

# Simulation of the Emergence of Language Groups

## Using the Iterated Learning Model on Social Networks

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**Abstract**—In evolutionary linguistics, the Iterated Learning Model (ILM) is often used for simulating the first language acquisition. Our purpose in this paper is to develop an agent-based model for language contact based on ILM. We put a learning agent on each node in the social network. Our experimental result showed that the language exposure rather deteriorates the emergence of local common languages, and grammars become non-compositional, which is different from our expectation. However, we have shown that an excessive string-clipping as well as a language exposure may constrain the appearance of local language community, independent of the shape of networks.

**Keywords**—Simulation, Language Acquisition, Iterated Learning Model, Social Network.

### I. INTRODUCTION

Thus far, simulation studies have played an important role in the field of the evolution of language [1]. Especially, a very important function of simulation is to prove if a prediction actually and consistently derives from a theory [2]. So far, there have been a variety of methodologies proposed on simulating the evolution of languages, each of which belongs to a different level of abstraction. Simulation studies for population dynamics alone include an agent-based model of language acquisition by Briscoe [3], which was developed toward a formal model of language acquisition device. On the other hand, Nowak [4] proposed a mathematical theory of the evolutionary dynamics of language called the language dynamics equation. The language dynamics equation is highly abstract, while agent-based model is considered to be a concrete, or less abstract.

Our goal is to provide a framework that represents the diachronic change in language by the contact among language communities. It would be useful not only for simulating typical language changes but also for novel phenomena taking place in the cyber world. In recent decades, the evolution of the Internet makes users possible to participate in discussions with anonymous people concerning their favorite topics beyond the physical distance. They do not only exchange small bits of information, but rather seem to establish a durable channel to communicate among people sharing common tastes on a chat or bulletin board system. They often employ spoken language instead of formal one after sharing common interests, and thus the expression tends to be spontaneous and haphazard. This phenomenon is often seen in language contact, but the time

and size of the language change on the internet are extremely fast and large, respectively [5]. Using the framework, it would be possible to deal with this rapid language change as a phenomenon of language evolution.

There have been simulation models dealing with language change. We employ them as possible. Thus far, Nakamura et al. [7] proposed a mathematical framework for the emergence of creoles [6] based on the language dynamics equation. Toward more concrete analysis, they introduced a spatial structure to the mathematical framework [8] [9], in which learning agents contact with neighbors according to the learning algorithm. Furthermore, the spatial structure was expanded into complex networks [10]. Their studies are based on a hypothesis about the emergence of creoles, that is, language contact is likely to stimulate creolization. However, their learning mechanisms are too simple to observe language changes from a linguistic aspect, as languages are defined as similarity measures in a numeric matrix.

We propose an agent based model to deal with grammatical changes in the language community. Therefore, our purpose in this paper is to show a relationship between communication among learning agents and grammatical changes. We employ Simon Kirby's Iterated Learning Model (ILM) [11], which shows a process of grammatical evolution through generations. Kirby's ILM has often been used in simulation models concerning language evolution [12]. One important reason for this is that ILM is robust against input sentences in terms of a syntactic learning. As long as learning from a single parent, its infant agent receives sentences derived from a consistent grammar, it is possible to acquire a concise grammar. Currently, the learning situation in ILM is extended to a multiple families connecting with a network. We can observe the language change, not only in diachronic situation, i.e., in parent-child relation, but also in synchronic situation.

Thus far, we have shown a pilot version [13], where we found a problem reported by Smith and Hurford [20], that is, in the case learning agents potentially have more than one teacher agent, the length of syntax rules tends to increase rapidly over generations due to the addition of symbols of meaningless terminal symbols. This problem causes an unnatural learning, which results in a fatal combinatorial explosion. We solved this problem and try again with a larger number of agents.

