First Steps towards Automated Synthesis of Tableau Systems for Interval Temporal Logics

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***Abstract*—Interval temporal logics are difficult to deal with in many respects. In the last years, various meaningful fragments of Halpern and Shoham's modal logic of time intervals have been shown to be decidable with complexities that range from NP-complete to non-primitive recursive. However, even restricting the attention to finite interval structures, the step from model-theoretic decidability results to the actual implementations of tableau-based decision procedures is quite challenging. In this paper, we investigate the possibility of making use of automated tableau generators. More precisely, we exploit the generator METTEL^2 to implement a tableau-based decision procedure for the future fragment of the logic of temporal neighborhood over finite linear orders. We explore and contrast two alternative solutions: a *concrete* tableau system, that operates on a concrete interval structure explicitly built over a finite, linearly-ordered set of points, and an *abstract* one, that operates on an interval frame which is forced to be isomorphic to a concrete interval structure by suitably constraining its accessibility relation.**

Keywords–Interval temporal logics; satisfiability; tableau systems; automated tableau system generation.

I. INTRODUCTION

In this paper, we make some initial steps towards the automated synthesis of tableau systems for interval temporal logics. It is well-known that turning (optimal) declarative, tableau-based systems for decidable temporal logics into effective decision procedures is far from being trivial. Such a transition turns out to be particularly complex in the case of interval temporal logics. In the last years, it has been experimented for two specific logics, namely, the temporal logic of subintervals D, interpreted over dense linear orders [1], and the future fragment of the logic of temporal neighborhood A, interpreted over finite linear orders [2]. However, in both cases the proposed solution is tailored to the logic under consideration, and thus it lacks generality. In this paper, we explore the possibility of exploiting a general tool for the automated synthesis of tableau systems, namely, the generator METTEL^2, to deal with interval temporal logics. Even though we will apply the proposed solution to the logic A only (as Bresolin et al. did in [2]), there is no any limitation that prevents its application to other interval temporal logics.

Propositional interval temporal logics play a significant role in computer science, as they provide a natural framework for representing and reasoning about temporal properties in a number of application domains [3]. This is the case, for instance, of computational linguistics, where significant interval-based logical formalisms have been developed to represent and reason about tenses and temporal prepositions [4]. As another example, the possibility of encoding and reasoning about various constructs of imperative programming in interval temporal logic has been systematically explored by Moszkowski in [5]. Other meaningful applications of interval temporal logics can be found in knowledge representation, systems for temporal planning and maintenance, qualitative reasoning, theories of action and change, specification and design of hardware components, concurrent real-time processes, event modeling, and temporal databases. Modalities of interval temporal logics correspond to binary relations between time intervals. In particular, Halpern and Shoham's modal logic of time intervals HS [6] features one modality for each Allen interval relation [7]. In [6], the authors showed that HS is undecidable over all meaningful classes of linear orders. Since then, a lot of work has been devoted to the study of HS fragments, mainly to disclose their computational properties and relative expressiveness. The classification of HS fragments with respect to the status (decidable/undecidable) of their satisfiability problem is now almost completed. In this paper, we focus our attention on the class of finite linear orders, which comes into play in a variety of application domains, e.g., in planning problems. A complete classification of HS fragments over finite linear orders is given in [8]. It shows that there are 62 non-equivalent (with respect to expressiveness) decidable HS fragments, which can be partitioned into four complexity classes, ranging from NP-complete to non-primitive recursive. For each decidable fragment, an optimal, tableau-based decision procedure has been devised. However, since each of such procedures has been given a declarative formulation, no one of them is available as a working system, apart from the tableau-based decision procedure for the fragment A reported in [2]. The only attempt to apply a generic theorem prover to an interval temporal logic can be found in [1], where a tableau-based decision procedure for the fragment D, interpreted over dense linear orders, has been developed in LoTREC [9][10]. LoTREC is a generic prover for modal and description logics that can be used to prove validity and satisfiability of formulas. Whenever a formula is satisfiable, it returns a model for it; whenever a formula is not valid, it returns a counter-model for it. In LoTREC, a tableau is a special kind of labeled graph that is built, and possibly revised, according to a set of user-defined rules. Every node of the graph is labeled with

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a set of formulae and can be enriched by auxiliary markings, if needed. Unfortunately, LoTREC, as well as most generic theorem provers, cannot be exploited to deal with other interval temporal logics because (i) it does not support an explicit treatment of world labels, and (ii) it manages closing conditions based on loop checks, but it does not allow explicit checks on the number of worlds generated during the construction of a tentative model. Such limitations are overcome by the current version of METTEL^2 [11], which provides the user with a flexible language for specifying propositional syntaxes and tableau calculi.

