A Hybrid Model Applied to the Vehicles Routing Problem With Simultaneous Pickups and Deliveries – VRPSPD

Pedro Pablo Ballesteros Silva
Facultad de Tecnología
Universidad Tecnológica de Pereira - UTP
Pereira, Colombia
e-mail: ppbs@utp.edu.co

Diana Paola Ballesteros Riveros
Centro de Comercio y Servicio
Servicio Nacional de Aprendizaje - SENA
Pereira, Colombia
e-mail: dipballesteros@sena.edu.co

Yanci Viviana Castro Bermudez
Facultad de Ingeniería Industrial
Universidad Tecnológica de Pereira - UTP
Pereira, Colombia
e-mail: yvcastro@utp.edu.co

Abstract— **Since many decades ago, one of the topics of greatest interest in research is that one related with the vehicle routing problem, present in many organizations. This, which is a transport problem, has multiple implications of economic, social, technological, and environmental order, when there is a provision of services to customers in the development and implementation of production processes, in the provisioning and distribution of goods and services, including carrying people within a determined time frame, with an adequate quality level. This paper presents a methodology to solve the homogeneous vehicles routing problem with simultaneous pickups and deliveries (VRPSPD) using matheuristics formed by the specialized genetic algorithm's Chu-Beasley and exact techniques of mixed integer linear programming, based on the Branch-and-Bound procedure. The VRPSPD problem considers a set of customers, whose demands of pick-up and delivery of products or people are known, and whose objective is to get the set of routes of minimal cost, which permit to satisfy the demand of the customers, considering the respective constraints of the system and the vehicles necessary for the completion of the same. Two new algorithms designed by the authors are implemented, which have been coded in C ++, obtaining good results in relatively short computing times, depending on the characteristics of the computers used.**

Keywords-constructive heuristic; exact techniques; genetic algorithm's Chu-Beasley; matheuristics.

I. INTRODUCTION

The issue related with the vehicles routing problem (VRP) constitutes a set of possibilities, which range from simple situations to high complexity problems that are currently subject of important research. The solution of VRP in practical terms consists of the identification of a series of routes of a set of vehicles to provide service to customers in the best way possible, in the development and implementation of the distribution and provisioning processes.

In its structure, the problem is oriented to the search of a set of optimal solutions, or solutions obtained through heuristics, metaheuristics or matheuristics, which are affected by diverse constraints related to the quantity of vehicles, their capacity, destination and demand sites, pick-up and delivery times, duration of routes, use of multiple depots, fleets of vehicles, among others.

The tours established in all the solutions start and finish in a depot, and the loading of the vehicles used is made up of the merchandise to be delivered and the merchandise which is simultaneously picked up from each customer, and in the combinatorial optimization, where many variables are considered with a good amount of parameters and the majority of their versions are of the NP-Hard class, as in their solving process the polynomial time is not worked.

One of the first researches concerning the vehicles routing problem with simultaneous pick-up and delivery (VRPSPD) arises by the year 1989, with the work carried out by Min [15], who developed a heuristic of three stages for a case study of a distribution system for the Franklin county public library, Ohio, where an important saving in time and distance was achieved through the application of a mathematical model.

The following is a list of the most important variants that this problem has presented from 1989 to 2019:

- Routing problems of multiple vehicles with simultaneous pick-up and delivery [15].
- Pick-up and delivery applying time windows [6].
- Pick-up and delivery systems [18].
- Routing problems with simple and multiple vehicles with simultaneous pick-up and delivery [12].
- Routing problems of multiple vehicles with pick-ups fractioning [13].
- Pick-up and delivery applying time windows and waiting times [7].
- Routing problems of a single vehicle with selective pick-up and delivery [11].

 ISBN: 978-1-61208-780-1

- Routing problems of multiple vehicles with load fractioning for pick up and deliveries [17].
- Routing problems of a single vehicle with pick-up and delivery based on customer satisfaction [8].

Vehicle routing problem with load fractioning for pick up and deliveries, applying time windows [1].

Flexible pick-up and delivery, applying time windows [23]

- Vehicles routing problems with pick-up and delivery VRPPD applying transport routes VRPPDSR [14].

Multiproduct inventory routing problem with transfer option and green approach IRP [25].

VRP with pick-up and delivery and transfer problem PDPT [26].

VRP with pick-up and delivery, time windows and contamination TWPDPRP [27].

- Oil routing and programming problems with fractioned pick-up and delivery GPDP [28].

VRP with synchronized pick-up and delivery SPD [29].

- VRP with pick-up and delivery with full loads FTPDP [30].
- VRP with simultaneous pick-up and delivery with constrained time [31].

VRP with pick-up and delivery, time windows and multiple products stacks PDPTWMS [32].

VRP with simultaneous pick-up and delivery and bi-dimensional loading constrains VRP2LSPD [33].

- Traveling Salesman problem with multi-product pick-up and delivery m-PDTSP [34].
- VRP with pick-up and delivery, time Windows, benefits and reserve requests PDPTWPR [35].

Other authors like [20] make a proposal of a general scheme of classification of the problems of delivery and pick up with their corresponding characteristics. The classification presents three groups:

- The first group is formed by the graph "many to many problems", where any vertex can be used as a source or as a destination. Its structure is similar to that of the Vehicle Routing Problem with simultaneous pick-up and delivery VRPDPD.
- The second group includes the problems of "one to many to one". Its equivalent is the Mixed Vehicle Routing Problem (MVRP). It is inferred here that the customer requests exclusively one of the services.
- The third group is formed by the one-to-one routing problems where each product is considered as a request coming from an origin and having a defined destination. An example of this case is the messenger operations and the door-to-door transport service.