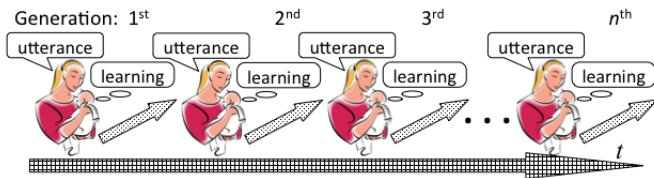


Figure 1. The iterated learning model

This paper is organized as follows. In Section 2, we introduce Kirby’s ILM. In Section 3, we propose an agent-based model for language contact. In Section 4, we examine our proposed method, and conclude in Section 5.

## II. ITERATED LEARNING MODEL FOR SOCIAL NETWORKS

In this section, we mention how to deal with ILM on the social networks. Firstly, we briefly explain Kirby’s ILM. After that, we introduce the modification for social networks by Matoba et al. [14] in order to avoid the combinatorial explosion, which enables us the expansion of ILM.

### A. Briefing Kirby’s Iterated Learning Model

Kirby [11] introduced the notions of compositionality and recursion as fundamental features of grammar, and showed that they made a human possible to acquire compositional language. Figure 1 illustrates ILM. In each generation, an infant can acquire grammar in his/her mind given sample sentences from his/her mother. When the infant has grown up, he/she becomes the next parents to speak to a newborn baby with his/her grammar. As a result, infants can develop more compositional grammar through the generations. Note that the model focuses on the grammar change in multiple generations, not on that in one generation. Although the poverty of stimulus explains the necessity of the universal grammar [15], Kirby [11] modeled it as learning through bottlenecks, which are rather necessary for the learning. Also, he adopted the idea of two different domains of language [16]–[19], namely, I-language and E-language; I-language is the internal language corresponding to speaker’s intention or meaning, while E-language is the external language, that is, utterances. In his model, a parent is a speaker agent and his/her infant is a listener agent. The speaker agent gives the listener agent a pair of a string of symbols as an utterance (E-language), and a predicate-argument structure (PAS) as its meaning (I-language). A number of utterances would form compositional grammar rules in listener’s mind, through learning process. This process is iterated generation by generation, and converges to a compact, limited number of grammar rules.

According to Kirby’s ILM, the parent agent gives the infant agent a pair of a string of symbols as an utterance, and PAS as its meaning. The agent’s linguistic knowledge is a set of a pair of a meaning and a string of symbols, as follows.

$$S/\text{love}(\text{john}, \text{mary}) \rightarrow \text{hjsbs}, \quad (1)$$

where the meaning, that is the speaker’s intention, is represented by a PAS  $\text{love}(\text{john}, \text{mary})$  and the string of symbols is the utterance “hjsbs”; the symbol ‘S’ stands for

**Verb:** admire, detest, hate, like, love  
**Noun:** john, mary, pete, heather, gavin  
 e.g.)  $\text{love}(\text{mary}, \text{john})$   
 (Identical arguments are prohibited.)

Figure 2. Words used in the experiment.

the category Sentence. The following rules can also generate the same utterance.

$$\begin{aligned} S/\text{love}(x, \text{mary}) &\rightarrow \text{h } N/x \text{ sbs} \\ N/\text{john} &\rightarrow \text{j} \end{aligned} \quad (2)$$

where the variable  $x$  can be substituted for an arbitrary element of category  $N$ .

The infant agent has the ability to generalize his/her knowledge with learning. This generalizing process consists of the following three operations [11]; *chunk*, *merge*, and *replace*.

