A Quantization Scheme based on Kullback-Leibler Divergence for Cooperative Spectrum Sensing in Cognitive Radio

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***Abstract*—Cognitive radio (CR) is one of the most promising next-generation communication systems due to its ability to improve spectrum utilization by the detection and the use of vacant channels of licensed users. Reliable detection of the licensed user signal is a pre-requirement for avoiding interference to the licensed user in a CR network. Cooperative spectrum sensing (CSS) is able to offer AN improved sensing performance compared to individual sensing. However, in CSS transmission of raw data from cognitive user (CU) to the fusion center (FC) requires large bandwidth of control channel, long latency and high consumption power. A quantization scheme, in which the raw data can be quantized with few bits at the CU and then be reported to the FC, can solve these problems of raw data transmission. In this paper, we proposed a quantization scheme based on Kullback-Leibler (KL) Divergence for reducing the number of quantization bits while keeping the similar sensing performance to the case that raw sensing data are used. Through simulation, it shown that the proposed scheme can achieve better sensing performance than that of uniform quantization scheme.**

***Keywords*-cognitive radio; quantization; Kullback-Leibler Divergence; cooperative spectrum sensing.**

I. Introduction

Recently, additional bandwidth and higher bitrates have been required in order to meet the users demands in wireless communication systems. As a result, available frequency bands have become a scarce resource. However, according to the Federal Communications Commissions spectrum policy task force report [1], actual utilization of the licensed spectrum varies from 15% to 80%. Cognitive radio technology [2] has been proposed to solve the problem of ineffective utilization of spectrum bands. The scarcity of spectrum bands can be relieved by allowing some CUs to opportunistically access the spectrum assigned to the Primary user (PU) whenever the channel is free. However, CUs must vacate their frequency when the presence of a PU is detected. Therefore, reliable detection of the PU signal is an essential requirement of CR networks.

To ascertain the presence of a PU, CUs can use one of several common detection methods, such as matched filter, feature, and energy detection methods [2], [3].Energy detection is the optimal method if the CR user has limited information about a PU signal (e.g., only the local noise power is known) [3]. With energy detection, the frequency energy in the sensing channel is received in a fixed bandwidth, *W*, over an observation time window, *T*, in order to compare with the energy threshold and determine whether or not the channel is being utilized. However, the received signal energy may severely fluctuate due to multipath fading and shadowing effects; therefore, it is difficult to obtain reliable detection with only one CR user. Fortunately, improved usage detection can be obtained by allowing some CR users to perform Cooperative Spectrum Sensing (CSS) [4].

In CSS, a quantization scheme can help to reduce the overhead in the control channels which is mainly due to transmission of raw data from CUs and the FC. KL-divergence theory often uses as a measure of the (dis)similarity between two distributions [5]. Subsequently, KL-divergence can be used to measure reliability of sensing information in CSS. In this paper, therefore we proposed a quantization scheme based on KL-divergence theory for CSS in CR network, in which KL-divergence is used as a reliable level of sensing data to decide boundary points for quantization scheme.

In particular, in Section II, we describe system model of the proposed scheme. In Section III, we present detail of the proposed quantization scheme based on KL-divergence. In Section IV, simulation results are shown for evaluating performance of the proposed scheme. Finally, we conclude the paper in Section V.

II. System Model

We consider a CR network composed of N CR users and *one* PU. All of the CR users use energy detectors to detect the presence of the PU signal. In order to perform CSS, sensing information will be quantized at CUs and after that they will be reported to the Fusion Center (FC) through a control channel. At the FC, all quantization levels obtained from the CUs will be combined to make a global decision concerning about the presence or absence of the PU signal by using equal gain combination (EGC) rule.

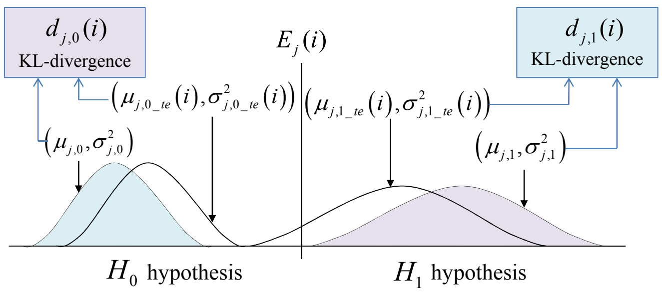

Figure 1. KL-divergences according to the presence or absence hypothesis of PU signal.