On the other hand, it should be kept in mind that the application of exact methods for the solution of VRPSPD has had difficulties concerning the use and consideration of many variables and restrictions [5] [24]. This situation is easily overcome with the use of matheuristics, getting good solutions, very close to optimal in relatively short computing times, depending on the characteristics of the computers used.

Many of the applications of VRPSPD are found in the different processes of inverse logistics, where the enterprises should carry out activities of inverse flux management both for finished products and raw materials. Examples of these operations are found in the beer or soft drink's bottling plants, when customers are visited to whom bottles full of the product are delivered and from whom empty bottles are picked up, guaranteeing the synchronization of deliveries and pick-ups, accomplishing the customer's satisfaction, and getting the optimal route with the minimal impact in costs in the supply chain.

Another application of the VRPSPD is observed in the transport of passengers when they are moved and picked up in different locations; in the home delivery service, where goods or documents are delivered and picked up, or where money is picked up; in data transmission and reception; in electric energy transmission and reception; in production systems when raw materials are delivered and finished or semi-finished products are picked up. It is important to consider that VRPSPD is also applied when new products are delivered and outdated or obsolete products are picked up, in order to give them an appropriate final deposition.

In practice, these processes are carried out mostly in empirical form, incurring in high transportation costs, in strong environmental impact, and in a questionable level of service to the final customer. Therefore, the efforts leading to the improvement and scientific solution of this situation are another of the contributions and objectives of this work.

In addition to the introduction, this article includes the following parts: 2. Description of the problem. 3. Formulation of the mathematical model. 4. Description of the implementation of the genetic algorithm of Chu-Beasley to solve the VRPSPD problem. 5. Experimental results of the matheuristics, where the genetic algorithm of Chu-Beasley and exact techniques for a depot, 50 clients and 4 vehicles are applied. 6. Analysis of sensibility of the genetic algorithm of Chu-Beasley, with variation of the size of the population. 7. Conclusions, with a final relationship of the consulted references.

II. DESCRIPTION OF THE PROBLEM

The Vehicle Routing Problem Pickup and Delivery (VRPPD) has been considered as an extension of the Vehicle Routing Problem -VRP), where each vehicle that visits each customer only once.

Once on the road, one must not only deliver, but also pickup certain products with a 100% service level (Full service). Vehicles of the project considers several scenarios: a depot, a vehicle and several clients; a depot, several vehicles and many clients; several depots, several vehicles and must exit and arrive at a depot or distribution center.

The scope many clients incorporating environmental effects in this last variant. In practical terms, it consists of identifying a series of routes for a set of vehicles to provide

 ISBN: 978-1-61208-780-1

delivery and pickup services to customers in the most appropriate way possible, in the development and execution of the supply and distribution processes.

The VRPSPD problem claims towards the search for a set of optimal solutions, or solutions obtained through heuristics (a procedure based on rules developed to determine a good quality solution to a specific problem), which are affected by different restrictions related to the number of vehicles, their capacity, places of destination and demand, delivery and pickup times, duration of the route, use of multiple depots, mixed vehicle fleet, among others.

The situations dealing with the pick-up and delivery of goods or people who must be transported between one origin and a destination, constitute a sort of vehicle routing problem, which must fulfill certain constrain of capability. Starting from the bibliographical revision made for this type of problem, it was found that there are three important groups, the description of which is presented below:

- Vehicle Routing Problem with Backhauls VRPBH. The situation outlined points out first to attend customers to whom merchandise is delivered and then from whom it is picked up [7].
- Mixed Vehicle Routing Problem MVRP. In this case, the customers require exclusively one of the services [15].
- Vehicle Routing Problem with Simultaneous Pickup and Delivery – VRPSPD. Here, the customers need the two services: deliver and pick up the merchandise or personnel, with a 100% service level, using one vehicle in one visit [2] [3] [24] [4].

In this work, we focus on this last group, where a deposit is considered: two sets, one of homogeneous capacity vehicles and another of customers whose delivery and pickup demands of merchandise or people must be attended simultaneously, with a 100% service level. The objective of the solution is to find the set of routes that guarantee the fulfillment of the restrictions shown below:

- The defined routes must start and end in the deposit.
- The requirements of all the customers must be satisfied at the 100% of the service level.
- Each customer can be visited just on time in the selected route.
- In each of the customers or nodes of the route, the total of the load transported by the vehicles should not exceed its capacity. That is, situations of no feasibility are not accepted.

Figure 1 shows the graphic configuration of the VRPSPD for three vehicles with their respective routes.

III. FORMULATION OF THE MATHEMATICAL MODEL

The feasibility of the problem VRPSPD depends on the sequence of the found route to visit the customers, and it is determined when upon verification of the customer's demand, this does not exceed the capacity of the vehicles.

Below is the description of the formulation of the mathematical model, proposed by Dell'Amico et al [4], applied by [21], with the following notation:

A = set of arcs which consist of the pairs *(i, j)* e *(j, i)* for each edge $\{i, j\}$ ε E_k.

G = (V_k, E_k) = complete graph with vertexes V= {0, 1, 2… *n*}, where the vertex 0 represents the depot and the rest corresponds to the customers. Each edge $\{i, j\}$ ε E_k has a non-negative cost and each customer *i* ε V'=V-{0} = {1, 2, 3….. *n*}.

d_i = amount of merchandise or product that has to be delivered to the customer *i*.

p_i = amount of merchandise or product that has to be picked up from customer *i*.

c_{ij} = matrix of travelling costs or distances, *i, j* ε V.