**Chunk** This operation takes pairs of rules and looks for the most-specific generalization.

$$\begin{aligned} &\left\{ \begin{array}{l} S/\text{love}(\text{john}, \text{pete}) \rightarrow \text{ivnre} \\ S/\text{love}(\text{mary}, \text{pete}) \rightarrow \text{ivnh0} \end{array} \right. \\ \Rightarrow &\left\{ \begin{array}{l} S/\text{love}(x, \text{pete}) \rightarrow \text{ivn } N/x \\ N/\text{john} \rightarrow \text{re} \\ N/\text{mary} \rightarrow \text{ho} \end{array} \right. \end{aligned} \quad (3)$$

**Merge** If two rules have the same meanings and strings, replace their nonterminal symbols with one common symbol.

**Replace** If a rule can be embedded in another rule, replace the terminal substrings with a compositional rule.

In Kirby’s experiment [11], five predicates and five object words shown in Figure 2 are employed. Also, two identical arguments in a predicate like  $\text{love}(\text{john}, \text{john})$  are prohibited. Thus, there are 100 distinct meanings (5 predicates  $\times$  5 possible first arguments  $\times$  4 possible second arguments) in a meaning space.

The key issue in ILM is to make the situation of *poverty of stimulus*. As long as an infant agent is given all sentences in the meaning space during learning, he/she does not need to make a compositional grammar; he/she would just memorize all the meaning-sentence pairs. Therefore, agents are given a part of sentences in the whole meaning space. The total number of utterances the infant agent receives during learning is parameterized. Since the number of utterances is limited, the infant agent cannot learn the whole meaning space, the size of which is 100; thus, to obtain the whole meaning space, the infant agent has to generalize his/her own knowledge by self-learning, i.e., *chunk*, *merge*, and *replace*. The parent agent receives a meaning selected from the meaning space, and utters it using her own grammar rules. When the parent agent cannot utter because of lack of her grammar rules, she invents a new rule. This process is called *invention*. Even if the invention does not work to complement the parent agent’s grammar rules to utter, she utters a randomly composed sentence.

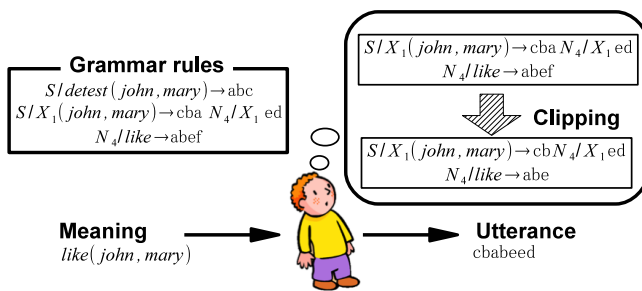


Figure 3. Image of clipping process

### B. Process for String Clipping

When an infant agent has a number of teacher agents consisting of his/her parent and neighbors, as the teacher agents have their own compositional grammar rules, they are inconsistent with each other. Although the infant agent tries to find a common chunk among utterances, it would be a short string. Since there is little probability of making a chunk from short strings, only long ones are likely to survive toward next generations. As a result, learning agents tend to have compositional rules with extremely long strings over generations [20].

Matoba et al. [14] proposed a clipping process in their model, which solves the above problem. This process is called *backclipping*. After learning process of the infant agent, he/she curtails symbols in his/her grammar rules from the tail of string, unless it contains ambiguity. As a result, when the infant agent becomes the new parent agent in the next generation, the grammar set does not contain extremely long rules any more.

Figure 3 illustrates the clipping process in our model. The infant agent tries to utter strings of  $like(john, mary)$  as shortly as possible. Firstly, he/she chooses a grammar rule from his/her grammar set for generating utterance of  $like(john, mary)$ , and deletes symbols one by one, i.e., “cba”, “ed”, “abef”. In case of “cba”, this string does not exist in the grammar rules of the infant agent, then the infant agent executes backclipping, and the string becomes “cba” to “cb”. The string “cb” does not exist in the grammar rules of the infant agent, so the infant agent executes backclipping, and the string becomes “cb” to “c”. Since “c” exists in the grammar rules of infant agent, the infant agent does not abridge it anymore, and adopts “cb” as the clipped word of “cba”. The same process is also applied to the other words. As a result, the sentence becomes shortened from “cbaabefed” to “cbabeed”.

