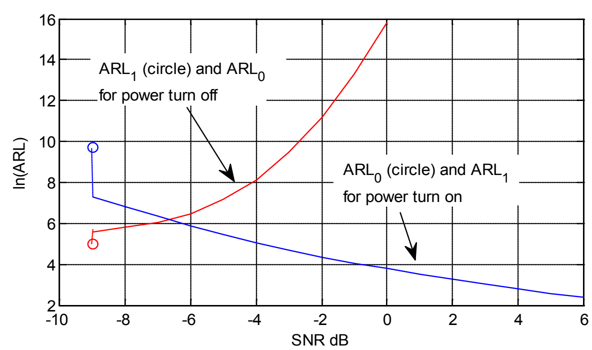

Figure 1. ARL for unknown power turn-on and off of two CUSUM detectors with γ=6 running equal but swapped sets of parameters θ.

One obvious difference for CUSUM is that detecting an unknown power turn-on gives improved detection speed if the power (SNR) is increased. Opposite, while detecting an unknown power turn-off the detection speed is almost constant independent of the apriori SNR.

A. *The SNR wall problem*

The CUSUM detector needs knowledge about θ_0 and θ_1 for optimal performance. The R-GLRT detector is able to pick the most likely θ_0 and θ_1 when they are not known in advance. Therefore unknown power levels both before and after the change point can be controlled. However, if the system noise floor is not properly calibrated it is impossible to distinguish between the power from an actual primary user or the unknown power from the detector itself. This eventually leads to the so-called SNR wall problem [17]. It is therefore obvious that some kind of automatic noise floor calibration is needed to prepare the detectors for accurate operation. In such a setting the R-GLRT detector is not only capable of giving stopping information but also valuable tracking information to refresh any estimate of the detector set noise σ_0. While important, this topic is anyway outside the scope of this paper.

B. *Primary user turn-on detection*

The *ARL* for an unknown power turn-on is presented in Fig. 2. As illustrated, the two CUSUMs are quite robust to SNR offsets from their most optimal design points. Therefore the discretization of the continuous parameter space can be done by a relatively small number of parallel CUSUMs when setting up the R-GLRT algorithm. The simulation shows that this R-GLRT algorithm reaches almost identical stopping times compared to each of the single CUSUMs at their best SNR.

C. *Primary user turn-off detection*

Fast and reliable turn-off detection is considered very important for cognitive radio applications. One complicating factor is the presence of multiple primary users at different power level turning off their transmitters at unknown change points.

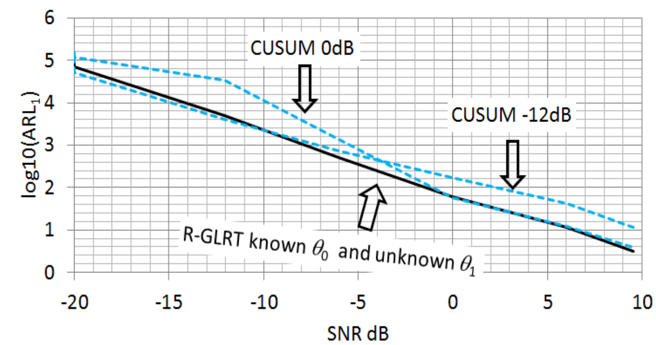

Figure 2. ARL_1 for unknown power turn-on of two CUSUM detectors optimal at -12dB and 0 dB compared with ARL_1 from R-GLRT containing three CUSUM detectors each optimal at -12, -6 and 0dB. ARL_0 for no shift equals 132000 for all.

It turns out that it is more complicated to detect the last (and sometimes the weakest) primary user power turn-off than the first primary user power turn-on. For example, assume 2 primary users, one strong and the other one at weak power. They transmit both for a long time. During this period the R-GLRT integrate prior statistics for all different $\theta_{0,u}$. The algorithm is however not capable of collecting any useful information about the weaker transmitter. If the stronger primary user turns off its carrier, the assumption of a constant (but unknown) θ_0 no longer holds, and the R-GLRT will have to trust the stopping condition from the CUSUM running on the weak power setting independent of the potential higher likelihood and faster stopping times from other CUSUMs.The R-GLRT detector is therefore not very well adapted to the task of detecting all primary users power turn off.

The CUSUM detector is on the other hand well adapted for such a task. In this case the mean detection speed versus different SNR becomes constant while the false alarm probability is reduced when the power is increased as illustrated in Fig. 1. This kind of detector behavior is conservative and not far from the wanted behavior. While it is possible to achieve faster detection speeds for higher SNR using the R-GLRT detector, the detection of power off has to be given with a certain confidence for the power being below a certain level. One single CUSUM is in fact capable of doing this task in ML optimal manner for all γ. Therefore we entirely focus on the CUSUM detector for power off detection.

