Estimation of Quality of Experience in 3G Networks with the Mahalanobis Distance

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***Abstract*—Quality of Experience is a parameter used to express the relationship between Quality of Service and the satisfaction of network service subscribers. The modeling of Quality of Experience demands for solving a multidimensional problem. In this paper, we present a Quality of Experience analysis of streaming videos. Related to this, we show that we can reduce the dimensions of the Quality of Experience modeling with the help of Principle Component Analysis techniques. We demonstrate that for our data set the Zero Throughput Time and the Packet Delay Variation are enough to get a picture of the state of the network. We further calculate the *Mahalanobis distance* to analyze the outliers in the data set. We illustrate that for our data set the 97.5 % quantile for the *Mahalanobis distance* is a good threshold that indicates low user perception. We also advocate the use of robust statistics in the analysis of Quality of Experience as we are dealing with contaminated data sets.**

***Index Terms*—Mahalanobis Distance; Quality of Experience; video streaming; robust statistics; 3G network measurements;**

I. Introduction

Quality of Experience (QoE) modeling is an important aspect for understanding the impact of Quality of Service (QoS) on service subscriber's satisfactions. Current state of the art QoE models are producing results that are approximately able to forecast the QoE for mobile streaming videos. Yet, a single highly effective metric for predicting the QoE in mobile networks valid in any context is to be established. This paper's aim is to contribute to a better understanding of the QoE and QoS relationship in 3rd Generation (3G) networks.

Linear weighting of QoS metrics has been popular in measuring the QoE [1], [2], [3]. The input of these algorithms are numerous QoS metrics that are collected during runtime and are linearly weighted against each other to produce a QoE estimate. The objective is to find a single metric that can be used to estimate the QoE. For this purpose, we introduce a new metric to QoE modeling, the *Mahalanobis distance* [4]. The Mahalanobis distance is a distance metric that expresses the distance of a measurement point to the center of a data set, taking into account the correlation of data set. We employ the Mahalanobis distance to compute a single value over multiple QoS measurements that tells us how a particular data point is related to the average state, i.e., the center of the data set, of the network. We show that the Mahalanobis distance is correlated to QoE even when computed over a subset of the measured QoS metrics.

To analyze the efficiency of the Mahalanobis distance in estimating the QoE we conducted a set of experiments where users watched a video that was streamed over a 3G network to a mobile device. The users rated the video on a scale from 1 to 5 with regards to the image quality, as specified by the ITU-T recommendation [5]. A different class of QoE research employs traditional point-based metrics, e.g., Peak Signal to Noise Ratio (PSNR), Moving Pictures Quality Metric (MPQM), or Mean Square Error (MSE), to assess the QoE of video images [6]. The PSNR is computed by comparing the original image before streaming and the actual image that was displayed on the end user's device. In contrast to PSNR, where computer algorithms are used to evaluate the satisfaction, we used humans to assess the video quality. This yields more realistic User Ratings (URs) but also introduces more noise to the measurement of the UR. Sources of noise originate from the human behavior, e.g., delay in rating, human forget factor, inaccurate rating.

We also demonstrate that the modeling can be simplified by only using a subset of the available QoS metrics without loosing much accuracy. We use Principle Component Analysis (PCA) techniques to reduce the dimensionality of the QoS metrics. Reducing the dimensionally of the QoS metrics simplifies the acquisition and reduces the computational efforts to prognosticate the QoE.

Moreover, we advocate the use of robust statistical methods. In our case the QoS data shows contaminated distributions. The contaminated part is of particular interest as we show that the low QoE ratings correspond to the contaminated part of the data set. Classical statistics fail to identify this contamination whereas robust statistics are designed to deal with anomalies in data sets. By means of an example we demonstrate that the Mahalanobis distance produces more accurate results with robust methods compared to classical methods.