Actually, such phenomenon occurs in the real world, as cutting the beginning and/or the end of a word off. The deletion of a part of a word constructs a new and shorter word;

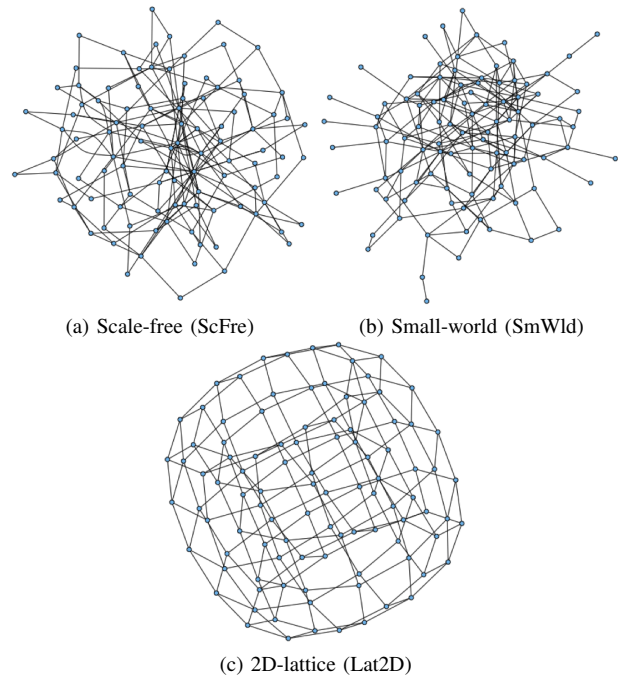
e.g.) **Hamburger** → **burger**, **Influenza** → **flu**,  
**Examination** → **exam**

A position of clipping is dependent on a phonological reason [21]. Since ILM does not deal with phonological information, we need to find an alternative way to shorten strings. In English, we often omit a few syllables of each word [22];

e.g.) **advertisement**, **doctor**, **laboratory**, **professor**,  
**demonstration**, **captain**, etc.

TABLE I. NETWORK CHARACTERISTICS

Network type ( $N = 100$ )	Average Degree	Average shortest path
Complete graph	99.00	1.00
Star	1.98	1.98
(a) Scale-free	3.96	3.28
(b) Small-world	4.00	3.41
(c) 2D lattice	4.00	12.88
Ring	2.00	25.25


 Figure 4. Examples of the networks ( $N = 100$ )

## III. AGENT-BASED MODEL FOR LANGUAGE CONTACT

In this section, we explain how language groups emerge in the agent-based model. Agents can get contact with neighbors on the network (Section III-A), who speak to the infant in a certain ratio of language exposure (Section III-B). The communication may affect agents' languages, which are classified into groups by the language similarity (Section III-C).

### A. Social Networks for Language Communities

Social networks play an important role of language change, regardless of whether they are connected by an actual or virtual relationship. Some simulation studies deal with complex networks [10] [23] [24]. There are several types of networks, each of which characterizes many real-world communities.

Table I shows network characteristics, in which each value is calculated based on 100 nodes [24]. The average degree denotes the average number of edges connected to a node. The average shortest path length stands for the average smallest number of edges, via which any two nodes in the network can be connected to each other. In this paper, we examine scale-free, small-world and 2D-lattice networks.

Figure 4 shows examples of networks, in which the preceding two networks are regarded as complex networks and the latter is for comparison. Each agent is assigned on a node in the networks.

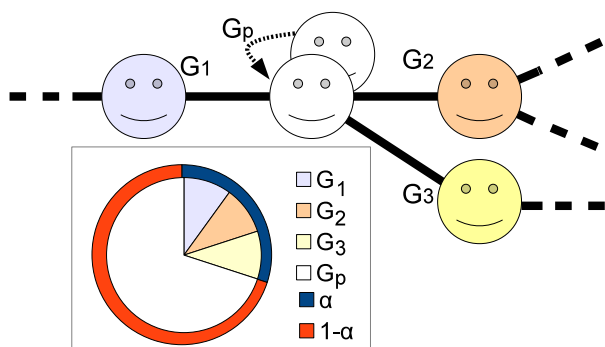


Figure 5. Language input from neighbors depending on the exposure ratio  $\alpha$