The CUSUM power off detector is simulated for three different SNR as depicted in Fig. 3, Fig. 4 and Fig. 5. In these Figs interpolated *ARL* values are fitted based on

$$\begin{aligned} ARL_0 &\approx k_0 e^{\gamma} \\ ARL_1 &\approx k_1 \gamma \end{aligned} \tag{14}$$

Closed form expressions of *ARL* are difficult to derive. It is however obvious that the mean likelihood growth is linear for constant θ. Therefore ARL_1 becomes asymptotically linear to γ. Our simulations also strongly indicate that ARL_0 is asymptotically linear to e^{γ} .

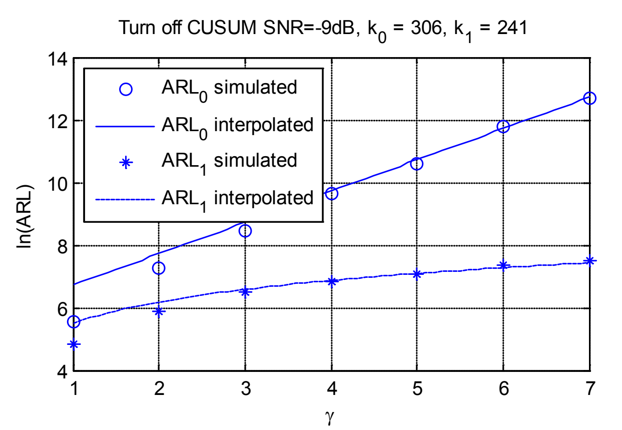

Figure 3. *ARL* for power turn off versus γ.

 ISBN: 978-1-61208-267-7

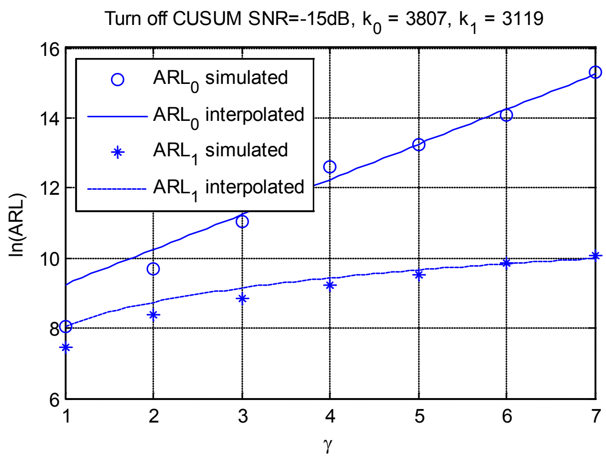

Figure 4. *ARL* for power turn off.

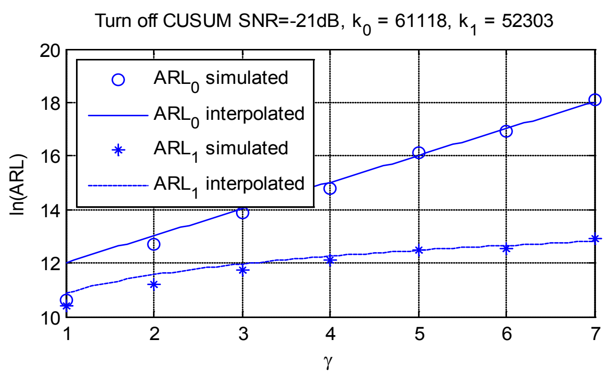

Figure 5. *ARL* for power turn off.

The empirical asymptotic solutions (14) are therefore well suited to find *ARL* at larger thresholds where simulations are cumbersome

V. COGNITIVE RADIO SENSING EFFICIENCY

In this chapter the sequential change detection algorithms are linked to cognitive radio opportunistic spectrum utilization. A spectral efficiency metric is defined that express how well the available spectrum is utilized. There are two terms that contribute to this metric. They are the correct transmissions factor η_c and the faulty transmissions penalty η_f. η_f is also to be understood as the maximum interference ratio into the primary system if a primary user is doing a continuous transmit operation. The total spectrum efficiency metric is defined by subtracting these two terms after weighting them equally

$$\eta = \eta_c - \eta_f \tag{15}$$

Correct transmissions are defined as transmissions done when the channel is idle during a free window of *W* consecutive samples. The utilization of this window is

