A Robust Periodogram-based IFO Estimation Scheme for OFDM-based Wireless Systems

Seung Goo Kang, Dahae Chong, Youngyoon Lee, and Seokho Yoon†
School of Information and Communication Engineering, Sungkyunkwan University, Suwon, Korea
Email: {general85, lvjs1019, news8876, and †syoon}@skku.edu
†Corresponding author

***Abstract*—In this paper, we propose a new periodogram-based integer frequency offset (IFO) estimation scheme robust to the fractional frequency offset (FFO) variation. We first observe the reason why the conventional IFO estimation scheme in [10] is sensitive to the variation of the FFO, and then, propose a new IFO estimation scheme using the modified maximum-likelihood (ML) metric. The numerical results demonstrate that the proposed scheme is more robust to the variation of the FFO and has better IFO estimation performance than the conventional scheme in [10].**

***Keywords*-estimation; frequency offset; OFDM; training symbol**

I. Introduction

Due to its immunity to multipath fading and high spectral efficiency, orthogonal frequency division multiplexing (OFDM) has been adopted as a modulation format in a wide variety of wireless systems such as digital video broadcasting-terrestrial (DVB-T), wireless local area network (WLAN), and worldwide interoperability for microwave access (WiMAX) [1]-[4]. However, the OFDM is very sensitive to the frequency offset caused by Doppler shift or oscillator instabilities, and thus, the frequency offset estimation is one of the most important technical issues in OFDM-based wireless systems [5], [6].

Various schemes [7]-[9] on the frequency offset estimation have been proposed so far. Schmidl and Cox (SC) proposed a frequency offset estimation scheme using a training symbol with two identical halves [7], whose estimation range is equal to the sub-carrier spacing. In [8], a new frequency offset estimation scheme that utilizes a training symbol with more than two identical parts was proposed, increasing the estimation range twice that of the SC scheme. However, the optimality for the estimation accuracy was not considered in the scheme in [8]. With the maximum-likelihood (ML) criterion, in [9], the optimal scheme for frequency offset estimation was derived using the same training symbol as in [8]. The scheme in [9] offers high estimation accuracy with the same estimation range as in the scheme in [8]. However, these schemes require a special training symbol structure, thus decreasing the transmission efficiency.

Recently, in [10], a periodogram-based frequency offset estimation scheme was proposed, which has the estimation range as large as the bandwidth of the OFDM symbol while maintaining the same performance as those of the schemes based on training symbols. However, its estimation performance for the integer part of the frequency offset (normalized to the sub-carrier spacing) rapidly changes according to the value of the fractional part of the frequency offset, eventually resulting in a significant variation in the overall frequency offset estimation performance.

Thus, in this paper, we propose a new integer frequency offset (IFO) estimation scheme robust to the fractional frequency offset (FFO) variation. We first investigate the influence of the FFO on the IFO estimation scheme in [10], and then, propose a modified ML IFO estimation scheme. The numerical results show that the proposed IFO estimation scheme is more robust to the variation of the FFO and has better performance than the IFO estimation scheme in [10].

II. Signal Model

The nth transmitted complex-valued OFDM sample $x(n)$ is generated by using the inverse fast Fourier transform (IFFT), and thus, can be expressed as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad n = 0, 1, \cdots, N-1, \tag{1}$$

where N is the size of the IFFT and X_k is a phase shift keying (PSK) or a quadrature amplitude modulation (QAM) symbol in the kth sub-carrier. The data part of the OFDM symbol has a duration of T seconds, and the cyclic prefix (CP), whose length is generally designed to be longer than the channel impulse response, is inserted in order to avoid the intersymbol interference (ISI).

The nth received OFDM sample $r(n)$ is obtained by sampling the received OFDM signal every $T_s = T/N$ seconds and is expressed as

$$r(n) = s(n)e^{j2\pi(\varepsilon_I + \varepsilon_F)n/N} + w(n), \tag{2}$$

where $s(n) = \sum_{k=0}^{L-1} h_k x(n-k)$ is the signal component with the kth channel filter tap coefficient h_k and the channel memory size L, ε_I and ε_F represent the IFO and FFO normalized to the sub-carrier spacing $1/T$, respectively, and $w(n)$ is the complex-valued additive white Gaussian

 ISBN: 978-1-61208-126-7

noise (AWGN) sample with mean zero and variance $\sigma_w^2 = \mathbf{E}\{|w(n)|^2\}$, where $\mathbf{E}\{\cdot\}$ and $|\cdot|$ denote the expectation and absolute value operators, respectively. In this paper, we assume that the channel is static during one OFDM signal period and timing synchronization is perfect.