C = {1, 2… m} = set of *m* homogeneous vehicles with capacity *Q*.

E_k = subset of *Vk*Vk* that comprises all the possible arcs.

Decision variables:

$$x_{ij} \begin{cases} 1, \text{ if the k vehicle travels the arc } (i, j) \text{ ε } V \text{ of the selected route} \\ 0, \text{ in any other case.} \end{cases}$$

D_{ij} = amount of products or merchandise pending to be delivered, which is transported in the arc *(i, j)*.

Q = capacity of the homogeneous vehicles.

P_{ij} = amount of products or merchandise pending to be picked up, which is transported in the arc (*i, j)*

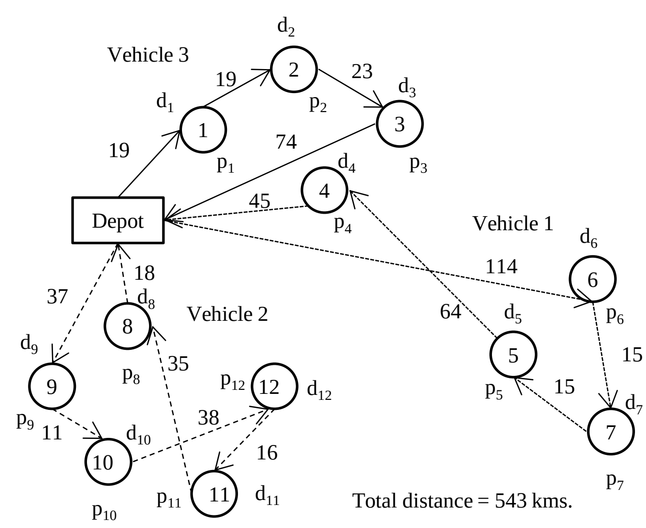

Figure 1. Graphic structure of the VRPSD for three vehicles.

The objective function and the constraints are presented below:

$$Z_{\min} = \min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \quad (1)$$

$$Z_{\min} = \min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \quad (2)$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V' \quad (3)$$

$$\sum_{j \in V'} x_{0j} \leq m \quad (4)$$

$$\sum_{j \in V} D_{ji} - \sum_{j \in V} D_{ij} = d_i \quad \forall i \in V' \quad (5)$$

$$\sum_{j \in V} P_{ij} - \sum_{j \in V} P_{ji} = p_i \quad \forall i \in V' \quad (6)$$

$$D_{ij} + P_{ij} \leq Q x_{ij} \quad \forall (i, j) \in A \quad (7)$$

$$D_{ij} \geq 0 \quad \forall (i, j) \in A \quad (8)$$

$$P_{ij} \geq 0 \quad \forall (i, j) \in A \quad (9)$$

$$x_{ij} \in \{0,1\} \quad \forall (i, j) \in A \quad (10)$$

$$d_j x_{ij} \leq D_{ij} \leq (Q - d_i) x_{ij} \quad \forall (i, j) \in A \quad (11)$$

$$p_i x_{ij} \leq P_{ij} \leq (Q - p_j) x_{ij} \quad \forall (i, j) \in A \quad (12)$$

$$D_{ij} + P_{ij} \leq (Q - \max\{0, p_j - d_j, d_i - p_i\}) x_{ij} \quad \forall (i, j) \in A \quad (13)$$

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j, i < j, \in V' \quad (14)$$

The objective function (1) minimizes the addition of the travelling costs or travelled distances in the selected route. With the constraint (2), there is a guarantee for each customer to be visited just once in the selected route. The constraint (3) makes each vehicle to leave each node or customer once in the route. With constraint (4) it is assured that, each vehicle is used once at the most. The expressions (5), (6) and (7) are constraints that guarantee the conservation of the flow of the delivered and picked up products in the established routes.

The nature of the variables of decision and the conditions of non-negativity are presented in the restrictions (8), (9), and (10).

If there is an attempt to get a stronger inequality for the non-negativity of the constraint (8), this can be substituted for inequality (11), as Gouveia holds in his work published [10], whose characteristic is the use of narrower limits.

Following the same strategy presented before of using stronger inequalities for P_{ij}, constraint (9) can be substituted for (12) and (7) for (13). With inequality (14), we get that each edge non-adjacent to the depot travels once as a maximum.

IV. Description of the Implementation of the Chu-Beasley Genetic Algorithm for the Solution of VRPSPD.

One of the reasons to apply the diverse techniques to the exact methods in the solution of NP-Hard problems, to which the VRPSPD belong, is that they require a lot of computation time as the size of the population or customers increases, since it is necessary to keep in mind a great number of variables and constraints. The application of metaheuristics like the Chu-Beasley genetic algorithm has generated good answers in relatively short computation times in comparison with the exact methods.

As shown in [36], the Chu-Beasley genetic algorithm has some features that make it more efficient as the following: It uses the objective function to identify the value of the best quality solution and considers the no feasibility in the process of substitution of one solution generated in the implementation of the algorithm; it just substitutes one individual at a time in each generational cycle; to avoid the premature convergence to optimal local solutions, each individual that joins the population has to be different to all those who make up the current population; it includes an aspiration criteria, though the new individual does not meet the requisite of controlled diversity; it incorporates an stage of improvement, after the recombination, that from certain intra-route and inter-route strategies there is an evaluation of a feasible solution before deciding if it makes part of the current population.

The implementation of the Chu-Beasley genetic algorithm includes the following stages:

A. *Construction of the Initial Population*

It can be constituted by two components: a first component is the configurations of the routes obtained from some constructive heuristics, and a second component is the configurations of the routes obtained I a controlled randomly way. For each configuration, there is an evaluation of the objective function (fitness) and the no feasibility associated to the load of each vehicle in the routes. The presentation of a configuration or solution of the VRPSPD for 20 nodes or customers and their codification is shown in Figure 2.