In the following, we make use of METTEL^2 to implement a tableau-based decision procedure for A over finite linear orders. We explore and contrast two alternative solutions: a *concrete* tableau system, that operates on a concrete interval structure explicitly built over a finite, linearly-ordered set of points, and an *abstract* one, that operates on an interval frame which is forced to be isomorphic to a concrete interval structure by suitably constraining its accessibility relation (using the specification language provided by METTEL^2). The main contributions of the paper can be summarized as follows: *(i)* it can be viewed as the first general attempt of using an automated generator to synthesize a tableau system for an interval temporal logic(D over dense linear orders is a very special case because, due to its properties, it bears strong resemblance to standard modal logic); *(ii)* while METTEL^2 works perfectly on a variety of other logics (see, e.g., [12] and Section III), it required a small, but not trivial, change to make it possible to formulate closing conditions for A; *(iii)* the abstract version of the tableau system, based on a suitable representation theorem, gives new insigths into the role of temporal knowledge representation and reasoning technique, and representation theorems [7][13][14].

The paper is structured as follows. In the next section, we introduce the logic A. In Section III, we provide an necessary overview of the system METTEL^2. In Section IV, we describe the proposed A-prover. Section V given an account of the experimental results. Section VI concludes the paper.

II. The Interval Temporal Logic A

Given a linearly ordered set $\mathbb{D} = \langle D, < \rangle$, a (strict) *interval* is a pair $[a, b]$, with $a, b \in D$ and $a < b$. There are 12 different relations (excluding the identity) between two intervals on a linear order, often referred to as *Allen's relations* [7]: the six relations depicted in Fig. 1, namely $R_A, R_L, R_B, R_E, R_D, R_O$, and the inverse ones, defined in the standard way, that is, $R_{\overline{X}} = (R_X)^{-1}$, for each $X \in \{A, L, B, E, D, O\}$. Intuitively, an interval structure over a linear order $\mathbb{D}$ consists of the set of all intervals over $\mathbb{D}$, together with a set of Allen's relations. We treat interval structures as Kripke structures [15], where Allen's relations play the role of accessibility relations, and we associate a modality $\langle X \rangle$ with each Allen relation R_X. Given a modality $\langle X \rangle$ associated with the relation R_X, with $X \in \{A, L, B, E, D, O\}$, its *transpose* is the modality $\langle \overline{X} \rangle$, corresponding to the inverse relation $R_{\overline{X}}$.

Syntax and (Concrete) Semantics. Halpern and Shoham's logic HS [6] is a multi-modal logic with formulae built from a finite, non-empty set $\mathcal{AP}$ of atomic propositions, the propositional connectives $\vee$ and $\neg$, and the complete set of

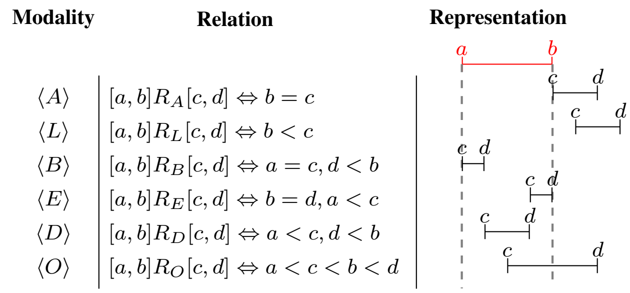

Figure 1. Allen's interval relations and the corresponding HS modalities.

modalities associated with all Allen's relations. With each subset $\{R_{X_1}, \ldots, R_{X_k}\}$ of this set of relations, we associate the fragment $\mathsf{X_1 X_2 \ldots X_k}$ of HS, whose formulae are defined by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle X_1 \rangle \varphi \mid \ldots \mid \langle X_k \rangle \varphi, \text{ with } p \in \mathcal{AP}.$$

The other propositional connectives and logical constants, e.g., $\wedge$, $\rightarrow$, and $\top$, can be derived in the standard way, as well as the dual modalities, e.g., $[A]\varphi \equiv \neg\langle A \rangle \neg\varphi$. In this paper, we focus our attention on the fragment A, whose formulae are generated by the following restricted grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle A \rangle \varphi, \text{ with } p \in \mathcal{AP}.$$

The concrete semantics of HS is given in terms of *concrete interval models*.