**B. Exposure Ratio  $\alpha$**

Nakamura et al. [7] has introduced an exposure ratio  $\alpha$ , which determines how often language learners are exposed to a variety of language speakers other than their parents. They modified the learning algorithm of Nowak et al. [4], taking the exposure ratio into account in order to model the emergence of creole community. They have shown that a certain range of  $\alpha$  is necessary for a creole to emerge. This parameter was further employed for the following network studies [8]–[10].

In some communities, a child learns language not only from his/her parents but also from other adults, whose language may be different from the parental one. In such a situation, the child is exposed to other languages, and thus may learn the most communicative language. In order to assess how often the child is exposed to other languages, let us divide the language input into two categories: one is from his/her parents, and the other is from other language speakers. The ratio of the latter to the total amount of language input is called an *exposure ratio*  $\alpha$ . This  $\alpha$  is subdivided into smaller ratios corresponding to those other languages, where each ratio is in proportion to the population of the language speakers.

An example distribution of languages is shown in Figure 5. Let  $G_i$  be the language of Agent  $i$ . Suppose a child has parents who speak  $G_p$ , he/she receives input sentences from  $G_p$  in the proportion of  $1 - \alpha$ , and from non-parental languages  $G_i (i \neq p)$  in the proportion of  $\alpha x_i$ , where  $x_i$  denotes a population ratio of  $G_i$  speakers in the neighbors.

**C. Distance between Languages and Language Groups**

In this section, we discuss how to deal with languages in the framework of ILM on a social network. An infant receives a meaning-signal pair from his parent and neighbors according to the exposure ratio  $\alpha$ . The number of utterances an infant receives is fixed, and he/she receives them in proportional to the language distribution for neighbors like the pie chart shown in Figure 5. The population consists of non-overlapping generations, that is, infants at each generation are born at the same time, become parents at the same time, and die at the same time. The network is fixed through generations.

In order to compare between languages, we define the distance in languages by the edit distance, known as the Levenshtein distance [25]; we count the number of insertion/elimination operations to change one word into the other. For example, the distance between “abc” and “bcd” becomes 2 (erase ‘a’ and insert ‘d’). Once the learning process has been

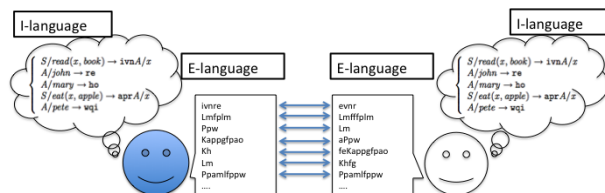


Figure 6. Calculation of the distance between languages

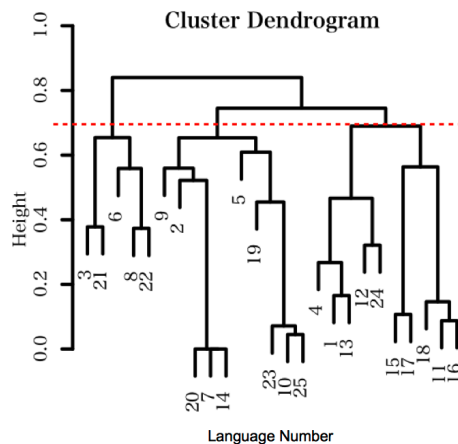


Figure 7. Clustering languages ( $N = 25$ )

finished, each agent has his/her own grammar rules. In other words, each agent can enumerate all the sentences he/she can utter as E-language derived from I-language. Figure 6 depicts an image of enumeration. Note that all the compositional grammar rules are expanded into a set of *holistic rules*, which do not include any variable, i.e., a rule consists of a sequence of terminal symbols. Since the Levenshtein distance between corresponding strings can be calculated, the average distance normalized at the range from 0 to 1 comes into the distance between languages.

