

Formal Verification of Parameterized Multi-agent Systems Using Predicate Diagrams*

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Abstract—This paper presents a formal diagram-based verification technique for multi-agent systems. A multi-agent system is a collection of intelligent agents that interact with each others and work together to achieve a goal. We view multi-agent systems as parameterized systems which are systems that consist of several similar processes whose number is determined by an input parameter. The motivation of this work is that by treating multi-agent systems as parameterized systems, the specification and verification processes can be done in the same way regardless of the number of agents involved in the multi-agent systems. In this paper, we show how predicate diagrams* can be used to represent the abstractions of parameterized multi-agent systems described by specifications written in TLA*. The verification process is done by integrating deduction verification and algorithmic techniques. The correspondence between the original specification and the diagram is established by non-temporal proof obligations; whereas model checker SPIN is used to verify properties over finite-state abstractions.

Keywords-multi-agent systems; parameterized systems; verification; predicate diagrams*; TLA*; TLA+.

I. INTRODUCTION

A multi-agent system is understood as a collection of intelligent agents, in this case are software or programs that interact with each others and work together to achieve a goal. Sycara [22] said that "Agent-based systems technology has generated lots of excitement in recent years because of its promise as a new paradigm for conceptualizing, designing, and implementing software systems. This promise is particularly attractive for creating software that operates in environments that are distributed and open, such as the internet." Because of this promise, many researches on multi-agent systems have been conducted. Most of these researches concentrate on the specification and verification of the agents' behaviors and the coordination among agents.

In this work, we focus on multi-agent systems which consist of several similar processes. Having this property, a multi-agent system can be viewed as a parameterized system, which is a system that consists of several similar processes whose number is determined by an input parameter. Many interesting systems are of this form. One of them is mutual exclusion algorithms for an arbitrary number of processes wanting to use a common resource.

The motivation of this work is that by treating multi-agent systems as parameterized systems, the specification and verification processes can be done more easily. Both processes are expected to be done in the same way regardless of the number of agents involved in the multi-agent systems.

Verification consists of establishing whether a system satisfies some properties, that is, whether all possible behaviors of the system are included in the properties specified. It is common to classify the approaches to formal verification into two groups, which are the deductive approach and the algorithmic approach. The deductive approach is based on theorem proving and typically reduces the proof of a temporal property to a set of proofs of first-order verification conditions. The most popular algorithmic verification method is model checking [6], [7], [21]. Although this method is fully automatic for finite-state systems, it suffers from the so-called state-explosion problem.

The need for a more intuitive approach to verification leads to the use of diagram-based formalisms. Basically, a diagram is a graph whose nodes represent sets of system states and whose edges represent the transition of the systems. Diagram-based approach combines the advantages of deductive and algorithmic approach, which are the process is goal-directed, incremental and can handle infinite-state systems.

In the context of parameterized systems, to provide methods for the uniform verification of such systems is a challenging problem. One solution of this problem is to treat or to represent a family of objects as a single syntactic object. This technique is called parameterization.

In [20], a diagram-based verification for parameterized systems is proposed. The diagrams, which are called predicate diagrams*, are variants of diagrams proposed by Cansell et al. In [5], they presented a class of diagrams called predicate diagrams and showed how to use the diagrams in formal verification. In [20], a little modification of the definition of the original predicate diagrams is made, in particular the definition related to the actions. Instead of actions, the new approach concentrates only on parameterized actions which are actions of the form $A(k)$. This form of actions is usually used in modeling actions of a particular process in

the system. TLA* [17] is used to formalize this approach and TLA+ [13] style is used to write specifications.

This paper is structured as follows. Section II explains briefly the formal specification of parameterized systems in TLA*. Section III describes the definition and the use of predicate diagrams* in the verification of parameterized systems. The next section describes the application of this approach on a case study which is block world problem. Discussion about the result and some related works are given in Section V. Finally, conclusion and future work will be given in Section VI.