The length of the configuration or solution remains defined by the quantity of customers or nodes attended by the vehicles. It is important to note that the routes are determined by the capacities of the vehicles and by the

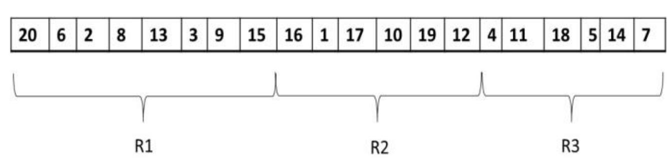

Figure 2 Genetic representation of a configuration of 20 nodes or customers.

quantities of the products that have to be delivered and picked up from each customer.

The input data are the matrix of costs or distances c_{ij}; the quantity of products to deliver d_i; the amount of products to pick up p_i; the amount of vehicles k with homogeneous capacity Q.

The parameters of the Chu-Beasley genetic algorithm: recombination rate = 0.80; mutation rate= 0.05.

In each configuration generated from the Chu-Beasley genetic algorithm, we get the value of the objective function (fitness), showing the feasibility or no feasibility of the route and the load of each vehicle assigned in the different subroutes or vehicles of the configuration. From the values of the objective function and the feasibility of the configurations or individuals, the genetic operators described in item *B* are used.

B. *Genetic Operators*

Three genetic operators are used in this work: Selection, recombination and mutation, whose procedures are detailed below.

1) *Selection*

In this stage, we use the method of selection by tournament. Two tournaments are carried out with the participation of all individuals of the current population *k* individuals are selected randomly, among whom the values of their objective functions and their no feasibilities are compared, selecting that who has the best value of the objective function if the *k* individuals are feasibility, or selecting that who has the least no feasibility if the *k* individuals are no feasibility. In the case of the presence of feasibility and no feasibility individuals, the feasibility individual with better objective function is selected. Two parents are left from these two tournaments, which pass on to the stage of recombination.

2) *Recombination*

This operator facilitates the exchange of information present in the two parents, and generates two descendants who own genetic material from parent 1 and parent 2. In this algorithm, which is elitist, just one of the children passes on to the current population. There are several techniques for the recombination, as: single-point crossover, two-point crossover, uniform crossover, partially mapped crossover (PMX), order-based crossover (OBX), Cycle crossover (CX), multi-parents crossover (MPX), longest common subsequence crossover (LCSX), among others. In this work, several forms of recombination are applied which explain the so-called PMX method, whose procedure is:

- Starting from the two parents P_1 and P_2 a common random segment is selected and copied in the child from P_1. From the first crossover point, elements *i* of this segment that have not been copied are sought in P_2. Para each element of these elements *i*, it is sought what elements *j* have been copied in its position from P_1.
- To place *i* in the taken position *j* from P_2, results in finding that said element is not there, since it has been copied before. If the place taken by *j* in P_2 has already been filled in the child *k*, *i* is placed in the position taken by *k* in P_2. The remaining elements are obtained from P_2, ending the process (See Figure 3).

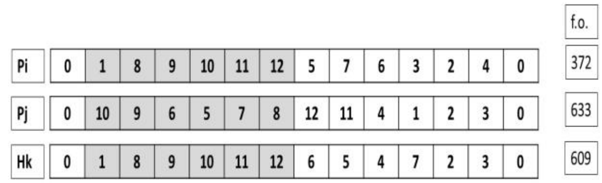

Figure 3 Application of the recombination PMX in a configuration of twelve customers. P_i, P_j are parents and H_k is the child resulting from the recombination.

Several recombination rates were used, 0.8 being the one with the best results.

3) *Mutation*

With this operator, some changes are made or parts of the resulting solution in the recombination are altered. This mechanism has the capacity to modify the current solution, applying small alterations. The mutation rate applied in this work is 0.05. There are several strategies that can help in this stage, like:

- Inter-routes local search through the shift (1, 0), shift (2, 0), shift (3, 0) strategy, with which 1, 2, 3 customers are transferred from one route to another.
- Inter-routes local search through the swaft (1, 1), swaft (2, 1), swap (2, 2), which facilitates the exchange of 1, 2 or 3 customers from one route to another.

Intra-routes local search. In this case, neighboring criteria from the customers are considered in order to make movements in the same route.

The strategies that can be applied are Rotation and 2-Opt. This is a local search algorithm proposed by Croes in 1958 to solve the problem of the travel agent. The idea is to consider a route that crosses over itself and reorder it to avoid such crossover.

In this work, several exchange strategies were used, and a swap (1, 1) application is described, for example, in order to exchange customer 3 of the route 3 for customer 4 of the route 1 of the next configuration, as it is shown in Figure 4.

As it can be observed, with the applied swap (1, 1) strategy in the mutation we find a configuration with an incumbent, which passes on to the process of improvement.

C. *Process of improvement of an individual*

After the selection, of the recombination and mutation, each configuration is subject to a stage of local improvement,

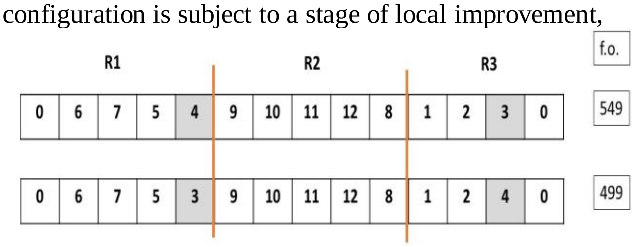

Figure 4 Swap application (1,1) for local inter-route search.

which consists of separating the routes individually and constructing sub-problems of a single route each one, and a deposit, with fewer customers that the complete problem, which are solved using the branch-and-cut exact technique, and the partial routes are constructed through optimal solutions. The resulting configuration passes on to the replacement stage.