Since agents independently invent languages, their acquired languages are different from each other. In order to classify agents into groups by the language similarity, we introduce a clustering method, recognizing a cluster as a language group. The relationship among languages is represented by a dendrogram shown in Figure 7. The vertical axis denotes the height of the tree, which generally depicts the mergers or divisions which have been made at successive level. We employed the complete linkage method throughout the experiments. The number of languages, therefore, depends on the cutting point of the tree. In this case, the community is regarded as consisting of three-language groups at the height of  $\theta = 0.7$ .

**IV. EXPERIMENTAL RESULTS**

Our purpose of these experiments is to examine how the configuration of networks affects the language learning by infant agents. We expect language groups to emerge depending on the types of networks and other conditions. Therefore, we examine three types of networks; Scale-free, Small-world, and 2D-lattice networks. Scale-free and small-world networks are drawn with BA [26] and WS [27] models, respectively. The number of nodes is fixed to  $N = 100$ . In BA networks, the



number of edges to add in each step is 2, and the power is set to 0.2. The generation of multiple edges is allowed. In WS, the number of neighborhood is 2, and the rewriting probability is set to 1.

We measure (a) the number of *language groups*, (b) the number of *grammar rules* and (c) *expressivity* of the grammar. (a) is calculated by setting the threshold to distinguish languages in the dendrogram to  $\theta = 0.7$ . (b) denotes the average number of grammar rules created in an agent at Generation 100. (c) is defined as the ratio of the number of utterable meanings derived from the grammar rules to the whole meaning space. Each infant agent receives 50 sentences, while the meaning space is 100 (5 predicates  $\times$  5 possible first arguments  $\times$  4 possible second arguments). Therefore, agents need to acquire a compositional grammar for high expressivity.

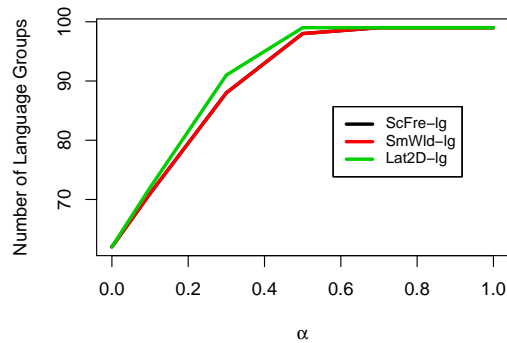
Since the exposure ratio  $\alpha$  activates communication with neighbors, we also expect local dialects to emerge through communication. We parameterize  $0 \leq \alpha \leq 1$ , where the larger the value  $\alpha$  is, the more frequently the neighbors speak to the infant agent. The situation  $\alpha = 1$  is an extreme case that the infant’s parent does not speak to him/her at all, but neighbors do.

Figure 8 shows experimental results. All the data are an average of 50 trials. Since scale-free and small-world networks are randomly drawn for each trial, no phenomenon peculiar to a specific network appears in the results.

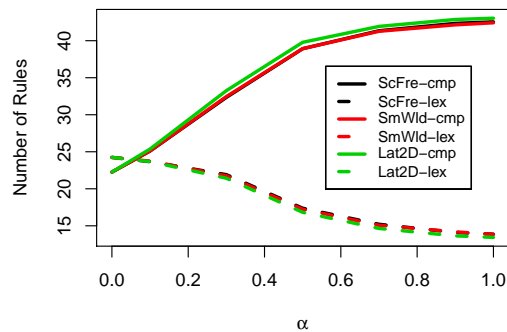
We classified languages into groups at 100th-Generation. Cutting the dendrogram at  $\theta = 0.7$ , we count the number of language groups in the community. Figure 8a shows the change in the number of language groups for every  $\alpha$ . The labels “ScFre-Ig,” “SmWld-Ig” and “Lat2D-Ig” denote the numbers of language groups in the scale-free, small-world, and 2D-lattice networks, respectively.