The paper is as follows; in section two, we elaborate on the mathematical concepts that we use during the analysis of our results, followed by an overview of the used QoS metrics. Section three applies the Mahalanobis distance to our data set. We discuss the distribution of the Mahalanobis distance, and identify and model the outliers of our QoS metrics. Section four briefly explains the practical use of the Mahalanobis distance in a real-time environment and we conclude in section five with the conclusion and future work.

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II. Background

In this section, we provide an overview of the *Mahalanobis distance* and provide an insight in the QoS metrics used for the assessment of the QoE.

A. Mahalanobis Distance

Mahalanobis distance is a distance metric that expresses the distance of a measurement point to the center of its data set taking into account the data set's correlation. The *Mahalanobis distance* differs from the *Euclidian distance* in the use of the correlation between the components of the data set.

The *Mahalanobis distance* is formally defined as

$$D_M(\mathrm{x}) = \sqrt{(\mathrm{x}-\mu)^T S^{-1}(\mathrm{x}-\mu)} \tag{1}$$

where $\mathrm{x} = (x_1, x_2, \ldots, x_N)$ is a multivariate vector containing a set of measurements, $\mu = (\mu_1, \mu_2, \ldots, \mu_N)$ is the center of the data set and S is the covariance matrix of the data set. It can be shown that if a data set is multinormal distributed then the Mahalanobis distances of the data set are distributed approximately as a chi-square distribution:

$$D_M^2 \sim \mathcal{X}_p^2, \tag{2}$$

with p degrees of freedom [7].

We further observe that our data set is contaminated. In such situations classical statistics yield biased estimates. To obtain more accurate estimates we employ the Minimum Covariance Determinant (MCD) algorithm to calculate the covariance matrix. The fast MCD algorithm is a highly robust estimator of multivariate location and scatter [8], [9]. In our paper, we used the `R` implementation of MCD with `alpha` equal to 0.75 and default remaining parameters. This corresponds to a breakdown point of 25 %, which ensures reasonable efficiency and high robustness against outliers [10].

B. QoS Metrics

During our experiments we assessed the following five QoS metrics: Packet Delay Variation (S_D), Packet Rate (PR), Packet Loss Rate (PL), Clumping Rate (CL), and Zero Throughput Time (T_Z). S_D is calculated as

$$S_D = \sqrt{\frac{1}{N-1}\left[\sum_{n=1}^{N}(D_n^2) - N\bar{D}^2\right]}, \tag{3}$$

where N is the number of received packets per interval, D_n the one-way-delay of packet n, and $\bar{D}$ the average one-way-delay. CL is an indicator that detects clumped packets sequences. T_Z is the duration of no observance of throughput. Additionally we count the number of packets, PR, and the packet loss, PL, per second. The T_Z, S_D and CL are different metrics that describe the variation in one-way-delays of packets. More detailed descriptions of the presented QoS metrics can be found in [11].

Before computing the Mahalanobis distances over the QoS metrics we applied an Exponentially Weighted Moving Average (EWMA) to the data set. The EWMA is used to address the human forgetfulness factor and the delay in rating of the objects under investigation. EWMA is defined as

$$y(j) = (1-\alpha)\,x(j) + \alpha\,y(j-1), \tag{4}$$

where α is the smoothing factor. In previous research $\alpha = 0.75$ yielded the most satisfying results [12]. Note that the EWMA needs a warm-up time to become effective.

III. Testbed Setup

We set up a testbed to record URs together with QoS metrics while streaming videos. We streamed a video from a Darwin Streaming Server (DSS) [13] to a HTC Dream over the Real Time Streaming Protocol (RTSP) [14]. The users that participated in the experiment watched a video alone in a darkened room and rated the quality of the video image whenever he or she felt was appropriate by pushing one of the five buttons on the screen.

The DSS is running on a Linux (2.6.27) Ubuntu 9.04 machine and streams a video MPEG-4 compressed with dimensions 176×144 pixels, 24 kHz AAC stereo sound, 23.97 fps, and streamed at a rate of 325 kbps. The HTC Dream runs a custom designed video streaming application on Android 1.5.