Definition 1: Let $\mathbb{D}$ be a linearly ordered set and $\mathbb{I}(\mathbb{D})$ be the set of all (strict) intervals over $\mathbb{D}$ (called *concrete interval structure*). A *concrete interval model* is a pair $M = \langle \mathbb{I}(\mathbb{D}), V \rangle$, where V is a *valuation function* $V : \mathcal{AP} \rightarrow 2^{\mathbb{I}(\mathbb{D})}$ that assigns to every atomic proposition $p \in \mathcal{AP}$ the set of intervals $V(p)$ on which p holds.

The *truth* of a formula is defined with respect to a concrete interval model M and an interval $[a, b]$ on it by structural induction on formulae as follows:

- $M, [a, b] \Vdash p$ iff $[a, b] \in V(p)$, for each $p \in \mathcal{AP}$;
- $M, [a, b] \Vdash \neg\psi$ iff it is not the case that $M, [a, b] \Vdash \psi$;
- $M, [a, b] \Vdash \varphi \vee \psi$ iff $M, [a, b] \Vdash \varphi$ or $M, [a, b] \Vdash \psi$;
- $M, [a, b] \Vdash \langle X \rangle \psi$ iff there is an interval $[c, d]$ such that $[a, b] R_X [c, d]$ and $M, [c, d] \Vdash \psi$, for each modality $\langle X \rangle$.

In the case of modality $\langle A \rangle$, the last semantic clause can be instantiated as follows:

$$M, [a, b] \Vdash \langle A \rangle \varphi \text{ iff there is } c > b \text{ such that } M, [b, c] \Vdash \varphi.$$

Formulae of HS can be interpreted in various interesting classes of concrete interval models, depending on the specific class of linear orders over which the models are built. As for the class of (concrete interval models built over) finite linear orders, the following small model theorem holds [16].

Theorem 1: Let φ be an A-formula. Then, φ is finitely satisfiable if and only if it is satisfiable on a model whose domain has cardinality strictly less than $2^m \cdot m + 1$, where m is the number of diamonds and boxes in φ.

The above result immediately provides a termination condition that can be used to implement a *fair* procedure that exhaustively searches for a model of size smaller than the bound.

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Abstract Semantics. As we already pointed out, METTEL^2 is flexible enough to allow one to provide an alternative, *abstract* version of the tableau system for A, based on a different, but equivalent, set of semantic conditions. To this end, we first define a suitable class of interval frames for A, called finite abstract interval A-structures, whose distinctive features are expressed by a set of first-order conditions, and then we show that any such frame is isomorphic to a concrete interval structure. It is worth noticing that such an abstract semantics, that takes intervals as first-class citizens, is quite common in the field of interval temporal logics, but not in those of modal and point-based temporal logics. In AI, the coexistence of concrete and abstract interval structures is well known since the early stages of interval-based temporal reasoning. The variety of binary relations between intervals in a linear order was first studied systematically by Allen, Hayes, and Ferguson [7][13][14], who explored their use in systems for time management and planning. The work by Allen and colleagues was based on the assumption that time can be represented as a dense line, and that points are excluded from the semantics. Both Allen and Hayes [17] and van Benthem [18] showed that interval temporal reasoning can be formalized as an extension of first-order logic with equality with one or more relations. As pointed out in [19], the characteristics of the proposed formalizations depend on basic choices about fundamental semantic parameters, such as the class of linear orders on which the interval structure is based (all dense linear orders, the rational numbers, etc.), and the set of interval relations added to the first-order language.

Given the dual nature of time intervals, that can be represented either as ordered pairs of time points over a linear order or as suitably-constrained, first-order individual objects, *representation theorems* have an important role in interval temporal logics. They can be described as follows (with respect to a specific class of linear orders). Given an extension of first-order logic with a set of interval relations, such as, for instance, $\{meets, during\}$, is there a set of axioms which constrain abstract models in this signature to be isomorphic to concrete ones? The problem can be alternatively stated as follows: can we define an isomorphism into concrete models whose domain is the set of intervals over the considered linear order and whose relations are the concrete interval relations? A number of representation theorems for interval logics can be found in the literature, including van Benthem [18], who considers the order of rational numbers and the interval relations *during* and *before*; Allen and Hayes [17], which refer to unbounded, dense linear orders, devoid of point intervals, and to the interval relation *meets* only; Ladkin [20], who takes into consideration point-based temporal structures with a 4-argument relation that encodes the interval relation *meets*; Venema [21], who considers dense linear orders with the interval relations *starts* and *finishes*; Goranko, Montanari, and Sciavicco [22], which deal with dense linear orders with the interval relations *meets* and *met-by*; and Coetzee [23], who refers to dense linear orders with the interval relations *overlaps* and *meets*.