Analyzing the results in Figure 8a, we can imply that each agent speaks a language different from others, as long as the number of language groups is almost the same as the population of the community. The language exposure is expected to make neighbors share a common language, but the result became different from our expectation. The reason is considered that the clipping process in ILM takes a chance for chunking from non-compositional sentences uttered by different agents. Finally, the most important thing is that there is no big difference between three networks. The exposure ratio  $\alpha$  seems too effective to show minor difference between them. Another reason comes from the number of agents, which is insufficient to show difference between the networks. Despite employment of the clipping process, it was difficult to increase the number of agents more than 100 in a practical computational time.

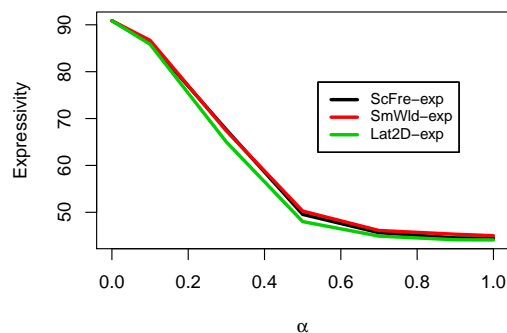
We also investigate acquired grammars. The number of grammar rules and its expressivity are shown in Figures 8b and 8c, respectively. Figure 8b shows that grammars become non-compositional according to  $\alpha$ . The suffix ‘-cmp’ denotes the number of compositional and holistic rules and the suffix ‘-lex’ is the number of lexical rules. The decrease of lexical rules implies holistic rules occupy agents’ knowledge, while an ideal compositional grammar consists of one compositional rule and ten lexical rules. Figure 8c is inevitably reflected by the compositionality. The language exposure negatively affects common languages. There is little difference between networks



(a) Language groups ( $\theta = 0.7$ )



(b) Grammar rules



(c) Expressivity (%)

Figure 8. Experimental results

in grammatical analysis as well as the result of language groups.

The series of experimental results differs from our expectation and from the former studies [7]–[10]. However, we have shown that an excessive string-clipping as well as a larger value of exposure ratio may constrain the appearance of local language community, independent of the shape of networks.

## V. CONCLUSION

In this paper, we proposed an agent-based model for language contact. We employed Kirby's iterated learning model and complex networks. Languages are measured with the Levenshtein distance of utterances, which enables us to show the language divergence by the clustering. The language exposure is expected to make neighbors communicate with each other. Totally, we succeeded to implement a linguistic community with learning agents connected with a social network. The network model makes it possible to observe not only diachronic but also synchronic changes in grammar. We achieved implementation of a large-scale, agent-based model where 100 processes of ILM run in parallel, which contributes to the simulation study on language evolution.

Although we had been faced with a serious problem in terms of constructing a network model with ILM, the new method for a string clipping solved the combinatorial explosion. We need to investigate the algorithm of a string clipping and grasp why it works wrong for language contact.

In the near future, we plan to run more different types of simulations toward the framework of for the diachronic change in languages by language contact.

