Spread Spectrum-Based Underlay Cognitive Radio Wireless Networks

Saed Daoud, David Haccoun, and Christian Cardinal

Department of Electrical Engineering
École Polytechnique de Montréal
Montréal, QC, Canada
Email: saed.daoud@polymtl.ca, david.haccoun@polymtl.ca, christian.cardinal@polymtl.ca

***Abstract*—In this paper, we investigate the performance of underlay cognitive radio (CR) systems that employ spread spectrum (SS). In particular, we consider a single-user secondary (cognitive) system that coexists with a multiple-user primary system. The quality of service of the primary system is protected by placing a maximum allowable interference power at the primary receiver (PR). We first derive the cumulative distribution function of the signal-to-interference-plus-noise ratio (SINR) at the secondary receiver (SR), which is then used to evaluate the outage probability and average bit error rate (ABER) of the secondary system. Simulation results verified by Monte-Carlo simulations show that SS-based underlay CR systems outperform conventional underlay CR systems by adapting the spreading factor (SF) of the spreading sequences.**

***Keywords*—Cognitive radio; spread spectrum; outage probability; average bit error rate.**

I. Introduction

Cognitive radio (CR) is a new promising technology, that makes an efficient use of the spectrum, by making the spectrum access process more dynamic, by adapting the transmissions' parameters to the surrounding environment, as well as to the users' demands [1]–[4]. In underlay CR mode [5], the adaptation is on the transmitted power from the unlicensed secondary (cognitive) system, such that the aggregate interference at the licensed primary receiver (PR) is below a certain threshold.

In [6], the author studied the capacity of underlay (also called *spectrum-sharing*) secondary system over additive white Gaussian noise (AWGN) channels, where the constraint is placed on the channel output signal, instead of the conventional problem formulation, where the constraint is placed on the channel input signal, mainly because of hardware limitation. In [7][8], the capacity of such system is studied over Rayleigh fading channels under peak and average interference power constraints placed at the primary receiver. In [9], the authors consider a multiuser secondary system which coexists with a primary system with one transmitter-receiver (TX-RX) pair. They found the outage probability at the best secondary receiver (SR) in terms of the received signal-to-interference-plus-noise ratio (SINR). The authors considered the effect of the primary system on the secondary receiver, which was something missing in the previously mentioned works.

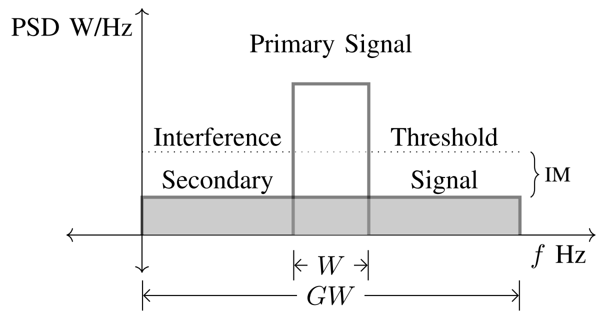

Figure 1. The primary and secondary signals's spectra at the PR.

In [10], the authors studied the problem of distributed frequency spectrum and power allocation and optimization for multicarrier direct sequence code division multiple access (MC DS-CDMA) system in an ad hoc setting. They considered the interweave mode, where the entire available spectrum is sensed, and only the subcarriers that are not being used are assigned to the secondary users. The optimization is done with a target data rate and available power constraints for each secondary user. In [11], the authors considered code division channelization, with joint transmit power and code assignment optimization, such that the interference from the secondary system to the primary system is considered acceptable, while SINR at the secondary receiver satisfies a pre-defined quality of service (QoS). In [12], the authors proposed a MC CDMA secondary system that aggregates non contiguous subbands such that the bandwidth of subbands isn't fixed.