More detailed description of the testbed and how the measurements are acquired refer to [15].

We collected data during forty minutes of video streaming over a 3G network. A total number of 106 URs were recorded. The ratings in relation to the five measured QoS metrics after EWMA are analyzed.

IV. Measurement Analysis

We start the analysis of the QoS starts with the identification of the metrics that are most influential in the data set and correlate most to the UR. Then we proceed to compute the Mahalanobis distance over the selected metrics and analyze its distribution. With this information we then try to model the QoS metrics of concern and attempt to optimize the QoE model.

A. Data set reduction

We first look at the pairwise Pearson's correlation coefficient $\rho_s(QoS_i, QoS_j)$ and place the values in the correlation matrix S_P. The correlation matrix (S_P) is presented in TABLE I. The correlation between the QoS metrics over the whole data set is given. The correlation between QoS metrics and URs is computed only over the rated measurement points. We identify

TABLE I
The correlation matrix (S_P) of the measured QoS metrics and the UR computed with Pearson's correlation coefficient (ρ).

	S_D	RP	PL	CL	T_Z	UR
S_D	**1.000**	-0.420	0.130	-0.158	-0.123	0.383
RP	-0.420	**1.000**	0.053	0.494	0.251	-0.442
PL	0.130	0.053	**1.000**	0.169	0.155	-0.382
CL	-0.158	0.494	0.169	**1.000**	0.698	-0.645
T_Z	-0.123	0.251	0.155	0.698	**1.000**	-0.662
UR	0.383	-0.442	-0.382	-0.645	-0.662	**1.000**

that some QoS metrics correlate moderately ~ 0.69 while others show less correlation ~ 0.42, or very low correlation. In particular the PL seems to be a very poor estimate of the UR, given its low correlation. As other QoS metrics show better correlation among each other and the UR, a Principle Component Analysis (PCA) is an appropriate technique to see if the dimensionality of the data set can be reduced without the loss of much information.

We apply the Robust Principal Component Analysis (ROBPCA) [16] to our QoS data set with $n = 2245$ measurement vectors and $p = 5$ dimensions. The loadings of the Principle Components (PCs) are shown in TABLE II together with their eigenvalues. The PCs can be seen as the spectrum of the data

TABLE II
THE DECOMPOSITION OF THE QOS DATA IN ITS PRINCIPLE COMPONENTS BY THE PCA TECHNIQUE. THE LOADINGS OF THE QOS METRICS ARE GIVEN PER PC TOGETHER WITH ITS EIGENVALUE λ.

	PC_1	PC_2	PC_3	PC_4	PC_5
λ	185.668	50.747	23.900	0.569	0.000
φ	0.712	0.906	0.998	1.000	1.000
S_D	-0.397	0.256	-0.881	0.022	0
PR	0.151	-0.928	-0.336	0.064	0
PL	0.000	0.000	0.000	0.000	1
CL	0.001	-0.054	-0.041	-0.998	0
T_Z	-0.905	-0.267	0.330	0.000	0

set where λ, the eigenvalues of the covariance matrix, are proportional to the importance of the PC. The MCD algorithm was also used to compute the covariance matrix for the ROBPCA. We used the R implementation with `alpha` set to 0.75. Reducing the dimension is achieved by disregarding PCs. A common selection criterion for PCs is

$$\sum_{j=1}^{k} \tilde{l}_j \Big/ \sum_{j=1}^{r} \tilde{l}_j > x, \tag{5}$$

where $\tilde{l}_j$ is the jth eigenvalue of the covariance matrix, $\tilde{l}_1 \geq \tilde{l}_2 \geq \ldots \geq \tilde{l}_r$ with $r = \text{rank}(S)$, and x is proportional to the compression of the data set after PC reduction. φ in TABLE II is the covariance matrix cumulative sum of the relative eigenvalues of the data set. By keeping PC_1 and PC_2 from TABLE II we obtain $x \approx 90\,\%$. The first two PCs are loaded, in order of significance, by T_Z, S_D, PR, CL, and PL. T_Z seems to be the main contributor in PC_1 and PR in PC_2. S_D is the second largest contributor to variation in both PCs and overall more influential than PR. Hence, the data set can be described by T_Z and S_D with regards to its variation, given that the loadings of T_Z and S_D are approximately $90\,\%$ and $35\,\%$ of PC_1 and PC_2, with weights 0.712 and 0.195, respectively.