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## REFERENCES

- [1] C. Lyon, C. Nehaniv, and A. Cangelosi, Eds., *Emergence of Communication and Language*. Springer, 2007.
- [2] A. Cangelosi and D. Parisi, Eds., *Simulating the Evolution of Language*. London: Springer, 2002.
- [3] E. J. Briscoe, "Grammatical acquisition and linguistic selection," in *Linguistic Evolution through Language Acquisition: Formal and Computational Models*, T. Briscoe, Ed. Cambridge University Press, 2002, ch. 9.
- [4] M. A. Nowak, N. L. Komarova, and P. Niyogi, "Evolution of universal grammar," *Science*, vol. 291, 2001, pp. 114–118.
- [5] D. Crystal, *Internet Linguistics: A Student Guide*, 1st ed. New York, NY, 10001: Routledge, 2011.
- [6] J. Arends, P. Muysken, and N. Smith, Eds., *Pidgins and Creoles*. Amsterdam: John Benjamins Publishing Co., 1994.
- [7] —, "Exposure dependent creolization in language dynamics equation," in *New Frontiers in Artificial Intelligence*, ser. Lecture Notes in Artificial Intelligence, A. Sakurai, K. Hasida, and K. Nitta, Eds., vol. 3609. Springer, 2006, pp. 295–304.
- [8] —, "Self-organization of creole community in spatial language dynamics," in *Proc. of 2nd IEEE International Conference on Self-Adaptive and Self-Organizing Systems (SASO2008)*, Venice, 2008, pp. 459–460.
- [9] —, "Prediction of creole emergence in spatial language dynamics," in *LATA 2009 (Proc. of 3rd International Conference on Language and Automata Theory and Applications)*, ser. Lecture Notes in Artificial Intelligence, A.H.Dediu, A.M.Ionescu, and C.Martin-Vide, Eds., vol. 5457. Tarragona: Springer, 2009, pp. 614–625.
- [10] —, "Self-organization of creole community in a scale-free network," in *Proc. of 3rd IEEE International Conference on Self-Adaptive and Self-Organizing Systems (SASO2009)*, San Francisco, 2009, pp. 293–294.
- [11] S. Kirby, "Learning, bottlenecks and the evolution of recursive syntax," in *Linguistic Evolution through Language Acquisition: Formal and Computational Models*, T. Briscoe, Ed. Cambridge University Press, 2002.
- [12] M. Delz, B. Layer, S. Schulz, and J. Wahle, "Overgeneralization of Verbs - the Change of the German Verb System," in *Proc. of EVOLANG9*, 2012, pp. 96–103.
- [13] —, "Multilayered formalisms for language contact," in *Proc. of WS on Constructive Approaches to Language Evolution*, Kyoto, 2012, pp. 145–147.
- [14] R. Matoba, H. Sudo, M. Nakamura, S. Hagiwara, and S. Tojo, "Process Acceleration in the Iterated Learning Model with String Clipping," *International Journal of Computer and Communication Engineering*, vol. 4, no. 2, 2014.
- [15] N. Chomsky, *Rules and Representations*. Oxford: Basil Blackwell, 1980.
- [16] D. Bickerton, *Language and Species*. University of Chicago Press, 1990.
- [17] N. Chomsky, *Knowledge of Language: Its Nature, Origin, and Use*. New York: Praeger, 1986.
- [18] J. R. Hurford, *Language and Number: the Emergence of a Cognitive System*. Oxford: Basil Blackwell, 1987.
- [19] S. Kirby, *Function, Selection, and Innateness: The Emergence of Language Universals*. Oxford University Press, 1999.
- [20] K. Smith and J. R. Hurford, "Language Evolution in Populations: Extending the Iterated Learning Model," in *Proc. of ECAL03*, 2003, pp. 507–516.
- [21] D. Jamet, "A Morphological Approach of Clipping in English. Can the Study of Clipping Be Formalized?" *Lexis*, no. 1, 2009, pp. 15–31.
- [22] A. Veisbergs, "Clipping in English and Latvian," *Poznan Studies in Contemporary Linguistics*, no. 35, 1999, pp. 153–163.
- [23] X. Castelló et al., "Modelling language competition: bilingualism and complex social networks," in *Proc. of EVOLANG7*, 2008, pp. 59–66.
- [24] T. Gong, L. Shuai, M. Tamariz, and G. Jäger, "Studying Language Change Using Price Equation and Pólya-urn Dynamics." *PLoS One*, vol. 7, no. 3:e33171, 2012.
- [25] D. Jurafsky and J. H. Martin, *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. Upper Saddle River, NJ, USA: Prentice Hall PTR, 2000.
- [26] A.-L. Barabasi and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, no. 5439, 1999, pp. 509–512.
- [27] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, 1998, pp. 440–442.