In [13], we investigated the performance of a secondary system when spreading is done by repetition channel coding as a simple means of spreading. Also, we investigated the performance when a combination of channel coding and spread spectrum is used over AWGN channels. In [14], we investigated the problem of maximizing the throughput of a secondary system using CDMA under some idealistic assumptions. In this paper, we consider the coexistence of a primary system with an underlay secondary system, where the secondary system is assumed to be using direct sequence spread spectrum (DS-SS) with a spreading factor (SF) G. See Figure 1. The primary system consists of multiple primary transmitters (PTs) and one primary receiver (PR), while the secondary system consists of one secondary transmitter (ST) and one secondary receiver (SR). The performance of the secondary system in terms of outage probability and average bit error rate (ABER) is investigated, by taking into account the effect of the primary system on SR. Simulation results verified by Monte-Carlo simulations show that SS-based CR systems outperform conventional underlay systems by adapting the spreading factor (SF) of the spreading sequences, which makes SS a promising technique to be used in such systems.

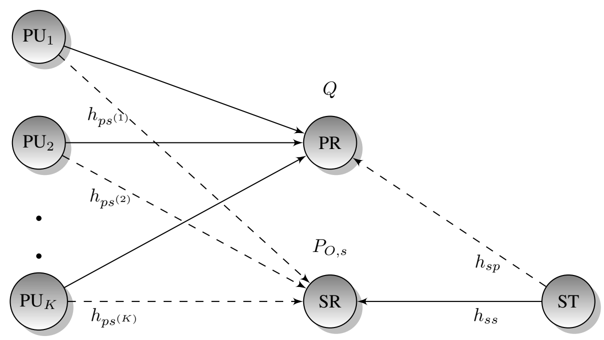

Figure 2. System Model. The dashed lines are interference links.

The rest of the paper is organized as follows: In Section II, the system and channel's model are presented, in Section III, the outage probability and ABER are investigated. In Section IV, simulation results are provided, and finally, we conclude in Section V.

II. System and Channel's Models

We consider the coexistence of a multiple primary user (PU) system with K users and a secondary cognitive system with one user. See Figure 2. It's assumed that all PUs communicate with one PR, i.e., multiple access uplink communication. Also, one secondary transmitter (ST) is communicating with one secondary receiver (SR). ST employs DS-SS with an SF of G, where we assume that $G \geq K$, while the primary system is assumed to employ frequency division multiple access (FDMA) such that there is no interuser interference (IUI) within the primary system. The channel between PU$_k$ and the SR is denoted by $h_{ps}^{(k)}$ for $k = 1, 2, \ldots, K$, while the channels between ST and PR and between ST and SR are denoted by h_{sp} and h_{ss}, respectively. All channels are assumed to be complex-valued Gaussian random variables with zero mean and unit variance. To protect the QoS of the primary system, a maximum interference threshold Q is set at the PR. PUs are assumed to transmit using a fixed power P, while ST is assumed to adapt its power such that the interference on PR is below Q.

III. Performance Analysis: Outage probability and Average Bit Error Rate

In this section, the performance of SS-based underlay CR system in the presence of multiple-user primary system is evaluated in terms of outage probability and ABER.

A. Outage Probability

Outage probability of the CR system is defined as, the probability that the instantaneous SINR falls below a ceratin threshold, γ_{th}. Hence, first, SINR at SR will be quantified mathematically and statistically. The statistics of SINR then will be used to evaluate the outage probability (and ABER in the next subsection). Since it is assumed that $G \geq K$, the SINR at SR is given by

$$\Gamma_s = \frac{|h_{ss}|^2 \, S(h_{sp})}{\frac{P}{G} \sum_{k=1}^{K} |h_{ps}^{(k)}|^2 + \sigma_n^2}, \tag{1}$$

where $S(h_{sp})$ is the transmit power from ST, and σ_n^2 is the AWGN power. The transmit power $S(h_{sp})$ must be adjusted such that the total interference at PR is less than or equal to the maximum allowable interference power. Mathematically, we need

$$\frac{1}{G} |h_{sp}|^2 S(h_{sp}) \leq Q, \tag{2}$$

where the factor $1/G$ is due to spreading the secondary signal's power over a bandwidth that is G times larger than the minimum required bandwidth. Since we didn't place any physical power budget on ST, we can set it to its maximum allowable value, which is given by