In the present work, we focus our attention on the class of finite linear orders and the interval relation *meets* (denoted by R_A), and we provide a representation theorem that forces any finite, suitably-constrained Kripke frame $\langle W, R_A\rangle$ to be isomorphic to a finite, concrete interval structure. As a matter of fact, some frame conditions will be explicitly forced by introducing specific first-order constrains (this is the case with irreflexivity, antisymmetry, composition, and linearity); other ones will be embedded into the definition of the tableau rules (this is the case with finiteness and connectedness).

Definition 2: Let W be a finite nonempty set and let $R_A \subseteq W \times W$ be such that for all $x, y \in W$, $x = y$ or xR_Ay or $xR_{\overline{A}}y$ or xR_Ly or $xR_{\overline{L}}y$, and so on (connectedness)[1]. The pair $\mathfrak{S} = \langle W, R_A\rangle$ is a *finite and connected, abstract interval* A-*structure* if and only if the following conditions are satisfied:

1) $\forall x \neg(xR_Ax)$ (irreflexivity);
2) $\forall x, y(xR_Ay \wedge yR_Ax \rightarrow x = y)$ (antisymmetry);
3) $\forall x, y(xR_Ay \rightarrow \exists z(\forall t(tR_Az \leftrightarrow tR_Ax) \wedge \forall t(zR_At \leftrightarrow yR_At)))$ (composition);
4) $\forall x, y, z, t((xR_Ay \wedge yR_At \wedge xR_Az \wedge zR_At) \rightarrow y = z)$ (linearity).

The next representation theorem shows that the above conditions suffice to force any finite and connected, abstract interval A-structure to be isomorphic to a finite concrete one. For the sake of readability, we introduce the relation R_A as an additional component of concrete interval structures, that is, we substitute $S = \langle \mathbb{I}(\mathbb{D}), R_A\rangle$ for $\mathbb{I}(\mathbb{D})$, Proving that any finite, concrete interval structure satisfies conditions 1–4, as well as connectedness, is trivial; proving that any finite and connected, abstract interval A-structure is isomorphic to a finite, concrete interval structure is definitely more involved. Such a result is formally stated by the following theorem, whose proof is omitted for space reasons.

Theorem 2: Every finite and connected, abstract interval A-structure is isomorphic to a finite, concrete interval structure.

Thanks to Theorem 2, we can interpret the logic A on finite and connected, abstract interval A-structures. To this end, we adapt the notion of model for A by defining it as a pair $M = \langle \mathfrak{S}, V\rangle$, where $\mathfrak{S}$ is a finite and connected, abstract interval A-structure and $V : \mathcal{AP} \mapsto 2^W$. Moreover, we accordingly revise the semantic clause for $\langle A\rangle$ as follows:

$$M, i \Vdash \langle A\rangle\psi \text{ iff there is } j \text{ such that } iR_Aj \text{ and } M, j \Vdash \psi.$$

In the following, we will show that one actually needs to explicitly encode conditions 1–4 only. As for finiteness, it can be forced by imposing a suitable cardinality constraint, that is, by providing an interval counterpart (that applies to $\mathfrak{S}$) of the constraint coming from Theorem 1 (that applies to concrete models). As for connectedness, it is guaraanteed by construction: all generated world are directly or indirectly connected to the initial one (no incomparable world is ever introduced).

III. AUTOMATED SYNTHESIS OF TABLEAU CALCULI AND METTEL^2

Tableau reasoning methods are powerful tools to reason about logical formalisms. They have been extensively used to develop decision procedures for description and modal logics [24][25], as well as for intuitionistic logics, conditional logics, metric and topological logics, and hybrid logics.

[1] In [17], Allen and Hayes showed that all Allen's relations are first-order definable in terms of the interval relation R_A (*meets*) only. As a matter of fact, the proof assumes the temporal domain to be dense and unbounded; however, it can be shown that such an assumption is not necessary.