The received signal of a CU under absence and presence hypothesis of PU signal can be formulated as follows:

$$\begin{cases} H_0 : x_j(k) = n_j(k) \\ H_1 : x_j(k) = h_j s(k) + n_j(k) \end{cases} \quad (1)$$

where H_0 and H_1 correspond to the hypothesis of the absence and presence, respectively, of the PU signal, $x_j(k)$ represents the signal received by the j^{th} CU, h_j denotes the amplitude gain of the channel, $s(k)$ is the signal transmitted from the PU, and $n_j(k)$ is the additive white Gaussian noise.

At the i^{th} sensing interval for the j^{th} CU, the received signal energy, $E_j(i)$, is given as

$$E_j(i) = \begin{cases} \sum_{k=k_i}^{k_i+M-1} |n_j(k)|^2, & H_0 \\ \sum_{k=k_i}^{k_i+M-1} |h_j x(k) + n_j(k)|^2, & H_1 \end{cases} \quad (2)$$

where M is the number of samples over one sensing interval, and k_i is the time slot at which the i^{th} sensing interval begins. When M is relatively large (e.g. $M > 200$), E_j can be closely approximated as a Gaussian random variable under both hypotheses such that

$$E_j \sim \begin{cases} \mathrm{N}(M, 2M), & H_0 \\ \mathrm{N}(M(\gamma_j+1), 2M(2\gamma_j+1)), & H_1 \end{cases} \quad (3)$$

where γ_j is the SNR of the channel between the PU and the j^{th} CU.

III. The Proposed Quantization Scheme based on Kullback-Leibler divergence for Cooperative Spectrum Sensing

A. Kullback-Leibler Divergence

The KL-divergence [5] is also known as the relative entropy between two probability density functions, $f(x)$ and $g(x)$, such that

$$D(f\|g) = \int f(x) \log\left[\frac{f(x)}{g(x)}\right] dx \quad (4)$$

It is obvious that the KL-divergence is always non-negative. Also, it is zero if and only if the two distributions coincide. KL-divergence is often used as a measure of the (dis)similarity between two distributions. The KL-divergence between two normal distributions with means and variance as $f \sim \left(\mu_f, \sigma_f^2\right)$ and $g \sim \left(\mu_g, \sigma_g^2\right)$ respectively, can be obtained such that [6]

$$\begin{aligned} D(f\|g) &= D\left(\mu_f, \mu_g, \sigma_f^2, \sigma_g^2\right) \\ &= \frac{1}{2}\left[\log\left(\frac{\sigma_g^2}{\sigma_f^2}\right) - 1 + \frac{\sigma_f^2}{\sigma_g^2} + \frac{(\mu_f - \mu_g)^2}{\sigma_g^2}\right] \end{aligned} \quad (5)$$

B. The Proposed Quantization Scheme based on Kullback-Leibler Divergence

Since KL-divergence is a useful tool to measure the (dis)similarity between two distributions, it can be a suitable measurement for reliability of sensing information (e.g. received signal energy) in spectrum sensing. Subsequently, KL-divergence will be utilized to decide quantization boundary for sensing information in CU.

Firstly, based on current received signal energy, the "*temporary mean*" and "*temporary variance*" of received signal energy under H_0, $\mu_{j,0\_te}(i)$ and $\sigma^2_{j,0\_te}(i)$, or H_1, $\mu_{j,1\_te}(i)$ and $\sigma^2_{j,1\_te}(i)$, hypothesis of the PU signal will be calculated as follows, respectively:

$$\begin{cases} \mu_{j,0\_te}(i) = \frac{m-1}{m}\mu_{j,0} + \frac{1}{m}E_j(i) \\ \sigma^2_{j,0\_te}(i) = \frac{m-1}{m}\sigma^2_{j,0} \\ \qquad + \frac{m-1}{m^2}\left[E_j(i) - \mu_{j,0}\right]^2, \end{cases} \quad (6)$$

$$\begin{cases} \mu_{j,1\_te}(i) = \frac{m-1}{m}\mu_{j,1} + \frac{1}{m}E_j(i) \\ \sigma^2_{j,1\_te}(i) = \frac{m-1}{m}\sigma^2_{j,1} \\ \qquad + \frac{m-1}{m^2}\left[E_j(i) - \mu_{j,1\_te}\right]^2, \end{cases} \quad (7)$$

where $\mu_{j,0}$, $\sigma^2_{j,0}$ and $\mu_{j,1}$, $\sigma^2_{j,1}$ are the means and variances of the received signal energy of the j^{th} CR user under H_0 and H_1 hypothesis of the PU signal respectively, and m is the pre-defined constant that represents the "*effecting level*" of the current received signal energy to its mean and variance. Based on numerical experiments, it is observed that the value of m should be chosen from 20 to 50 for keeping the accuracy of the sensing performance.