$$S(h_{sp}) = \frac{G\,Q}{|h_{sp}|^2}. \tag{3}$$

Substituting (3) into (1) yields to

$$\Gamma_s = \frac{\frac{|h_{ss}|^2}{|h_{sp}|^2} \, G \, \gamma_Q}{\frac{\gamma_P}{G} \sum_{k=1}^{K} |h_{ps}^{(k)}|^2 + 1}, \tag{4}$$

where $\gamma_Q = Q/\sigma_n^2$ and $\gamma_P = P/\sigma_n^2$. Having the SINR expression at SR as in (4), the outage probability at SR can be expressed as

$$\begin{aligned} P_{O,s} =& \mathrm{Pr}\left[\Gamma_s \leq \gamma_{th}\right] \\ =& \mathrm{Pr}\left[\frac{\alpha_1 \, G \, \gamma_Q}{\frac{\gamma_P}{G}\alpha_2 + 1} \leq \gamma_{th}\right], \end{aligned} \tag{5}$$

where γ_{th} is the threshold below which the system will be in outage, $\alpha_1 = \frac{|h_{ss}|^2}{|h_{sp}|^2}$ and $\alpha_2 = \sum_{k=1}^{K} |h_{ps}^{(k)}|^2$. Let $\alpha_{XY} = |h_{XY}|^2$, then the CDF of α_1 can be expressed as

$$\begin{aligned} F_{\alpha_1}(x) =& \mathrm{Pr}\left\{\frac{\alpha_{ss}}{\alpha_{sp}} \leq x\right\} \\ =& \int_0^{\infty} \mathrm{Pr}\left\{\alpha_{ss} \leq x\beta | \alpha_{sp} = \beta\right\} \, f_{\alpha_{sp}}(\beta) \, d\beta, \end{aligned} \tag{6}$$

where $F_Y(y)$ and $f_Y(y)$ are the CDF and probability distribution function (PDF) of the random variable Y. Since the channels' coefficients are assumed to be complex-valued Gaussian random variables with zero mean and unit variance, the channels' magnitude squares are exponentially distributed with unit mean, i.e.,

$$F_{\alpha_{ss}}(x) = 1 - \exp(-x) \quad (7a)$$
$$f_{\alpha_{sp}}(x) = \exp(-x). \quad (7b)$$

It is straightforward to show that the CDF of the random variable α_1, denoted by $F_{\alpha_1}(x)$ to be

$$F_{\alpha_1}(x) = \Pr\left[\alpha_1 \leq x\right] = 1 - \frac{1}{x+1}. \quad (8)$$

Also note that α_2 is the summation of K squared *complex-valued* Gaussian random variables with variance $1/2$ per dimension, i.e., α_2 is central Chi-square random variable with $2K$ degrees of freedom. Thus, its PDF is given by [15]

$$f_{\alpha_2}(\alpha_2) = \frac{1}{(K-1)!}\,\alpha_2^{K-1}\,e^{-\alpha_2}. \quad (9)$$

Then, the outage probability in (5) can be re-written as

$$\begin{aligned} P_{O,s} =& \int_0^\infty F_{\alpha_1}\left(\frac{\gamma_{th}}{G\,\gamma_Q}\left[\frac{\gamma_p}{G}\alpha_2 + 1\right]\right) f_{\alpha_2}(\alpha_2)\,d\alpha_2 \\ =& 1 - \frac{1}{(K-1)!\frac{\gamma_{th}\gamma_P}{G^2\gamma_Q}}\underbrace{\int_0^\infty \frac{\alpha_2^{K-1}\,e^{-\alpha_2}}{\alpha_2 + \frac{G^2\gamma_Q}{\gamma_{th}\gamma_P}\left[\frac{\gamma_{th}}{G\gamma_Q}+1\right]}\,d\alpha_2}_{I}, \end{aligned} \quad (10)$$

where $F_{\alpha_i}(.)$ and $f_{\alpha_i}(.)$ are the CDF and PDF of the random variable α_i. From [16, eq. 3.353.5], the integral I can be expressed as

$$I = (-1)^{n-1}\beta^n e^{\beta\mu}\mathrm{Ei}\left(-\beta\mu\right) + \sum_{k=1}^{n}(k-1)!(-\beta)^{n-k}\mu^{-k}, \quad (11)$$

where $n = K-1$, $\mu = 1$, and $\beta = \frac{G^2\gamma_Q}{\gamma_{th}\gamma_P}\left[\frac{\gamma_{th}}{G\gamma_Q}+1\right]$.