Schmidt and Tishkovsky [26] devise a method for automatically generating tableau calculi from a first-order specification of the formal semantics of a logic. The idea is that of turning such a specification into a set of inference rules giving rise to a sound, complete, and terminating deduction calculus for the logic, provided that the logic has the finite model property.

The tableau synthesis method works as follows [26]. The user defines the formal semantics of the given logic in a many-sorted first-order language so that certain well-definedness conditions hold. The semantic specification of the logic is then automatically reduced to Skolemised implicational forms, which are subsequently transformed into tableau inference rules. Combined with a set of default closure and equality rules, the generated rules provide a sound and complete calculus for the logic. Under certain conditions, the generated set of rules can be further refined [27]. If the logic has the finite model property, the generated calculus can be automatically turned into a terminating calculus by adding a suitable blocking mechanism.

The tableau prover generator METTEL^2 [11] has been implemented to complement the theoretical tableau synthesis framework given in [26]. METTEL^2 produces Java code of a tableau prover from specifications of a logical syntax and a tableau calculus for a given logic. It aims at providing an easy-to-use system for non-technical users and it allows technical users to improve/extend the implementation of generated provers. METTEL^2 has been successfully employed to produce tableau provers for modal logics, description logics, epistemic logics, and temporal logics with cardinality constraints. It is worth pointing out that prior implementations of systems for automated synthesis of tableau calculi already existed. Among them, we would like to mention LoTREC [9], [10] and The Tableau Work Bench (TWB) [28], which are the prover engineering platforms most closely related to METTEL^2. Although METTEL^2 does not give the user the same possibilities for programming and controlling derivations as these systems, its specification language is more expressive. As an example, Skolem terms are allowed both in premises and conclusions of rules. The expressive specification language also allows one to specify the syntax of arbitrary propositional logics and it makes METTEL^2 able to deal with the interval temporal logic A (which we focus on in this paper) and possibly with most of the other fragments of HS.

IV. TABLEAU PROVERS FOR A

In this section, we describe the specifications of two tableau provers for the logic A, which are based on the concrete and the abstract semantics, respectively.

The steps for obtaining the specifications are common to both provers. They can be summarized as follows. First, we apply the tableau synthesis framework [26] to the semantics of A. Since both concrete and abstract semantics for A consist of connective definitions and the background theory, the well-definedness conditions given in [26] are trivially fulfilled for both of them. Therefore, the generated calculi are automatically sound and (constructively) complete for the logic A. Next, we apply the atomic refinement [27] to the rules of the obtained calculi by moving negated atomic formulae in the rule conclusions to its premises while changing their signs. While retaining soundness and (constructive) completeness of the calculi, this reduces branching factor of the rules and makes tableau algorithms based on the calculi more efficient. Finally, we extend the tableau languages with additional constructs which replace the first-order predicates in the original calculi. This further simplifies the calculi, making them more readable and specifiable in METTEL^2.

The tableau specifications for the concrete and abstract semantics of A in METTEL^2 specification language are listed in Fig. 2. The symbol / separates premises of a rule from its conclusions and the symbol || separates branches of the rule. A priority value is assigned to each rule with the keyword $priority$. The less the value the more eagerly the rule is applied during derivation.

The tableau specification for the concrete semantics of A is based on two logical sorts: the sort of points and the sort of logical formulae. Disjunction $p \vee q$ is represented in the specification as p|q, negation $\neg p$ is represented as ~p, and <A> represents the modal operator $\langle A \rangle$. Constructs which extend the language of the logic are the ordering predicate $<$ on the sort of points ($a < b$ is represented as {a<b}), the equality predicate ({{a=b}} stands for $a = b$), a Skolem function f, to generate new terms of the sort of points, and expressions of the form $[a, b] : \varphi$, which are formulae φ of A labeled by intervals $[a, b]$, where a and b are points. The rules at lines 1–8 of the concrete tableau enforce $<$ to be a strict linear ordering. The rule at line 10 ensures that all the intervals are not degenerative. The remaining rules are standard rules for modal-like logics. It is worth pointing out that the rules at lines 1–8 and at line 15 are obtained by atomic refinement from the rules generated by the tableau synthesis framework. As an example, the rule [a,b]:~(<A>p) {b < c} / [b,c]:~p is obtained by the refinement from the generated rule [a,b]:~(<A>p) / ~{b < c} || [b,c]:~p. As a consequence of the results in [27], the calculus is sound and (constructively) complete for the standard interval semantics of the fragment A.