III. Proposed Scheme

A. Influence of the FFO on the IFO estimation

In [10], the estimates $\hat{\varepsilon}_I$ and $\hat{\varepsilon}_F$ of the IFO and FFO are obtained as

$$\hat{\varepsilon}_I = \arg\max_{f_k} \left\{ I(f_k) + I(f_k+1) \right\} \tag{3}$$

and

$$\hat{\varepsilon}_F = \frac{\sqrt{I(\hat{\varepsilon}_I+1)}}{\sqrt{I(\hat{\varepsilon}_I)} + \sqrt{I(\hat{\varepsilon}_I+1)}}, \tag{4}$$

respectively, where 'arg' is the argument operation and $I(f_k)$ is the signal periodogram defined as

$$I(f_k) = \left| \sum_{n=0}^{N-1} r(n)e(n)e^{-j2\pi f_k n/N} \right|^2, \tag{5}$$

where $f_k \in \{-\frac{N}{2}, -\frac{N}{2}+1, \cdots, \frac{N}{2}-1\}$ is the kth IFO candidate and $e(n) = \frac{x(n)^*}{\|x(n)\|^2}$ with the complex conjugate '$*$' and Euclidean norm $\|\cdot\|$ is the envelope equalized processing factor employed to remove the data modulation effect.

In the absence of the noise, $\hat{\varepsilon}_F$ is given by $\frac{Z(\hat{\varepsilon}_I)}{Z(\hat{\varepsilon}_I)+Z(\hat{\varepsilon}_I+1)}$, where $Z(\alpha) = |\sin(\pi(\varepsilon-\alpha)/N)|$, and is drawn as a function of $\varepsilon - \hat{\varepsilon}_I$ as shown in Fig. 1, where $\varepsilon = \varepsilon_I + \varepsilon_F$ is the real frequency offset. It is seen from the Fig. 1 that the FFO can be correctly estimated only when $0 \le \varepsilon - \hat{\varepsilon}_I < 1$, that is, when $\hat{\varepsilon}_I \in (\varepsilon - 1, \varepsilon]$.

Fig. 2 shows the IFO metric $\{I(f) + I(f+1)\}$ normalized to $N^2\|h_0\|^2$ as a function of the frequency $f \in [-N/2, N/2)$ for $\varepsilon_F = 0.4$ and 0.8 when $\varepsilon_I = 1$, $N = 8$, and the noise is absent, where '$\circ$' represents the IFO metric value corresponding to each f_k and the shaded region represents the range of $\hat{\varepsilon}_I \in (\varepsilon - 1, \varepsilon]$ for a correct estimation of the FFO. In this paper, the correct estimation probability of the IFO is defined as the probability that the maximum IFO metric corresponds to f_k within the shaded region. From the Fig. 2, we can clearly see that the correct estimation probability of the IFO would be very sensitive to the variation of the FFO, since the ratio of the IFO metric value corresponding to f_k within the shaded region to the largest one among the IFO metric values corresponding to f_ks' outside the shaded region rapidly changes according to the value of the FFO: specifically, the ratio when $\varepsilon_F = 0.4$ is larger than that when $\varepsilon_F = 0.8$, and thus, the correct estimation probability of the IFO would be higher when $\varepsilon_F = 0.4$ than when $\varepsilon_F = 0.8$.

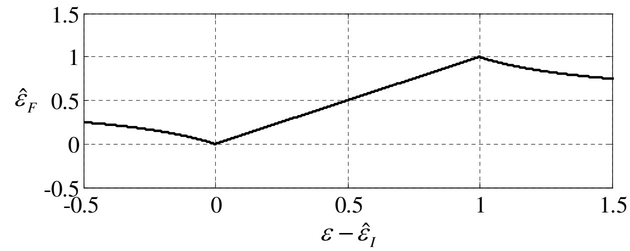
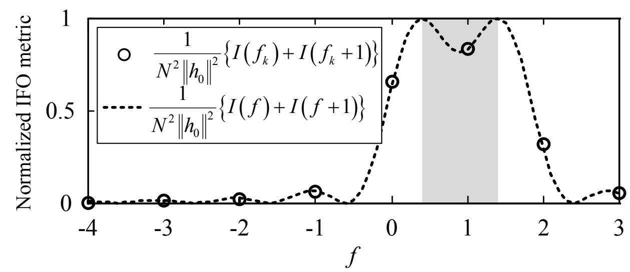
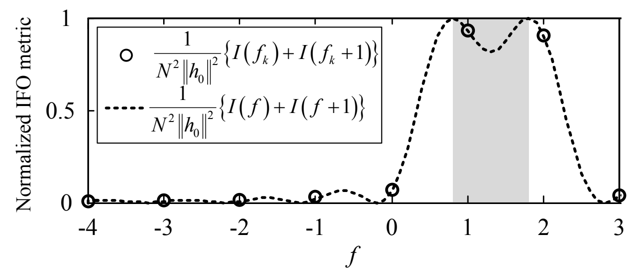