D. Replacement stage

Here, there is a comparison of the resulting configurations of the previous stages with the individuals of the current population. A replacement is made for one of the individuals of the population, privileging the feasibility over the no feasibility and objective function when feasible solutions are compared.

V. EXPERIMENTAL RESULTS OF THE MATHEURISTICS WHERE THE CHU- BEASLEY GENETIC ALGORITHM AND EXACT TECHNIQUES ARE APPLIED FOR A DEPOT, 50 CUSTOMERS AND 4 VEHICLES.

In the literature associated to the VRPSPD three types of testing problems are known: Dethloff proposed 40 reference instances with 50 customers and the quantity of vehicles was 4,9 and 10 [5], Salhi and Nagy worked 14 instances, the quantity of customer was in the range of 50-199 and the vehicles used were 3,4,5,6,7 and 10 [19], and Montané and Galvão used 12 instances with 100-200 customers and the quantity of vehicles were 3,5,9, 10,12,16,23, and 28 [16]. The experimental tests were carried out with some of the instances proposed by Dethloff and the solutions with the best optimal limits have been published in the specialized literature [22] [9].

The applied methodology is, consequently, a hybrid of the Chu-Beasley genetic algorithm and mixed integer linear programming. The respective computing results are presented in Table I, using the Dethloff CON 3-8 instance. As can be observed in the Table I, out of 24 experiments, two configurations with values of the objective function of **537.63** and **537.36** were obtained, which compared with **523.05** that is the reported value by [20] as the optimal value of the instance, show good results in relatively short computing times (13.77 min and 14.45 min respectively) against the 32.05 minutes requires in [20]. Then, starting from the configuration obtained applying the Chu-Beasley genetic algorithm with the value of the objective function of 537, 36, the four routes travelled by the four vehicles are considered.

Each route is treated as a small linear programming problem, which may be solved easily with the corresponding mathematical model. In other words, we are applying the matheuristics, which supported by a good configuration produced by the Chu-Beasley genetic algorithm, with two constructive algorithms designed by the authors.

TABLE I. COMPUTATIONAL RESULTS IN THE IMPLEMENTATION OF THE CHU-BEASLEY GENETIC ALGORITHM USING THE CON3-8 INSTANCE OF DETHLOFF

Number of experiments	Population size	Last generation in which the best objective function was maintained	A	B	C(s)
1	200	9,546,568	539.28	539.22	826.58
2	200	169,195,900	631.47	634.56	6022.13
3	200	14,797,376	604.41	606.86	621.71
4	200	18,281,904	601.59	604.66	865.83
5	200	25,751,135	566.4	569.15	1296.69
6	200	3,875,938	618.01	620.88	272.66
7	200	21,615,848	606.92	608.98	750.13
8	200	13,634,095	538.78	543.38	612.28
9	200	6,847,952	543.45	545.59	413.04
10	200	2,136,193	690.19	692.52	196.78
11	200	1,557,888	601.54	604.33	101.57
12	200	1,795,437	610.41	614.69	107.84
13	200	139,029	676.67	701.18	51.40
14	200	7,860,846	612.69	613.77	216.72
15	200	6,991,854	546.82	550.75	411.33
16	200	594,774	605.87	609.91	134.55
17	200	16,078,477	644.01	646.70	523.78
18	200	41,765,529	593.76	597.38	1131.61
19	200	44,159,790	551.33	553.53	1378.16
20	200	12,396,611	584.14	586.64	803.85
21	200	17,381,286	537.36	541.40	866.87
22	200	19,271,996	602.20	604.85	851.25
23	200	105,346,307	567.76	579.51	4786.89
24	200	50,972,211	571.41	573.79	1836.32

A: Best value of the objective function in the respective generation,
B: Worst value of the objective function in the respective generation.
C: Computation time for the related number of generations (seconds).

The first algorithm generates the traveling matrices and the amount of the products to be delivered and picked up from each customer (node) of each of the routes. The second algorithm permits the control of the load for each route, data necessary for the mixed integer linear mathematical programming model to be applied. The resulting solutions in this case for the four routes are observed in Table II.

Next, the description of the matrix generator called array generation algorithm and clustering algorithm are presented.

A. Array generator algorithm

This algorithm is applied to the best configuration or the incumbent configuration obtained from the AGCB. Its description is:

- Start: the best configuration of the AGCB is taken.
- Considering the capacity of each vehicle, customers are assigned sequentially until their capacity is exhausted.
- The previous action is repeated for the rest of the vehicles until the last customer of the AGCB configuration.
- The configurations established for each vehicle are the basis for generating the distance matrices and the quantities to be delivered and collected in each sequence.
- Once the matrices for each vehicle have been generated, they become small problems to which the mathematical model encoded in GAMS is applied separately, obtaining the optimal solution for each vehicle.

• With the integration of the solutions for each vehicle, a new configuration.

B. Clustering algorithm

This algorithm is applied before using the Chu - Beasley AGCB genetic algorithm). It consists of the following phases:

Phase 1:

• Be based on known instances for multiple depots. If the instances are given in coordinates, the distance between customers and between them and the depot is calculated.
• The closest neighbor heuristic is applied, assigning the closest customers to each depot, regardless of their capacity.