The tableau specification for the abstract semantics is also based on two sorts: the sort of intervals and the sort of logical formulae. The additional constructs are two Skolem functions f and g, the equality predicate, and a binary relational symbol R on the sort of intervals (for the sake of simplicity, we use R for R_A). The tableau operates on labeled formulae $@_i\varphi$ (@i p in the specification), where φ is a formula of A and i is an interval. The lines 1–7 of the abstract tableau define the theory of the relation R and correspond to the conditions 1–4 in Definition 2. While the rest of the rules are similar to standard rules for modal-like logics and they can be specified in tableau development platforms like LoTREC and TWB, the four rules listed at lines 3–6 are special. All the four rules make use of the same Skolem function g; moreover, the rules at lines 3 and 5 have the Skolem function g in their premises. Allowing specifications of tableau rules where Skolem functions occur in the rule premises is a distinctive feature of METTEL^2 prover generator, which demonstrate the expressiveness of the METTEL^2 specification language. Similarly to the case of the concrete tableau, the rules at lines 1–7 and at line 13 are obtained by atomic refinement. Therefore, the calculus is sound and (constructively) complete for the relational semantics of the fragment A.

Termination of both provers is achieved by a modification

```
{a < a} / priority 0;
{a < b} {b < c} / {a < c} priority 3;
{a < b} {c < d} /
      {{c = a}} || {c < a} || {a < c} {c < b} ||
      {{c = b}} || {b < c} priority 7;
{a < b} {c < d} /
      {{d = a}} || {d < a} || {a < d} {d < b} ||
      {{d = b}} || {b < d} priority 7;
[a,b]:p [a,b]:~p / priority 0;
[a,b]:p / {a < b} priority 1;
[a,b]:~(~p) / [a,b]:p priority 1;
[a,b]:(p|q) / [a,b]:p || [a,b]:q priority 5;
[a,b]:~(p|q) / [a,b]:~p [a,b]:~q priority 3;
[a,b]:<A>p / [b,f(b,p)]:p priority 9;
[a,b]:~(<A>p) {b < c} / [b,c]:~p priority 4;
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```
R i i / priority 0;
R i j R j i / priority 0;
R i j R k g(i,j) / R k i priority 4;
R i j R k i / R k g(i,j) priority 10;
R i j R g(i,j) k / R j k priority 4;
R i j R j k / R g(i,j) k priority 10;
R i j R j k R i l R l k / {{ j = l }} priority 6;
@i p @i ~p / priority 0;
@i ~(~p) / @i p priority 1;
@i (p|q) / @i p || @i q priority 5;
@i ~(p|q) / @i ~p @i ~q priority 3;
@i <A>p / R i f(i,p) @f(i,p) p priority 9;
@i ~(<A>p) R i j / @j ~p priority 4;
```

Figure 2. Tableau specifications for concrete (left) and abstract (right) semantics.

to the generated Java code to ignore branches which exceed the allowed limit of points or intervals (see Theorem 1).

V. Testing and Results

We have tested our implementations against the same benchmark of problems used in [2], although the absolute speed results cannot be immediately compared since the two experiments used a different hardware. These problems are divided into two classes. First, we tested the scalability of the implementation with respect to a set of combinatorial problems of increasing complexity (COMBINATORICS), where the n-th combinatorial problem is defined as the problem of finding a model for a formula that contains n conjuncts, each one of the form $\langle A\rangle p_i$ $(0 \leq i \leq n)$, plus $\frac{n(n+1)}{2}$ conjuncts of the form $[A]\neg(p_i \wedge p_j)$, with $i \neq j$. (Notice that there are $n(n+1)$ different conjuncts of the pointed out form. However, a conjunct with indices i, j is equivalent to another one with indices j, i. This is why $\frac{n(n+1)}{2}$ is posed.) Then, we considered the set of 72 purely randomized formulas used in [29] to evaluate an evolutionary algorithm for the same fragment (RANDOMIZED). Table I summarizes the outcomes of the experiments. For each class of problems, the corresponding table shows, for each instance n, the time (in milliseconds) necessary to solve the problem taking into account, when appropriate, the specific policy that has been used. In particular, the concrete version has been run under both the 'breadth first' and the 'depth first' (left branch first) policies. A time-out of 1 minute was used to stop instances running for too long.