Figure 1. $\hat{\varepsilon}_F$ as a function of $\varepsilon - \hat{\varepsilon}_I$ in [10].



(a) When $\varepsilon_F = 0.4$



(b) When $\varepsilon_F = 0.8$

Figure 2. IFO metric $\{I(f) + I(f+1)\}$ normalized to $N^2\|h_0\|^2$ as a function of the frequency $f \in [-N/2, N/2)$ for $\varepsilon_F = 0.4$ and 0.8 when $\varepsilon_I = 1$, $N = 8$, and the noise is absent.

B. Proposed IFO Estimation Scheme

From the discussions in the previous section, we can see that the periodogram-based IFO estimation problem can be modeled as a detection problem of a single tone with the maximum energy. Then, the IFO estimate can be obtained as

$$\hat{\varepsilon}_I = \arg\max_{f_k} I(f_k), \tag{6}$$

which is in fact the ML solution for detecting a single tone with the maximum energy [11] and its normalized metric value is shown as a function of f in Fig. 3, where it is clearly observed that the correct estimation probability of the IFO would be still very sensitive to the FFO variation as that corresponding to (3), since (6) is an ML solution that does not take the FFO into account. Also, we can observe that the shaded region is always equal to the left slope of the normalized IFO metric graph in Fig. 3, and thus, through the some shift operation, the normalized IFO metric value in the shaded region can be increased regardless of the value

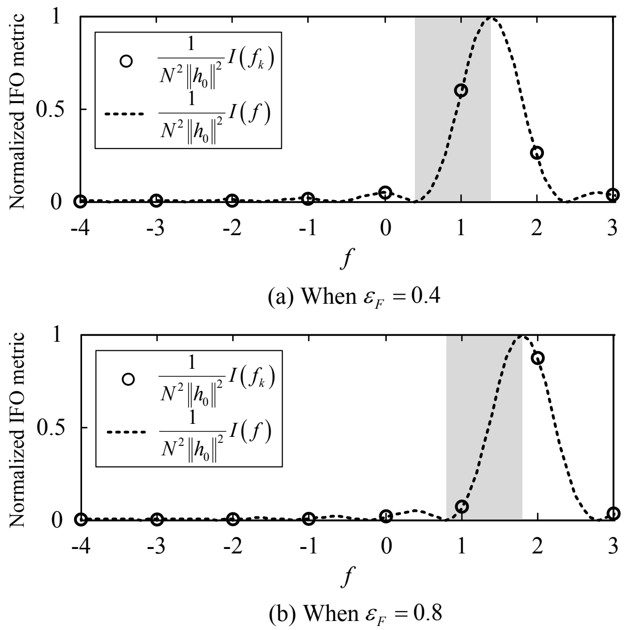


Figure 3. IFO metric $I(f)$ normalized to $N^2\|h_0\|^2$ as a function of the frequency $f \in [-N/2, N/2)$ for $\varepsilon_F = 0.4$ and 0.8 when $\varepsilon_I = 1$, $N = 8$, and the noise is absent.