Phase 2:

• With the previous result, the capacity of each depot is evaluated taking into account the assigned customers. That is, if depots are overloaded, the difference between the current depot and the other depots is calculated.

• Determine if there are reassignable depots. If the answer is affirmative, the depot with the shortest distance is chosen and it is analyzed whether said depot has the capacity to receive one or more clients.

• If the answer is affirmative, the client (s) are reassigned to said depot, considering the capacity of the depot.

• The previous action is repeated until all clients are assigned to depots without exceeding their capacity.

• Each depot remains with its respective clients, obtaining feasible assignments. Once the allocation for each depot is known with its respective clients, the AGCB is applied for each depot.

TABLE II. SOLUTIONS OBTAINED WITH MIXED INTEGER LINEAR PROGRAMMING.

Routes	Sequence of the routes	Solution	Best possible solution
Route vehicle 1	dep 25 27 42 36 37 24 4 9 2 43 50 31 29 39 dep	194.71	190.26
Route vehicle 2	dep 14 21 22 12 6 41 18 15 26 19 48 49 3 dep	128.74	116.52
Route vehicle 3	dep 35 45 16 5 10 30 1 13 17 32 44 28 33 7 46 20 47 8 11 dep	183.73	183.73
Route vehicle 4	dep 40 23 38 34 dep	31.30	31.30
	Total distance traveled	**538.48**	
	Total distance from Chu Beasley	**539.28**	
	Optimal solution according to Subramanian (20)	**523.05**	

VI. Analysis of sensibility of the Chu-Beasley genetic algorithm varying the size of population

The analysis of sensibility may be made for different scenarios. For example, it can be made for evaluating the performance of the algorithm varying the recombination rate, the mutation rate, modifying the techniques for selection of parents for the recombination, or varying the size of the population.

In this research for reasons of space in this paper, the analysis is done to evaluate the performance of the algorithm by varying the size of the population. In Table IV shows the results of several generations with their corresponding objective function (fitness) for three population sizes: 100, 200 and 300 configurations.

It can be observed both in Table III and in Figure 5 concerning the results of the evolutionary process in these tests or experiments that for the size of the population of 300 the value of the objective function differs significantly from the values for the other two sizes.

Between the size of 200 and 100 the incumbent (best solution accomplished in the process) is obtained for the size of 200, result which gets stable starting from 6.000.000 generations, placing itself with likelihood in a local optimal solution (537,36) which at any rate is very close to the optimal solution found by [20].

The implementation of the Chu-Beasley genetic algorithm was made in C++ from the configurations obtained with the best objective functions, each of the routes defined in the solution was considered as a sub-problem which was solved by the exact method, using the CPLEX Optimization Studio software, version 12.4, and was run in

TABLE III. ANALYSIS OF SENSIBILITY FOR DIFFERENT POPULATION SIZES IN MULTIPLE GENERATIONS IN THE IMPLEMENTATION OF THE CHU – BEASLEY ALGORITHM USING THE DETHLOFF CON 3-8 INSTANCE.

Population size			Generation of evolutive process	Objetive function . OF (100)	Objetive function . OF (200)	Objetive function . OF (300)
100	200	300	500,000	668,06	670,73	677,91
100	200	300	1,000,000	594,55	591,68	665,57
100	200	300	1,500,000	589,76	589,35	658,70
100	200	300	2,000,000	555,06	589,35	652,20
100	200	300	2,500,000	550,81	589,35	651,09
100	200	300	3,000,000	550,82	586,48	651,09
100	200	300	3,500,000	550,83	579,25	650,88
100	200	300	4,000,000	550,84	572,81	650,88
100	200	300	4,500,000	550,85	571,37	650,17
100	200	300	5,000,000	550,86	555,82	650,17
100	200	300	5,500,000	550,87	547,78	650,17
100	200	300	6,000,000	550,88	537,36	650,17
100	200	300	6,500,000	550,89	537,36	650,17
100	200	300	7,000,000	550,90	537,36	650,17
100	200	300	7,500,000	550,91	537,36	650,17
100	200	300	8,000,000	537,36	537,36	650,17
100	200	300	8,500,000	537,37	537,36	650,17
100	200	300	9,000,000	537,38	537,36	650,17

 ISBN: 978-1-61208-780-1

TABLE IV: CHARACTERISTICS OF THE COMPUTERS USE

Lenovo B40 Laptop	Dell Latitude E6500 Laptop	Lenovo Personal Computer
Intel processor core (TM) 1.70 GHz – 2.40 GHz x4.	Intel processor core (TM)2 Duo 2.80 GHz – 2.80 GHz.	Intel Processor Core (TM) 3.00 GHz – 3.00 GHz x4.
RAM memory: 4.00 GB	RAM memory: 4.00 GB	RAM memory: 8.00 GB
64 bits OS.	64 bits OS.	64 bits OS.

three computers with the following characteristics. (See Table IV).

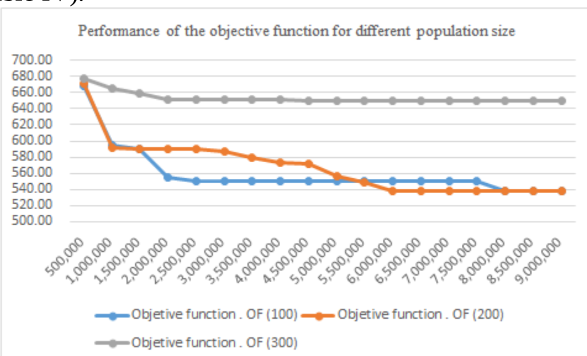

Figure 5 Performance of the objective function for different population size

VII. CONCLUSIONS

The proposed matheuristic and applied in this research, which combined the Chu - Beasley genetic algorithm and the exact technique based on the Branch and Cut algorithm, incorporated in the GAMS software, was a very good alternative to solve the VRPSPD for large sizes of clients or nodes with a single depot or multiple depots, where due to the nature of NP-Hard problems, mixed linear integer programming (MILP) does not solve them within an acceptable time frame. One of the critical stages in the implementation of the genetic algorithm of Chu & Beasley [20] was the synchronization of the parameters used.