B. Average Bit Error Rate (ABER)

Another useful performance metric is the ABER, which will be derived next. Toward that end, let

$$X = \frac{\alpha_1\,G\,\gamma_Q}{\frac{\gamma_P}{G}\alpha_2 + 1}. \quad (12)$$

Without any loss of generality, and for simplicity of exposition, coherent binary phase shift keying (BPSK) is assumed. In this case, the conditional BER is given by

$$\begin{aligned} \varepsilon_s(x) =& Q\left[\sqrt{2x}\right] \\ \leq& \frac{1}{2}e^{-x}, \end{aligned} \quad (13)$$

where we used the Chernoff upper bound of the Gaussian Q-function in the second line of (13). Then the ABER is upper bounded as

$$\varepsilon_s \leq \frac{1}{2}\int_0^\infty e^{-x}\,f_X(x)\,dx \quad (14)$$

where $f_X(x)$ is the PDF of the random variable X. It can be shown by integration by parts that

$$\int_0^\infty e^{-x} f_X(x)\,dx = \int_0^\infty e^{-x}F_X(x)\,dx, \quad (15)$$

where $F_X(x)$ is the CDF of the random variable X. This implies that we don't need to derive the PFD of the random variable X, but instead, we can use the CDF directly in evaluating the ABER. The ABER in (14) can be re-written as

$$\varepsilon_s \leq \frac{1}{2}\int_0^\infty e^{-x}\,F_X(x)\,dx. \quad (16)$$

The CDF to evaluate the ABER has the same expression as the outage probability in (10) by replacing γ_{th} with x. The ABER can be re-written then as

$$\varepsilon_s = \frac{1}{2} - \frac{G^2\gamma_Q}{2(K-1)!\gamma_p}\int_0^\infty \frac{1}{x}e^{-x}I(x)\,dx, \quad (17)$$

where

$$\begin{aligned} I(x) =& (-1)^{n-1}\left[\frac{G}{\gamma_p} + \frac{G^2\gamma_Q}{x\gamma_p}\right]^n e^{\frac{G}{\gamma_p}+\frac{G^2\gamma_Q}{x\gamma_p}}\,\mathrm{Ei}\left(-\left[\frac{G}{\gamma_p}+\frac{G^2\gamma_Q}{x\gamma_p}\right]\right) \\ &+ \sum_{k=1}^{n}(k-1)!\left(-\left[\frac{G}{\gamma_p}+\frac{G^2\gamma_Q}{x\gamma_p}\right]\right)^{n-k}, \end{aligned} \quad (18)$$

where $n = K-1$.

IV. NUMERICAL RESULTS

In this section, numerical evaluation verified by Monte-Carlo simulations are provided for the above mathematical derivations.

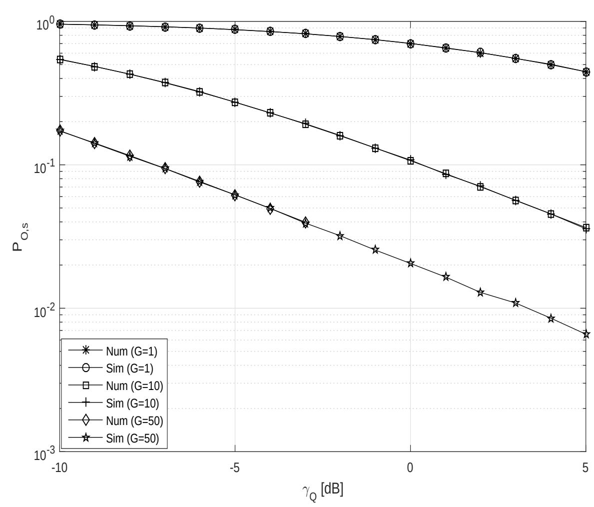

Figure 3. Outage probability at SR vs. γ_Q [dB] for $\gamma_P = 3$ dB, $\gamma_{th} = 0$ dB, $G = \{1,\,10,\,50\}$, and $K = 1$.