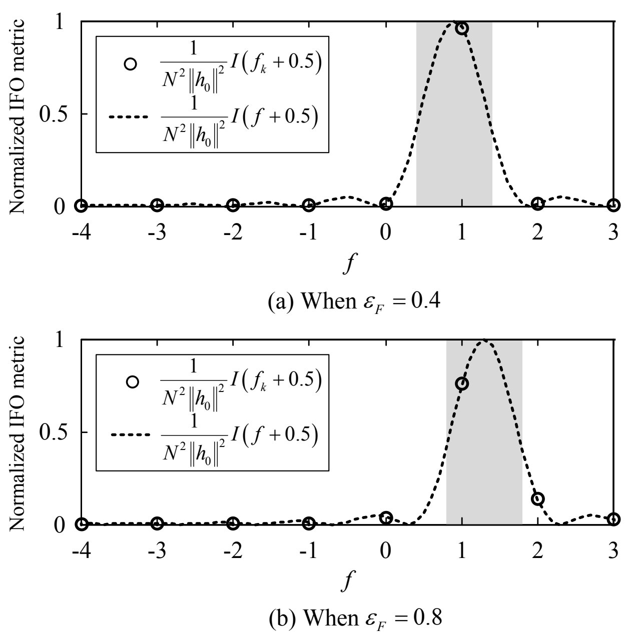


Figure 4. IFO metric $I(f + 0.5)$ normalized to $N^2\|h_0\|^2$ as a function of the frequency $f \in [-N/2, N/2)$ for $\varepsilon_F = 0.4$ and 0.8 when $\varepsilon_I = 1$, $N = 8$, and the noise is absent.

of the FFO. Based on these observations, we propose the following modified ML IFO estimation scheme

$$\hat{\varepsilon}_I = \arg\max_{f_k} I(f_k + 0.5), \tag{7}$$

which is the shifted version of (6) to the left by 0.5 and its metric $I(f+0.5)$ always has the maximum value within the shaded region regardless of the value of the FFO as shown in Fig. 4 since the length of the shaded region is 1 and the FFO is distributed uniformly over $[0, 1)$. Thus, the correct estimation probability of the IFO in the proposed scheme is expected to be robust to the FFO variation.

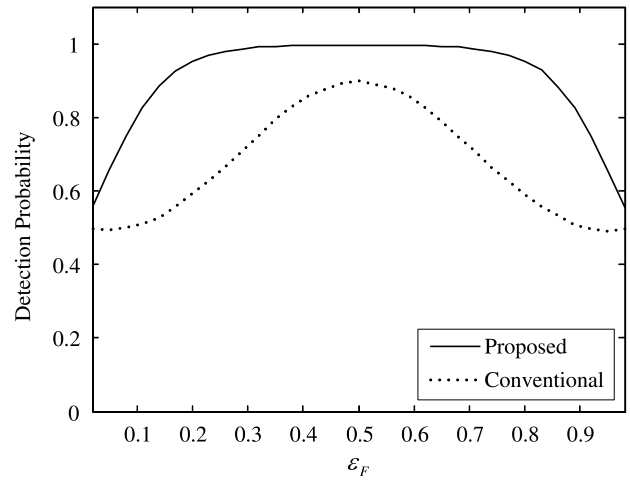


Figure 5. Correct estimation probabilities of the IFO as a function of the FFO for the proposed and conventional schemes in the AWGN channel model when SNR is 0 dB.

IV. Numerical Results

In this section, we compare the performance of the proposed IFO estimation scheme with the conventional scheme in [10]. In the simulation, we assume the following parameters: quadrature PSK (QPSK) modulated data sequence $\{X_k\}_{k=0}^{N-1}$, the FFT size of $N = 64$, a CP with a length of 8 samples, and the maximum Doppler shift of 125 Hz (corresponding to a mobile speed of 54 km/h and a carrier frequency of 2.5 GHz used for WiMAX). The signal to noise ratio (SNR) is defined as σ_s^2/σ_w^2 with $\sigma_s^2 \triangleq \mathbf{E}\{|s(n)|^2\}$. We consider AWGN and four-path Rayleigh fading channel models with path delays of 0, 2, 4, and 6 samples and exponential power delay profile of $\mathbf{E}\{A_l^2\} = \exp(-0.768l)$ (i.e., the power ratio of the first and last paths is set to be 10 dB).

Figs. 5 and 6 show the correct estimation probabilities of the IFO as a function of the FFO for the proposed and conventional schemes. As expected, the proposed scheme is more robust to the FFO variation than the conventional scheme. In addition, we can see that the proposed scheme significantly outperform the conventional scheme. This can be explained as follows. Since the proposed IFO metric is based on a single periodogram, it has only a single peak and can exploit it for detection. On the other hand, the conventional scheme uses the sum of two periodograms, and thus, has two peaks in the metric. The two peaks increase the metric value corresponding to the correct IFO candidate; however, they also increase the metric values