At first sight, the relational (abstract) version of the tableau system looks more (time) efficient than the standard (concrete) one. However, the number of instances that generated a memory error indicates that the latter uses less memory, which can be considered an interesting result on its own. All the experiments were executed on Java 1.7.0_25 OpenJDK 64-Bit Server VM under the Java heap size limit of 3Gb on a hardware based on Intel® Core™ i7-880 CPU (3.07GHz, 8Mb), with a total memory of 8Gb (1333MHz), under the 64-bit Fedora Linux 17 operating system.

VI. Conclusions and Future Work

In this paper, we illustrated the outcomes of a first experiment in automated generation of tableau-based decision procedures for interval temporal logics using the automatic prover generator MetTeL^2. Thanks to its expressive power and flexibility, we explored and contrasted two alternative implementations: a concrete and an abstract one (at the best of our knowledge, this is the first tableau-based decision procedure for interval temporal logics based on an abstract frame semantics). Even though the performance of the developed systems is not particularly exciting, the use of generators like MetTeL^2 provides a general and effective way of implementing tableau systems for interval temporal logics. We believe it possible to make the concrete tableau system more efficient, provided that we represent the linear order by a list of points. This would remedy the exponential blow-up of inequality formulae in the tableau derivation, but, unfortunately, lists cannot be represented in the language of MetTeL^2 yet. The addition of such a feature to MetTeL^2 and the analysis of its actual impact are left for future work. As for the abstract tableau system, in principle, it allows us to compare alternative, but equivalent, formulations of the first-order constraints for a given fragment. Last but not least, we are going to validate the proposed approach on other, more expressive HS fragments.

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Table I. EXPERIMENTAL RESULTS (IN MILLISECONDS; '-': "OUT OF TIME"; 'M': "OUT OF MEMORY"; 'Y': "SATISFIABLE"; 'N': "UNSATISFIABLE").

COMBINATORICS

	CON		ABS	
n	DF	BF		sat
1	10	10	0	y
2	60	100	0	y
3	270	420	10	y
4	920	1360	30	y
5	2930	4010	70	y
6	7890	9850	150	y
7	19420	23670	300	y
8	47220	51220	560	y
9	-	-	1000	y
10	-	-	1790	y
11	-	-	3440	y
12	-	-	4660	y
13	-	-	7600	y
14	-	-	11560	y
15	-	-	17170	y
16	-	-	25160	y
17	-	-	35610	y
18	-	-	50740	y
19	-	-	-	-
20	-	-	-	-

RANDOMIZED

	CON		ABS	
n	DF	BF		sat
1	-	-	-	-
2	0	0	0	y
3	10	0	0	y
4	0	10	0	y
5	-	-	-	-
6	0	10	0	y
7	-	-	-	-
8	10	10	0	y
9	20	20	10	y
10	10	10	0	y
11	-	-	-	-
12	10	10	0	y
13	10	10	0	y
14	10	10	0	y
15	-	-	-	-
16	10	20	0	y
17	30	50	10	y
18	-	-	-	-
19	30	50	0	y
20	-	-	-	-
21	20	50	10	y
22	-	-	-	-
23	-	-	-	-
24	20	20	0	y
25	-	-	-	-
26	-	-	-	-
27	-	-	-	-
28	-	-	-	-
29	-	-	-	-
30	-	-	-	-
31	10	10	10	n
32	-	-	-	-
33	-	-	M	-
34	60	70	10	y
35	-	-	-	-
36	-	-	-	-
37	-	-	M	-
38	-	-	M	-
39	-	-	M	-
40	-	-	M	-
41	-	-	-	-
42	-	-	-	-
43	-	-	-	-
44	-	-	-	-
45	-	-	M	-
46	-	-	-	-
47	-	-	-	-
48	-	-	-	-
49	-	-	-	-
50	-	-	M	-
51	-	-	M	-
52	-	-	-	-
53	-	-	M	-
54	-	-	-	-
55	-	-	M	-
56	-	-	M	-
57	-	-	M	-
58	-	-	-	-
59	-	-	M	-
60	-	-	M	-
61	-	M	M	-
62	-	-	-	-
63	M	-	-	-
64	-	-	-	-
65	-	-	M	-
66	-	-	-	-
67	M	-	-	-
68	-	-	-	-
69	M	-	-	-
70	-	-	M	-
71	M	M	M	-
72	-	-	-	-

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