With the two-phase methodology (genetic algorithm of Chu & Beasley [20] with the exact technique based on linear programming) and the incorporation of constructive heuristic algorithms, provided by the author for the generation of routes from the configuration with the incumbent of Chu & Beasley [20], the route matrices were obtained with the determination of the number of customers per route from the load control per vehicle. This methodology considerably facilitated the application of the matheuristic proposed in this research and allowed through certain sensitivity analyzes the evaluation of the performance of the algorithm used, being able to verify that its implementation was efficient and worked well.

It should be taken into account that the mathematical statistics do not guarantee the obtaining of an optimal global solution of the problems, but they did generate good solutions in very reasonable computation times, depending on the characteristics of the computer equipment used.

This research showed a classification of the VRPSPD, according to some of its variants and applied solution methods. As the complexity of the problem increases, the interest of researchers to apply metaheuristics or hybrid methods in its solution grows, including, as in this case, the proposed matheuristic.

The vast majority of the articles reviewed referred to real situations, which makes the application of solution techniques more interesting.

The trend of heuristics for the VRPSPD allowed the development of new algorithms with a good level of performance that, in general, required significant computation times in their processes.

The proposed matheuristic considered the costs of the routes (measured in distances) and their environmental impact (CO_2 generation), from which it was deduced that the amount of fuel used depends on the number of depots and vehicles used in the execution of deliveries and pickups.

The results achieved with the applied matheuristic can become a starting point for new research, since at present there are no known research that include the proposed matheuristic.

In its objectives, this research contributed to knowledge through the discovery of new contributions, also in the application of existing knowledge to new situations, or the connection of previously unrelated events in the specialized literature, and with it important contributions were made, which were described in this document.

REFERENCES

[1] P. Belfiorea and H. Yoshizaki. “Heuristic methods for the fleet size and mix vehicle routing problem with time windows and split deliveries”. *European Journal of Operational Research-ELSEVIER.* Part B 40. pp. 589-601, 2013.

[2] N. Bianchessi and G. Righini. “Heuristic algorithms for the vehicle routing problem with simultaneous pickup and delivery” [Journal] // *Computer and Operation Research Elsevier*, vol.36 (12), pp.3215-3223, 2007.

[3] E. Cao and M. Lai.2009. “An improved differential evolution algorithm for the vehicle routing problem with simultaneous pickups and deliveries and time windows”. *European Journal of Operational Research-ELSEVIER. Engineering Applications of Artificial Intelligence*, pp. 188-195, 2009.

[4] M. Dell’Amico, G. Righini, and M. Salani. “A branch-and-price approach to the vehicle routing problem with simultaneous distribution and collection”. *Transportation Science*, 40(2), pp. 235–247, 2006.

[5] J. Dethloff. “Vehicle routing problem and reverse logistics: the vehicle routing problem with simultaneous delivery and pick-up” [Journal]. - Springer Berlin: *Operation Research Spectrum*, vol. 23, pp.79 -96, 2001.

[6] Y. Dumas, J. Desrosiers and F. Soumis. “The pickup and delivery problem with time windows”. *European Journal of Operational Research-ELSEVIER.* Vol. 54, pp. 7-22, 1991.

 ISBN: 978-1-61208-780-1

[7] A. Fabri and P. Recht. "On dynamic pickup and delivery vehicle routing with several time windows and waiting times". *European Journal of Operational Research-ELSEVIER.* Part B 40. pp. 335–350, 2006.

[8] J. Fan. "The vehicle routing problem with simultaneous pickup and delivery based on customer satisfaction". *European Journal of Operational Research-ELSEVIER*, pp. 5284- 5289, 2011.

[9] Y. Gajpal and P. Abad. "An ant colony system (acs) for vehicle routing problem with simultaneous delivery and pickup". Computers & Operations Research, vol. 36 (12), pp. 3215–3223, 2009.

[10] L. Gouveia. "A result on projection for the vehicle routing problem". *European Journal of Operational Research*, pp. 610–624, 1995.

[11] I. Gribkovskaiaa, G. Laporte and A. Shyshou, "The single vehicle routing problem with deliveries and selective pickups". *European Journal of Operational Research-ELSEVIER.* Part B 40. pp. 2908-2924, 2008.

[12] I. Karaoglan, F. Altiparmak, I. Kara, I., & Dengiz, B. "A branch and cut algorithm for the location routing problem with simultaneous pickup and delivery. *European Journal of Operational Research-ELSEVIER.* pp. 318-332, 2011.

[13] C. G. Lee, M. A. Epelman, C. C. White III and Y. A. Bozer. "A shortest path approach to the multiple-vehicle routing problem with split pickups". *European Journal of Operational Research-ELSEVIER.* Part B 40. pp. 265-284, 2006.

[14] R. Masson, S. Ropke, F. Lehuédé and O. Péton. "A branch, cut, and price approach for the pickup and delivery problem with shuttle routes". *European Journal of Operational Research-ELSEVIER*, vol. 236, Issue 3, pp. 849-862, 2014.

[15] H. Min. "The multiple vehicle routing problem with simultaneous delivery and pick up points". *Transportation Research*. Vol. 23. No. 5, pp. 377-386, 1989.