 ISBN: 978-1-61208-126-7

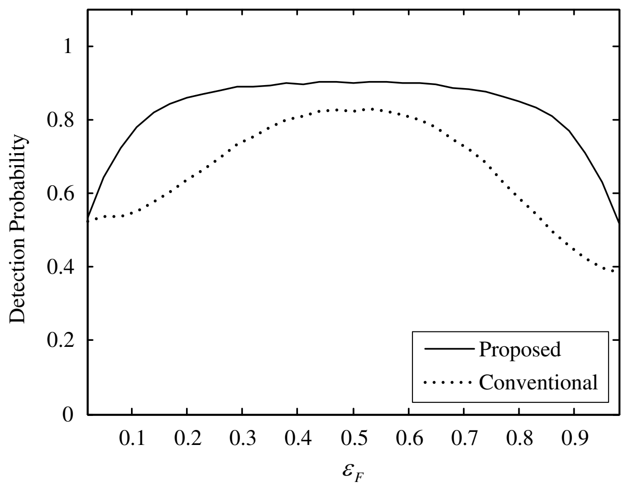


Figure 6. Correct estimation probabilities of the IFO as a function of the FFO for the proposed and conventional schemes in the Rayleigh fading channel model when SNR is 5 dB.

corresponding to the incorrect IFO candidates located on both sides of the correct IFO candidate, thus resulting in a higher incorrect estimation probability (i.e., a lower correct estimation probability) of the IFO.

V. Conclusion and future works

In this paper, we have proposed a new IFO estimation scheme robust to the FFO variation. We have first studied the influence of the FFO on the conventional IFO estimation scheme in [10], and then, proposed a modified ML IFO estimation scheme. By using efficiently shifted version of ML metric, the proposed scheme always has the maximum metric within the correct estimation range regardless of the value of the FFO. From the numerical results, it is confirmed that the proposed scheme is more robust to the variation of the FFO and has better IFO estimation performance than the conventional scheme in [10].

Acknowledgment

This research was supported by the National Research Foundation (NRF) of Korea under Grant 2010-0014610 with funding from the Ministry of Education, Science and Technology (MEST), Korea, and by the Information Technology Research Center program of the National IT Industry Promotion Agency under Grant NIPA-2010-C1090-1011-0005 with funding from the Ministry of Knowledge Economy, Korea.

References

[1] K.-T. Lee and J.-S. Seo, "Pilot-aided iterative frequency offset estimation for digital video broadcasting (DVB) systems," *IEEE Trans. Consumer Electron.*, vol. 53, no. 1, pp. 11-16, Feb. 2007.

[2] M. Morelli, C.-C. J. Kuo, and M. O. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): a tutorial review," *Proc. IEEE*, vol. 95, no. 7, pp. 1394-1427, July 2007.

[3] A. Filippi and S. Serbetli, "OFDM symbol synchronization using frequency domain pilots in time domain," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3240-3248, June 2009.

[4] K. Cai, X. Li, J. Du, Y.-C. Wu, and F. Gao, "CFO estimation in OFDM systems under timing and channel length uncertainties with model averaging," *IEEE Trans. Wireless Commun.*, vol. 9, no. 3, pp. 970-974, Mar. 2010.

[5] T. Pollet, M. V. Bladel, and M. Moeneclaey, "BER sensitivity of OFDM to carrier frequency offset and Wiener phase noise," *IEEE Trans. Commun.*, vol. 43, no. 2, pp. 191-193, Feb. 1995.

[6] M. Morelli and M. Moretti, "Fine carrier and sampling frequency synchronization in OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1514-1524, Apr. 2010.

[7] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.*, vol. 45, no. 12, pp. 1613-1621, Dec. 1997.

[8] S. Chang and E. J. Powers, "Efficient frequency-offset estimation in OFDM-based WLAN systems," *Electron. Lett.*, vol. 39, no. 21, pp. 1554-1555, Oct. 2003.

[9] M.-H. Cheng and C.-C. Chou, "Maximum-likelihood estimation of frequency and time offsets in OFDM systems with multiple sets of identical data," *IEEE Trans. Sig. Process.*, vol. 54, no. 7, pp. 2848-2852, July 2006.

[10] G. Ren, Y. Chang, H. Zhang, and H. Zhang, "An efficient frequency offset estimation method with a large range for wireless OFDM systems," *IEEE Trans. Vehic. Technol.*, vol. 56, no. 4, pp. 1892-1895, July 2007.

[11] A. Milewski, "Periodic sequences with optimal properties for channel estimation and fast start-up equalization," *IBM J. Res. Develop.*, vol. 27, no. 5, pp. 426-431, Sep. 1983.