$$\eta_c = \frac{W - ARL_1}{W} \tag{16}$$

where the length of the secondary user transmission is $W\text{-}ARL_1$. If ARL_1 approaches zero, the utilization approaches unity which is the highest possible. If $ARL_1 \geq W$, no mean successful utilization is possible. The second term of the spectral efficiency is the penalty when interfering into the primary user spectrum

$$\eta_f = \frac{W - ARL_1}{W - ARL_1 + ARL_0} \tag{17}$$

where $W\text{-}ARL_1\text{+}ARL_0$ is the average length between each faulty transmission. The efficiency metric η approaches its optimal value unity if $ARL_1 << W << ARL_0$. Note that the idle window size *W* is assumed either known or estimated by the secondary users. Once the primary user turn-off change point $\hat{m}$ is detected at the stopping time instant *T*, the secondary user is assumed to be capable of transmitting a packet with duration $W+\hat{m}-T$ before the free slot of opportunity closes after *W* samples. If in doubt about the size of *W* and the accuracy of $\hat{m}$, the secondary user may decide to turn off early to allow itself to search for the primary user turn-on to update estimates of *W*. The cognitive radio spectral efficiency in case of correct knowledge of *W* and negligible variance of $\hat{m}$ becomes

$$\eta \approx \frac{W - k_1\gamma}{W} - \frac{W - k_1\gamma}{W - k_1\gamma + k_0 e^{\gamma}} \tag{18}$$

From this equation it is possible to find the maximum efficiency for a given SNR and *W* by varying γ. The result is given in Fig. 6. It is thinkable to utilize short windows of opportunity if the SNR from all primary users is known to be high. However, at an SNR of -21 dB, at least 12,000,000 free samples are needed to give an utilization higher than 95%. All different CUSUMs, independent of SNR optimality set point, surprisingly need the same threshold γ for a given efficiency. For example at 95% efficiency a threshold of 10.8 is needed. At 97.5% efficiency the threshold has to be increased to 12.4 regardless of CUSUM SNR setting.

It is also important to check out η_f versus η. This is given in Fig. 7. With a total efficiency of 95% the maximum interference ratio into the primary system becomes 0.43%.

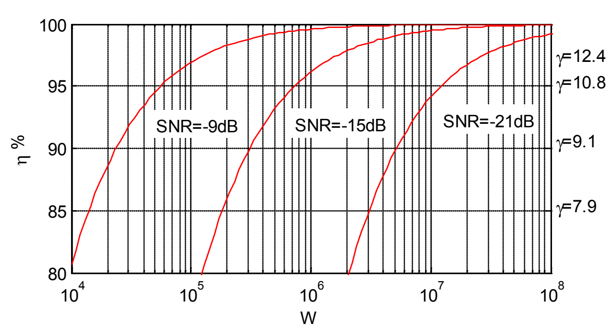

Figure 6. Maximum spectrum efficiency utilization η versus *W* for three different CUSUMs and their individual SNR at optimal γ.

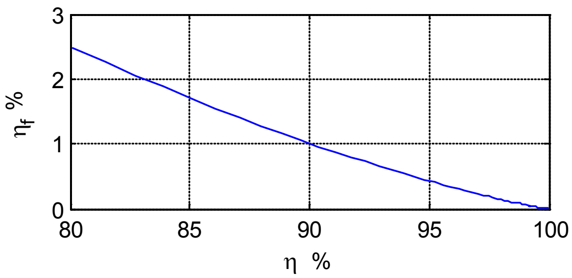

Figure 7. Primary system interference η_f versus total spectral efficiency η in percent at optimal γ.

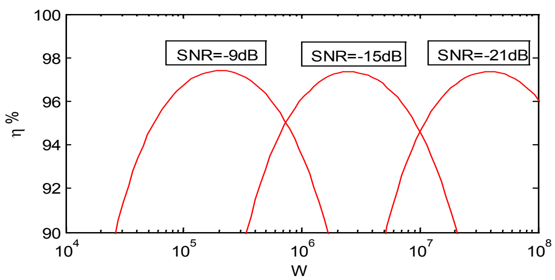

Figure 8. Spectrum efficiency utilization η versus W and SNR for three different CUSUM having equal thresholds γ=10.8.

If on the other hand the thresholds of all CUSUMs are frozen to for example 10.8, then the spectrum efficiency versus W becomes modified as shown in Fig. 8. Because the threshold now is constant, the efficiency will begin to drop when W increase above a certain maximum length. In this situation the penalty η_f due to longer faulty transmissions starts to increase. To avoid this burden for the primary user system, an adaptive threshold setting versus W is necessary.