In Figure 3, outage probability vs. γ_Q in [dB] is shown for single user system (i.e., $K = 1$), and for $G = \{1,\,10,\,50\}$, $\gamma_P = 3$ dB, and $\gamma_{th} = 0$ dB. Monte-Carlo simulations are also shown, where 10^5 channel realizations were generated for each γ_Q point. Two observations can be made. First, underlay cognitive radio system that employs spread spectrum (i.e., $G > 1$) has better performance than conventional systems that don't employ SS (i.e., $G = 1$). This is because of two reasons, a) by spreading the secondary signal's spectrum over larger bandwidth using spreading sequences, the interference caused at PR is reduced per unit bandwidth, and that allows ST to transmit at higher power (see Figure 1), and b) the interference caused by PT at SR is also reduced, because depreading the secondary received signal by a synchronized replica of the spreading sequence has the effect of spreading the primary signal's power over larger bandwidth, and thus its effect within the secondary signa's bandwidth is significantly reduced, which contributed more to better performance. That is why the performance as seen in Figure 3 is improved as G increased. In conventional systems, the secondary system cannot do anything beyond adapting its power to meet the interference threshold requirement at PR. Because of this, secondary systems usually don't have enough interference margin that allows the secondary system to be operational, by transmitting at an acceptable power level. On the other hand, in SS-based underlay CR systems, SF G can be adapted such that the interference threshold at PR is met, and making the secondary system operational. The second observation is that, Monte-Carlo simulations are in agreement with the numerical evaluation, which implies that our mathematical derivations are correct.

In Figure 4, outage probability vs. γ_Q in [dB] is shown for multiple primary user system for $K = 5$, and for $G = \{10, 50\}$ (note that we didn't include $G = 1$ for non SS systems, because it is assumed that $G \geq K$, such that the effect of the secondary system is the same for all primary users), $\gamma_P = 3$ dB, and $\gamma_{th} = 0$ dB. Monte-Carlo simulations are also shown, where 10^5 channel realizations were generated for each γ_Q point. The same observations as before can be made.

In Figure 5, outage probability vs. γ_Q [dB] is shown for $K = \{1, 5, 10\}$, $G = 50$, $\gamma_p = 10$ dB, and $\gamma_{th} = 5$ dB. In this case, when G is fixed, and K is variable, we note that, as K is increased, the performance deteriorates. Which is expected, because, although the primary signals' power are despread at SR, the interference from the primary system at SR is the sum of the interference from all primary users. We notice that, the difference in performance as K is increased is not significant. Maybe this due to that fact that G is large enough to make the interference from the primary system to be small, and in the limit when $G \to \infty$, the system performance approaches that of point-to-point system with no interference.

In Figure 6, ABER vs. γ_Q [dB] is shown for $K = 2$ users, and $G = \{10, 100\}$. The corresponding Monte-Carlo simulation curves are shown as well. It can be observed that there is an error floor. This is due to the fact that, although the primary signals are spread at the SR, the interference is the sum of the primary spread signals. However, as G is increased,