The KL-divergences will be calculated between the PDF and *temporary* PDF (with *temporary* mean and variance) of the received signal energy of each CR user under the presence or absence hypothesis of the PU signal, denoted as $d_{j,1}(i)$ and $d_{j,0}(i)$, respectively. Base on Eqn. (5), we are able to define

$$\begin{aligned} d_{j,0}(i) &= D\left[\mu_{j,0\_te}(i), \mu_{j,0},\ \sigma^2_{j,0\_te}(i), \sigma^2_{j,0}\right] \\ d_{j,1}(i) &= D\left[\mu_{j,1\_te}(i), \mu_{j,1},\ \sigma^2_{j,1\_te}(i), \sigma^2_{j,1}\right] \end{aligned} \quad (8)$$

Fig. 1 shows an example of KL-divergence calculations of a CU.

Here, we define the "*extended KL-divergence*" as follows

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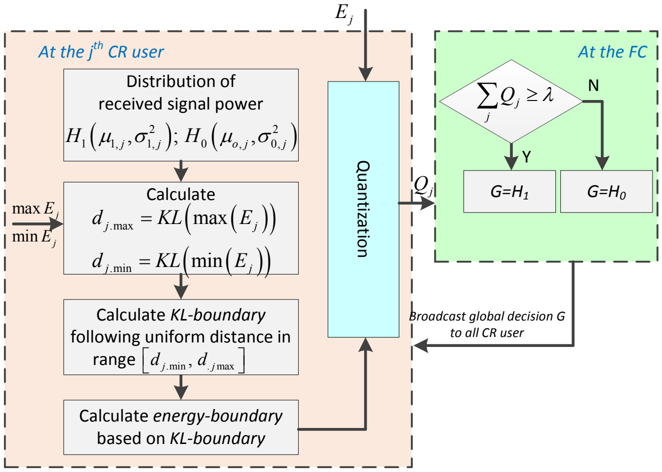

Figure 2. Flow-chart of the proposed scheme.

$$
\begin{aligned}
d_j(i) &= d_{j,0}(i) - d_{j,1}(i) \\
&= KL\left[E_j(i), \mu_{j,0}, \sigma^2_{j,0}, \mu_{j,1}, \sigma^2_{j,1}\right] \qquad (9)
\end{aligned}
$$

It can be seen that $d_j(i)$ can obtain higher value when the value of $d_{j,1}(i)$ is smaller and the value of $d_{j,0}(i)$ is higher. It means that if the value of $d_j(i)$ is higher, the current received signal energy $E_j(i)$ will "near" H_1 distribution than H_0 distribution. In contrary, if the value of $d_j(i)$ is smaller, the current received signal energy $E_j(i)$ will "near" H_0 distribution than H_1 distribution. Subsequently, $d_j(i)$ can represent for reliable level of the sensing information, e.g. with higher absolute value of $d_j(i)$, we have more reliable sensing information.

We propose a quantization scheme based on the "*extended KL-divergence*" of which flow-chart is shown in Fig. 2.

At the first state, the values range of d_j $[d_{j.min}, d_{j.max}]$ will be splited into 2^{Nb} equal parts by $(2^{Nb}+1)$ "*KL-boundary*" poits, Bd_j, as

$$
Bd_j(n) = d_{j.\min} + \frac{n-1}{Nb}\left(d_{j.\max} - d_{j.\min}\right) \qquad (10)
$$

where $n = 1, ..., 2^{Nb}+1$, Nb is number of quantization bits, $d_{j.\max} = KL(\max E_j)$ and $d_{j.\min} = KL(\min E_j)$. At the first sensing interval, the maximum value, $\max E_j$, and minimum value, $\min E_j$, of received signal energy of the j^{th} CU can be set as $(\mu_{j,1}+\sigma_{j,1})$ and $(\mu_{j,0}-\sigma_{j,0})$ respectively. After that, they are updated based on the real received signal energy, that is, if the current received signal energy is higher than $\max E_j$, $\max E_j$ will be set to be equal to the current received signal energy. On the other hand, if the current received signal energy is smaller than $\min E_j$, $\min E_j$ will be set to be equal to the current received signal energy.

For the next state, the "*energy-boundary*" will be calculated according to the values of "*KL-boundary*" as follows:

$$
BE_j(n) = KL^{-1}\left(Bd_j(n)\right) \qquad (11)
$$

Based on "*energy-boundary*", each CU quantizes the received signal energy and reports quantization level to the FC. We define $u_j(i)$ as the quantization decision and $E_j(i)$ as the quantization input. The quantization process $Q(.)$ can be expressed as

$$
\begin{aligned}
u_j(i) &= Q\left(E_j(i)\right) \\
&= m \text{ if } E_j(i) \in \Delta_j(m),\ m = 1, 2, ..., 2^{Nb} \qquad (12)
\end{aligned}
$$

where $\Delta_j(m) = [BE_j(m), BE_j(m+1))$.