The spectral efficiency found here is close to the results given in [6] using a predefined fixed window length. The main difference and the benefit of recursive sequential detection compared to block based sensing is that the sensing window length is not chosen in advance but rather is decided by the algorithm itself as soon as the predefined likelihood is reached. As a consequence, sequential detection is more robust to variable conditions. Another complicating factor of block based sensing is the need for synchronization or sliding window techniques to maintain an optimum start with respect to the unknown change point.

VI. CONCLUSION

Two algorithms for ML optimal change detection have been presented and compared for cognitive radio application. The R-GLRT achieves as expected better performance under unknown SNR conditions particularly for power on detection. Power off detection is very well handled by the traditional CUSUM detector. For 95% spectrum efficiency utilization a free window of 50,000 samples is needed at SNR=-9dB, 700,000 free samples is needed for SNR=-15dB and 12,000,000 free samples is needed for SNR=-21dB. It is shown that the interference level against the primary users in this case is 0.43% independent of SNR.

The processing burden of the R-GLRT as simulated in this paper, is about 20 additions per input sample and unlike traditional GLRT completely independent of observation window size and threshold setting. Therefore both the traditional CUSUM and the R-GLRT algorithm are particularly useful in cognitive radio when doing lengthy estimations at low SNR and low probability of false alarm.

REFERENCES

[1] E.S. Page, "Continuous inspection schemes," Biometcrica, 1954, vol. 41, pp.100-115.

[2] S. M. Kay, Fundamentals of statistical signal processing Volume II Detection theory, Prentice hall, 2009.

[3] M. Basseville, I. V. Nikiforov, Detection of Abrupt Changes: Theory and Application, Previously published by Prentice-Hall Inc.

[4] H. Vincent Poor and O. Hadjiliadis, Quickest Detection, Cambridge University Press, 2009.

[5] K. Husby, A. Lie, and J. E. Håkegård, "Quickest recursive GLRT Maximum Likelihood change detection," 2012, unpublished.

[6] A. M. Wylinski, M. Nekovee, and Y. T. Hou, Cognitive radio communications and networks, Principles and Practice, Academic Press, 2010.

[7] G. Lorden, "Procedures for reacting to a change in distribution," The Annals of Mathematical Statistics 1971, vol. 42, No. 6, 1897-1908.

[8] G. J. Ross, D. K. Tasoulis, and N. M. Adams, "Sequential monitoring of a Bernoulli sequence when the pre-change parameter is unknown," Comput Stat, doi: 10.1007/s00180-012-0311-7, Springer-Verlag, 2012.

[9] H. Li, C. Li, and H. Dai, "Quickest Spectrum Sensing in Cognitive Radio," 42nd Annual Conference on Information Sciences and Systems, doi: 10.1109/CISS.2008.4558521, pp. 203-208, 2008.

[10] L. Lai, Y. Fan, and H. Vincent Poor, "Quickest Detection in Cognitive Radio: A Sequential Change Detection Framework," doi: 10.1109/GLOCOM.2008.ECP.567, pp. 1-5, IEEE GLOBECOM, 2008.

[11] G. Verdier, N. Hilgert, and J. P. Vila, "Optimality of CUSUM Rule Approximations in Change-Point Detection Problems: Application to Nonlinear State–Space Systems," IEEE Transactions on information theory, vol. 54, No. 11, pp. 5102-5112, November 2008.

[12] F. Gustafsson, "The marginalized likelihood ratio test for detecting abrupt changes," Automatic Control, IEEE Transactions on 41(1): pp. 66-78, 1996.

[13] T. Kerr, "Decentralized Filtering and Redundancy Management for Multisensor Navigation," Aerospace and Electronic Systems, IEEE Transactions on AES-23(1): pp. 83-119, 1987.

[14] J. M. Lucas and R. B. Crosier, "Fast Initial Response for CUSUM Quality-Control schemes: Give your CUSUM a head start," Technometrics, vol. 24, NO. 3, August 1982

[15] J. Lee, S. Pullen, and P. Enge, "Sigma-mean monitoring for the local area augmentation of GPS," Aerospace and Electronic Systems, IEEE Transactions on 42(2): pp. 625-635, 2006.

[16] G. V. Moustakides, "Quickest detection of abrupt changes for a class of random processes," Information Theory, IEEE Transactions on 44(5): pp. 1965-1968, 1998.

[17] R. Tandra and A. Sahai, "SNR Walls for Signal Detection," IEEE Journal of selected topics in signal processing, vol. 2, NO. 1, pp. 4-17, doi: 10.1109/JSTSP.2007.914879, Feb 2008.