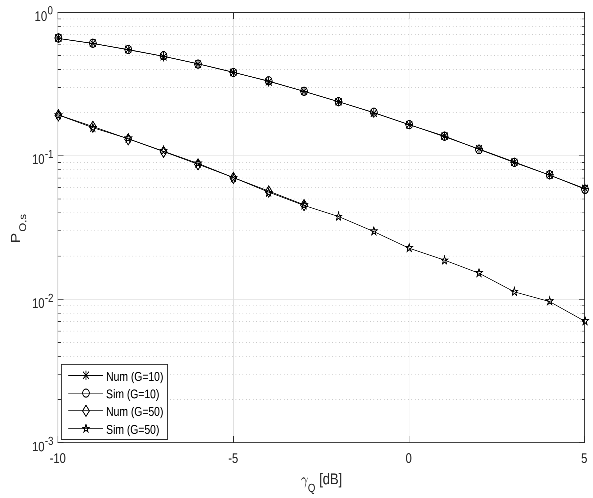

Figure 4. Outage probability at SR vs. γ_Q [dB] for $\gamma_P = 3$ dB, $\gamma_{th} = 0$ dB, $G = \{10, 50\}$, and $K = 5$.

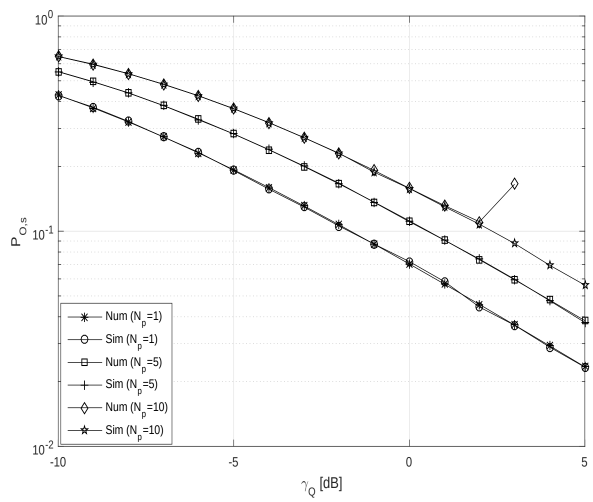

Figure 5. Outage probability at SR vs. γ_Q [dB] for $\gamma_P = 10$ dB, $\gamma_{th} = 5$ dB, $G = 50$, and $K = \{1, 5, 10\}$.

the performance is improved, which is again attributed to the fact that the interference level from each PU is decreased within the secondary signal's bandwidth of interest. We can also observe that there is a small constant difference between the numerical evaluation and Monte-Carlo simulations, which we believe is due to an inherit error in the numerical evaluation of the integral (17).

V. Conclusions and Future Work

In this paper, we considered SS-based underlay cognitive radio systems, and the performance of such systems was evaluated. In particular, first the CDF of the SINR at SR was derived, which was then used to evaluate the outage probability and ABER of the secondary system. Numerical results verified by Monte-Carlo simulations showed that, deploying SS in

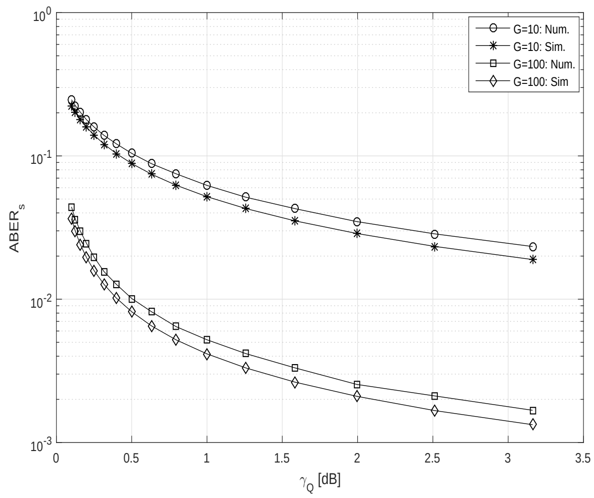

Figure 6. ABER vs. γ_Q [dB] for $\gamma_P = 5$ dB, $G = [10\ \ 100]$, and $K = 2$.

underlay CR systems, can improve the performance significantly compared to non-SS underlay CR systems. This study showed that underlay CR systems can be considered a viable option besides the interweave mode, because a limiting factor in underlay systems was that the transmit power is too low for the secondary system to be operational. Spread spectrum can alleviate this limitation.

As a future work, we will consider the case when both the primary and secondary systems consist of multiple users. Furthermore, the effect of channel estimation and synchronization for the spreading sequences will be considered.

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