At the FC, all quantization levels received from CUs will be combined to make a global decision concerning about the presence or absence of the PU signal by using equal gain combination rule (EGC) as

$$
\begin{cases}
G(i) = 1, \ if \ \sum\limits_j u_j(i) \geq \lambda \\
G(i) = 0, \ if \ otherwise
\end{cases} \qquad (13)
$$

where λ is the threshold for global decision.

IV. Simulation Results

We consider a CR network with $N = 20$ CUs, in which all CUs have the same SNR $= -15$dB. As references, sensing performances of following CSS scheme are provided: EGC with raw sensing information (denoted as EGC-raw data), EGC with the proposed quantization scheme (denoted as EGC-the proposed scheme) and EGC with the uniform quantization scheme (denoted as EGC-uniform quantization).

Fig. 3 illustrates the ROC curves of the proposed scheme and the considered comparison schemes. The figure shows that EGC-the proposed scheme with 3bits can obtain the similar sensing performance to EGC-raw data. On the other

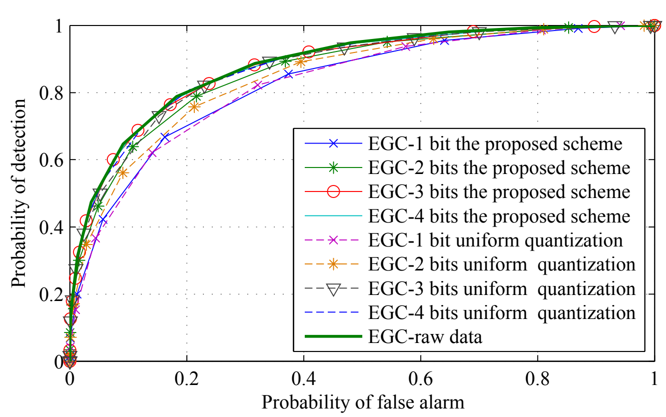

Figure 3. ROC curves of cooperative spectrum sensing when SNR of all CUs are given as -15dB.

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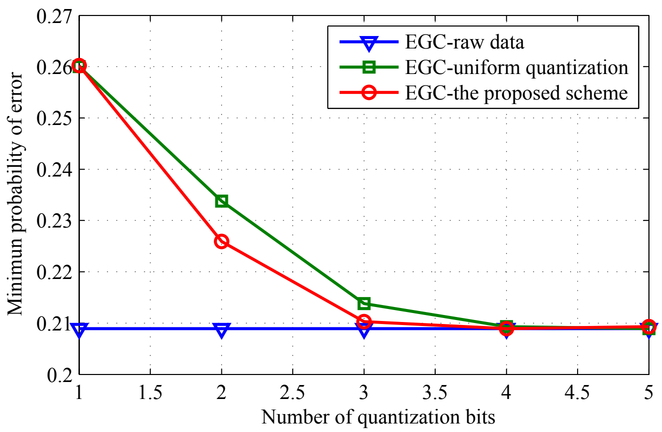

Figure 4. Minimum probability of error of cooperative spectrum sensing when SNR of all CUs are given as -15dB.

hand, EGC-uniform quantization needs 4 bits to achieve the similar performance.

Fig. 4 shows the minimum probability of error of the proposed scheme and above reference schemes, where probability of error is defined as:

$$P_e(\lambda) = P_f(\lambda)\Pr(H_0) + P_m(\lambda)\Pr(H_1) \tag{14}$$

where $P_f(\lambda)$ and $P_m(\lambda)$ are probability of false alarm and probability of miss detection of the global decision, respectively. Since $P_f(\lambda)$ and $P_m(\lambda)$ are function of λ, $P_e(\lambda)$ is also a function of λ. We define λ_{opt} as the optimal threshold at the FC with which probability of error of global decision is minimized. Subsequently, minimum probability of error can be given as $P_{e.\min} = P_e(\lambda_{opt})$. Fig. 4 shows the effect of the quantization bits on the minimum probability of error. From Fig. 4, we can observe that the EGC-the proposed scheme can achieve same performance to that of EGC-raw data with more than 3 quantization bits while the EGC-uniform quantization requires more than 4 quantization bits to achieve same performance.

V. Conclusion

In this paper, we have proposed a quantization scheme based on KL-divergence for cooperative spectrum sensing in CR network. The proposed scheme can reduce the necessary quantization bits comparison with the uniform quantization scheme while still keeping the same sensing performance with the raw data.

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