II. PARAMETERIZED SYSTEMS SPECIFICATION

This work is restricted to a class of parameterized systems that are interleaving and consist of a finitely, but arbitrarily, discrete components. Let M denote a finite and non-empty set of processes running in the system. A parameterized system can be described as a formula of the form:

$$parSpec \equiv \forall k \in M : Init(k) \wedge \square [Next(k)]_{v[k]} \wedge L(k).$$

where

- $Init$ is a state predicate describing the global initial condition,
- $Next(k)$ is an action characterizing the next-state relation of a process k ,
- v is a state function representing the variables of the system and
- $L(k)$ is a formula stating the liveness conditions expected from the process or subsystem k .

III. PREDICATE DIAGRAMS*

Basically, a predicate diagram* is a finite graph whose nodes are labeled with sets of (possibly negated) predicates, and whose edges are labeled with actions as well as optional annotations. This section gives a brief description of predicate diagrams*. For a detail explanation for predicate diagrams*, the readers may consult [20].

A. Definition

It is assumed that the underlying assertion language contains a finite set \mathcal{O} of binary relation symbols \prec that are interpreted by well-founded orderings. For $\prec \in \mathcal{O}$, its reflexive closure is denoted by \preceq . We write $\mathcal{O}^=$ to denote the set of relation symbols \prec and \preceq for $\prec \in \mathcal{O}$.

Definition 1. Assume given two finite sets \mathcal{P} and \mathcal{A} of state predicates and parameterized action names. A predicate diagram* $G = (N, I, \delta, o, \zeta)$ over \mathcal{P} and \mathcal{A} consists of :

- a finite set $N \subseteq 2^{\overline{\mathcal{P}}}$ of nodes where $\overline{\mathcal{P}}$ denotes the set of literals formed by the predicates in \mathcal{P} ,
- a finite set $I \subseteq N$ of initial nodes,
- a family of δ_A where $A \in \mathcal{A}$ of relations $\delta_{A \subseteq N \times N}$; we also denote by δ the union of the relations δ_A , for

$A \in \mathcal{A}$ and write $\delta^=$ to denote the reflexive closure of the union of these relations,

- an edge labeling o that associates a finite set $\{(t_1, 1), \dots, (t_k, k)\}$, of terms t_i paired with a relation $\prec_i \in \mathcal{O}^=$ with every edge $(n, m) \in \delta$, and
- a mapping $\zeta : \mathcal{A} \rightarrow \{NF, WF, SF\}$ that associates a fairness condition with every parameterized action in \mathcal{A} ; the possible values represent no fairness, weak fairness, and strong fairness.

A parameterized action $A \in \mathcal{A}$ can be taken at node $n \in N$ iff $(n, m) \in \delta_A$ holds for some $m \in N$. The set of nodes where A can be taken is denoted by $En(A)$.

A *run* of a predicate diagram* is a sequence of triples, $\rho = (s_0, n_0, A_0)(s_1, n_1, A_1) \dots$ where s_i is a state, $n_i \in N$ is a node and $A_i \in \mathcal{A} \cup \{\tau\}$ (τ denotes stuttering transition). A *trace* through a predicate diagram*, $\sigma = s_0 s_1 \dots$, is defined as the set of those behaviors that correspond to fair runs satisfying the node and edge labels.

B. Verification using predicate diagrams*

The verification process using predicate diagrams* is done in two steps. The first step is to find a predicate diagram* that conforms to the system specification. Theorem 1 is used to prove the conformance.

Theorem 1. Let $G = (N, I, \delta, o, \zeta)$ be a predicate diagram* over \mathcal{P} and \mathcal{A} , let $parSpec = Init \wedge \square[\exists k \in M : Next(k)]_v \wedge \forall k \in M : L(k)$ be a parameterized system. If all the following conditions hold then G conforms to $parSpec$:

- 1) For all $n \in I$, $\models Init \rightarrow n$.
- 2) $\models n \wedge [\exists k \in M : Next(k)]_v \rightarrow n' \vee \bigwedge_{(m, A(k)) : (n, m) \in \delta_{A(k)}} \langle \exists k \in M : A(k) \rangle_v \wedge m'$
- 3) For all $n, m \in N$ and all $(t, \prec) \in o(n, m)$:
 - a) $\models n \wedge m' \wedge \bigwedge_{A(k) : (n, m) \in \delta_{A(k)}} \langle \exists k \in M : A(k) \rangle_v \rightarrow t' \prec t$.
 - b) $\models n \wedge [\exists k \in M : Next(k)]_v \wedge n' \rightarrow t' \preceq t$.
- 4) For every parameterized action $A(k) \in \mathcal{A}$ such that $\zeta(A(k)) \neq NF$:
 - a) If $\zeta(A(k)) = WF$ then $\models parSpec \rightarrow WF_v(\exists k \in M : A(k))$.
 - b) If $\zeta(A(k)) = SF$ then $\models parSpec \rightarrow SF_v(\exists k \in M : A(k))$.
 - c) $\models n \rightarrow ENABLED \langle \exists k \in M : A(k) \rangle_v$ holds whenever $n \in En(A(k))$.
 - d) $\models n \wedge \langle \exists k \in M : A(k) \rangle_v \rightarrow m'$ holds for all $n, m \in N$ such that $(n, m) \notin \delta_{A(k)}$.

The second step of the verification process is to prove that all traces through a predicate diagram* satisfy some property F . In this step, a diagram predicate* is viewed as a finite transition system. As a finite transition system, its

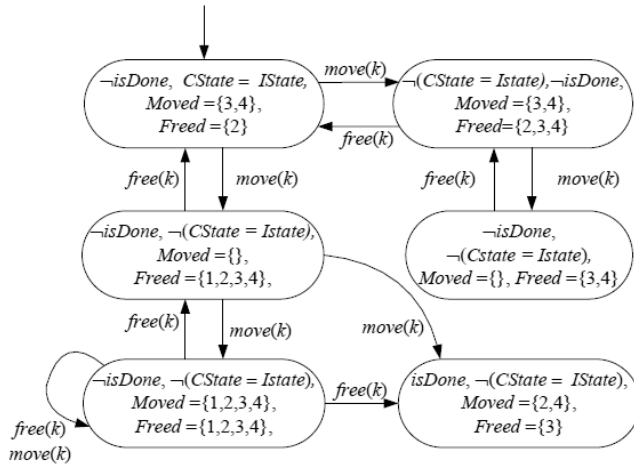


Figure 3. Predicate diagrams* for Block World Problem.

- $FState \equiv \langle \langle 0, 3 \rangle, \langle 0, 0 \rangle, \langle 1, 0 \rangle, \langle 0, 0 \rangle \rangle$

The verification can be stated as to prove the following theorem:

$$BWorld \rightarrow \Box(Init \rightarrow \Diamond isDone).$$

For the first step, we have to find a suitable predicate diagram* for the *BWorld*. Figure 3 depicts one of the possible predicate diagrams* for *BWorld*.

We set \mathcal{P} to contain six predicates. The union of those predicates define the properties of system's states that hold on every node. There are two predicates that are not written explicitly, which are $:k \in \{1, 2\}$ and $\forall k \in M : ag_type \in \{1, 2\}$. These predicates hold on every node. We also use two sets *Moved* and *Freed* which are sets of integers to indicate the blocks that can be moved or can be freed, respectively. It is assume that the following conditions hold:

- $\forall i \in Blocks : i \in Moved \leftrightarrow (\exists j \in Blocks : i \neq j \wedge CState[i] = \langle 0, 0 \rangle \wedge CState[j][2] = 0)$
- $\forall i \in Blocks : i \in Freed \leftrightarrow (\exists j \in Blocks : i \neq j \wedge CState[i][j, 0] \wedge CState[j][2] = i)$

Using Theorem 1 it can be shown that the predicate diagram* in Figure 3 conforms to the specification in Figure 2. This is done by proving the following formulas:

- $Init \rightarrow \neg isDone \wedge \neg(CState = IState) \wedge Moved = \{3, 4\} \wedge Freed = \{2\}$
- $\neg isDone \wedge \neg(CState = IState) \wedge Moved = \{3, 4\} \wedge Freed = \{2\} \wedge [\exists \in M : Next(k)]_v \rightarrow (\neg isDone' \wedge \neg(CState' = IState') \wedge Moved' = \{3, 4\} \wedge Freed' = \{2\}) \vee ((\exists k \in M : free(k))_v \wedge \neg isDone' \wedge \neg(CState' = IState') \wedge Moved' = \{3, 4\} \wedge Freed' = \{2, 3, 4\}) \vee ((\exists k \in M : free(k))_v \wedge \neg isDone' \wedge \neg(CState' = IState') \wedge Moved' = \{1, 2, 3, 4\})$
- $\neg isDone \wedge \neg(CState = IState) \wedge Moved = \{1, 2, 3, 4\} \wedge Freed = \{1, 2, 3, 4\} \wedge [\exists \in M : Next(k)]_v \rightarrow (\neg isDone' \wedge \neg(CState' = IState') \wedge Moved' = \{1, 2, 3, 4\}) \vee ((\exists k \in M : free(k))_v \wedge \neg isDone' \wedge \neg(CState' = IState') \wedge Moved' = \{2, 4\} \wedge Freed' = \{3\}) \vee \neg isDone \wedge \neg(CState = IState) \wedge Moved = \{1, 2, 3, 4\} \wedge Freed = \{1, 2, 3, 4\})$

- $((\exists k \in M : move(k))_v \wedge isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{2, 4\} \wedge Freed' = \{3\} \vee ((\exists k \in M : move(k))_v \wedge \neg isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{1, 2, 3, 4\} \wedge Freed' = \{1, 2, 3, 4\})$
- $\neg isDone \wedge \neg(CState = IState) \wedge Moved = \{1, 2, 3, 4\} \wedge Freed = \{1, 2, 3, 4\} \wedge [\exists \in M : Next(k)]_v \rightarrow (\neg isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{1, 2, 3, 4\} \wedge Freed' = \{1, 2, 3, 4\}) \vee ((\exists k \in M : free(k))_v \wedge (isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{1, 2, 3, 4\} \wedge Freed' = \{1, 2, 3, 4\}) \vee ((\exists k \in M : free(k))_v \wedge \neg isDone' \wedge (\neg(CState' = IState')) \wedge Moved' = \{1, 2, 3, 4\} \wedge Freed' = \{1, 2, 3, 4\}) \vee ((\exists k \in M : free(k))_v \wedge \neg isDone' \wedge (\neg(CState' = IState')) \wedge Moved' = \{2, 4\} \wedge Freed' = \{3\}) \vee \neg isDone \wedge \neg(CState = IState) \wedge Moved = \{1, 2, 3, 4\} \wedge Freed = \{1, 2, 3, 4\})$
- $(isDone \wedge \neg(CState = IState) \wedge Moved = \{2, 4\} \wedge Freed = \{3\}) \wedge [\exists \in M : Next(k)]_v \rightarrow (isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{2, 4\} \wedge Freed' = \{3\})$
- $(\neg isDone \wedge \neg(CState = IState) \wedge Moved = \{3, 4\} \wedge Freed = \{2, 3, 4\}) \wedge [\exists \in M : Next(k)]_v \rightarrow (\neg isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{3, 4\} \wedge Freed' = \{2, 3, 4\}) \vee ((\exists k \in M : free(k))_v \wedge (\neg isDone' \wedge CState' = IState' \wedge Moved' = \{3, 4\} \wedge Freed' = \{2\})) \vee ((\exists k \in M : move(k))_v \wedge \neg isDone' \wedge (\neg(CState' = IState')) \wedge Moved' = \{1, 2, 3, 4\} \wedge Freed' = \{3, 4\})$
- $(\neg isDone \wedge \neg(CState = IState) \wedge Moved = \{1, 2, 3, 4\} \wedge Freed = \{3, 4\}) \wedge [\exists \in M : Next(k)]_v \rightarrow (\neg isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{1, 2, 3, 4\} \wedge Freed' = \{3, 4\}) \vee ((\exists k \in M : free(k))_v \wedge (\neg isDone' \wedge \neg(CState' = IState')) \wedge Moved' = \{3, 4\} \wedge Freed' = \{2, 3, 4\}))$

The next step we encode the diagram in the input language of SPIN. We use 12 variables, which are:

- *action* and *node* to indicate the last action taken and the current node,
- *done* to indicate whether *isDone* is true or not,
- *cistate* to indicate whether $CState = IState$ is true or not,
- *m1, m2, m3*, and *m4* to represent predicate $1 \in Moved$, $2 \in Moved$, $3 \in Moved$ and $4 \in Moved$, respectively, and
- *f1, f2, f3*, and *f4* to represent predicate $1 \in Freed$, $2 \in Freed$, $3 \in Freed$ and $4 \in Freed$, respectively,

The theorem to be proven is now can be written as $\Box((\neg done \wedge cistate \wedge \neg m1 \wedge \neg m2 \wedge m3 \wedge m4 \wedge \neg f1 \wedge \neg f2 \wedge \neg f3 \wedge \neg f4) \rightarrow \Diamond(done \wedge \neg cistate \wedge \neg m1 \wedge m2 \wedge \neg m3 \wedge m4 \wedge \neg f1 \wedge \neg f2 \wedge f3 \wedge \neg f4))$. Last, by using SPIN we model-checked the resulted transition system. As result, we concluded that the specification satisfies the property we want to prove.

V. RELATED WORK

Many works are devoted to the formal specification of multi-agent systems. Most of these works concentrate on the specification of the agents' behaviors and the coordination among agents. Abouaissa et al. [1] presented a formal

approach for specification of multi-agent systems. This approach is based on roles and organization notions and high-level Petri nets and is applied on a multi-modal platform associating combined rail-road transportation system. Merayom et al. [16] proposed a formalism called Utility State Machines for specifying e-commerce multi agent systems. Brazier et al. [3] used the DESIRE framework to specify a real-world multi-agent application on a conceptual level. Originally DESIRE is designed for formal specification of complex reasoning systems. Fischer and Wooldridge [9] described first step towards the formal specification and verification of multi-agent systems, through the use of temporal belief logics. This work is closed to the one of Taibi [23]. The similarity is that we use TLA-based to formalize our approaches. However, Taibi's work did not treat the multi-agent systems as parameterized systems.

Besides formal specification, formal verification is also a popular topic in the field of multi-agent systems. Several approaches can be found in [2], [10], [11], [15], [23]. In [11], Giese et al. presented an approach for making complex multi-agent system specifications. Every specification includes a detailed environment model that amenable to verification. The verification process is done by means of simulation and formal verification. Taibi used TLC, the TLA model checker, to verify the specification [23]. Gaud et al. proposed a formal framework based on multi-formalisms language for writing system specification and used abstraction to reduce the state space of the system [10]. Ayed et al. proposed a diagram-based verification by using AUML protocol diagrams for representing multi-agent systems. These diagrams are then translated into event-B language for verification purpose [2]. In [15], Massaci et al. concerned about the use of access control for limiting the agent capability of distributed systems. They presented a prefixed tableau method for the calculus of access control. The calculus was the basis for the development and the verification of an implemented system.

Because diagrams can reflect the intuitive understanding of the systems and their specifications, they are proposed to be used for verification. A diagram can also be seen as an abstraction of the system, where properties of the diagram are guaranteed to hold for the systems as well. In particular, the use of diagrams in verification of distributed systems can be found; for example in [5] the author proposed the use of predicate diagrams, introduced in [4], for analyzing a self-stabilizing algorithm.

VI. CONCLUSION

We have shown how a multi-agent system can be viewed and thus can be formally specified and verified as a parameterized system. In particular, we define a general form for specification of parameterized multi-agent systems in TLA*. By considering a case study, we have shown that a multi-agent system whose agents are not homogenous still can be

specified as a parameterized system.

In this paper, we have successfully write specification and verify the block world problem. This problem is an example of multi-agent system whose agents are not homogeneous. In order to fulfill the homogeneity requirement, we add preconditions to actions in the specification to guarantee that only the appropriate agent may take a particular action. For verification process we use predicate diagram* to represent the abstractions of the systems. The correspondence between the original specification and the diagram is established by non-temporal proof obligations; whereas model checker SPIN is used to verify properties over finite-state abstractions.

In the context of parameterized systems, there are two classes of properties that may be considered, namely the properties related to the whole processes and the ones related to a single process in the system. It is planned to investigate those properties of the block world problem. The verification will be conducted by using a diagram-based verification called parameterized predicate diagrams [18], [19].

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