[16] F.A.T Montané and R.D Galvao. "A tabu search algorithm for vehicle routing problem with simultaneous pickup and delivery services". *European Journal of Operational Research,* 33, 3 // *Computers and Operation Research Elsevier*, pp. 595 -619, 2006.

[17] M. Nowak, O. Ergun and C. White III Chelsea. "An empirical study on the benefit of split loads with the pickup and delivery problem". *European Journal of Operational Research-ELSEVIER.* Part B 40, pp. 734-740, 2009.

[18] S. N. Parragh, K. F. Doerner, R. F. Hartl. "A survey on pickup and delivery problems" Part II: Transportation between pickup and delivery locations. *Institut fur Betriebswirtschaftslehre*, Universitat Wien Brunnerstr. 72, 1210 Wien, Austria, 2008.

[19] S. Salhi and G. A. Nagy. "A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling" [Journal]. - [s.l.]: *Journal of the Operational Research Society*, Vol. 50, no. 10, pp. 1034 – 1042, 1999.

[20] A. Subramanian. (2012) "Heuristics exact and hybrid approaches for vehicle routing problems". Universidade Federal Fluminense. Tesis Doctoral. Niteroi. pp. 13, 17, 19.

[21] A. Subramanian, L. Satoru, E. Uchoa. "New Lower Bounds for the Vehicle Routing Problem with Simultaneous Pickup and Delivery". *9th International Symposium, SEA* Ischia Island, Napoles, Italy, may 20/22, pp. 276-287, 2010.

[22] A. Subramanian, L. M. A. Drummond, C. Bentes, L. S. Ochi, and R. Farias. "A parallel heuristic for the vehicle routing problem with simultaneous pickup and delivery". *Computers & Operations Research*, Vol. 37, Issue 11, pp. 1899-1911, 2010.

[23] H. Wang and Y. Chen. "A coevolutionary algorithm for the flexible delivery and pickup problem with time windows". *European Journal of Operational Research-ELSEVIER.* International Journal of Production Economics. pp. 4-13, 2013.

[24] E. Zachariadis, C. Tarantilis and C. Kiranoudis. "A hybrid methaheuristic algorithm for the vehicle routing problem with simultaneous delivery and pick - up service" [Journal]. - [s.l.]: *Expert System with Aplications*, Vol. 36, Issue 2, part 1, pp. 1070 -1081, 2009.

[25] E. Zachariadis, C. D. Tarantilis, and C.T. Kiranoudisb, "The vehicle routing problem with simultaneous pickups and deliveries and two dimensional loading constraints", *European Journal of Operational Research – ELSEVIER,* vol. 251, pp. 369-386, 2016.

[26] A. Subramanian, E. Uchoa, A. Alves Pessoa, and L. Satoru Ochi, "Branch and cut with lazy separation for the vehicle routing problem with simultaneous pickup and delivery". *European Journal of Operational Research-ELSEVIER*, vol. 39, Issue 5, pp. 338-341, 2011.

[27] Y. Li, H. Chen, and C. Prins, "Adaptive large neighborhood search for the pickup and delivery problem with time windows, profits, and reserved requests", *European Journal of Operational Research – ELSEVIER,* vol. 252, pp. 27-38, 2016.

[28] F. Hennig, B. Nygreena, K. C. Furmanb, and J. Song, "Alternative approaches to the crude oil tanker routing and scheduling problem with split pickup and split delivery", *European Journal of Operational Research – ELSEVIER,* vol. 243, pp.41-51, 2015.

[29] T. Gschwind, "A comparison of column generation approaches to the synchronized pickup and delivery problem", European Journal of Operational Research – ELSEVIER, vol. 247, pp. 60-71, 2015.

[30] M. Gendreaua, J. Nossackb, and E. Pesch, "Mathematical formulations for a 1- full truckload pickup and delivery problem". *European Journal of Operational Research – ELSEVIER*, vol. 242, pp. 1008-1016, 2015.

[31] O. Polata, C. B. Kalaycia, O. Kulaka, and H. Otto, "A perturbation based variable neighborhood search heuristic for solving the vehicle routing problem with simultaneous pickup and delivery with time limit", *European Journal of Operational Research - ELSEVIER*, vol. 242, pp. 369-382, 2015.

[32] M. Cherkesly, G. Desaulniers, S. Irnich, and G. Laporte, "Branch price and cut algorithms for the pickup and delivery problem with time windows and multiple stacks", *European Journal of Operational Research – ELSEVIER*, vol. 250, pp. 782-793, 2016.

[33] E. Zachariadis, C. D. Tarantilis, and C.T. Kiranoudisb, "The vehicle routing problem with simultaneous pickups and deliveries and two dimensional loading constraints", *European Journal of Operational Research – ELSEVIER,* vol. 251, pp. 369-386, 2016.

[34] H. Hernández, I. Rodríguez, and J. J. Salazar, "A hybrid heuristic approach for the multi-commodity pickup and delivery traveling salesman problem", *European Journal of*

Operational Research – ELSEVIER, vol. 251, pp. 44-52, 2016.

[35] Y. Li, H. Chen, and C. Prins, "Adaptive large neighborhood search for the pickup and delivery problem with time windows, profits, and reserved requests", *European Journal of Operational Research – ELSEVIER*, vol. 252, pp. 27-38, 2016.

[36] R. Gallego, E. Toro y A. Escobar, "Técnicas Heurísticas y Metaheurísticas", *Colección de trabajos de Investigación Editorial UTP*, pp.158-162, 2015.

 ISBN: 978-1-61208-780-1