We observe that the n dimensions of the QoS data set can be reduced while approximately maintaining variability. We show that retaining only T_Z and S_D after dimension reduction is appropriate. By using PCA techniques we concluded that mainly T_Z but also S_D are the QoS metrics that best describe the data set with regards to its variability.

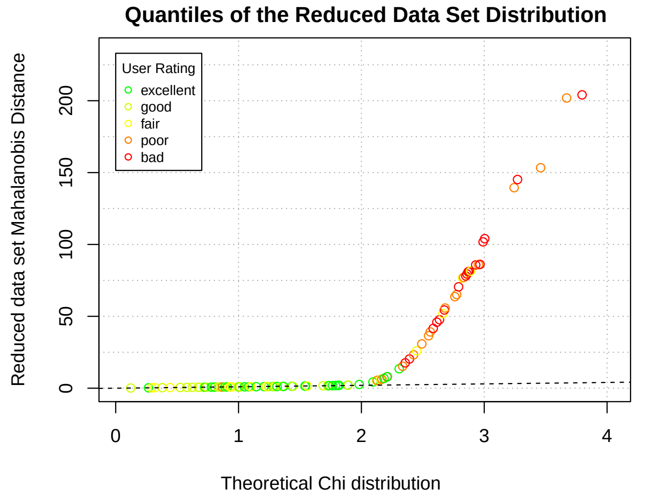

Fig. 1. The Mahalanobis distance Q-Q plot of the reduced data set (T_Z and S_D) against a $\mathcal{X}_{p=2}$ distribution. Only the rated measurements are shown and colored by their User Rating (UR).

B. Mahalanobis distance quantile analysis

We model the QoE with a reduced data set retaining the two most influential QoS metrics on the variance of the data set, i.e., T_Z and S_D. The Mahalanobis distance distribution computed over T_Z and S_D of the whole data set is shown in Figure 1. This figure shows a Q-Q plot where the rated data points are colored to their UR. *Red* is a bad rating, whereas *green* is an excellent rating. The Mahalanobis distance distribution is plotted against a theoretical $\mathcal{X}$ distribution with two degrees of freedom.

The Q-Q plot of the reduced data set against the theoretical $\mathcal{X}_2$ disribution starts of as it seems linearly but it diverges at a given point. The Mahalanobis distance shows approximately linear relationship with a $\mathcal{X}_2$ distribution when the data set of concern is multinormal distributed. We analyze the distribution of T_Z and S_D in more detail later on. We can now observe that their distribution is contaminated, which, in particular, gives rise to the tail. The contamination is manifested in the Q-Q plot through divergence of the quantile equality line (the dashed line in Figure 1). We observe that the *bad* and *poor* URs correspond to the tail of the distributions, and they are separated from the main body of the distribution. Therefore the distribution can be modeled as being contaminated, where the measurement points in the tail are part of a contamination distribution and considered to be anomalies or outliers of the QoS average behavior. The contaminated part is of particular interest as this is an indicator of poor satisfaction.

C. Mahalanobis Distance Tolerance

Given that the squared Mahalanobis distance is approximately $\mathcal{X}^2_{p=2}$ distributed, points larger than $\sqrt{\mathcal{X}^2_{p,0.975}}$ can theoretically be considered as outliers [7]. To get a better insight we construct a set of vectors in $\mathbb{R}^2$ that define the $97.5\,\%$ tolerance ellipse on the T_Z-S_D plane based on the

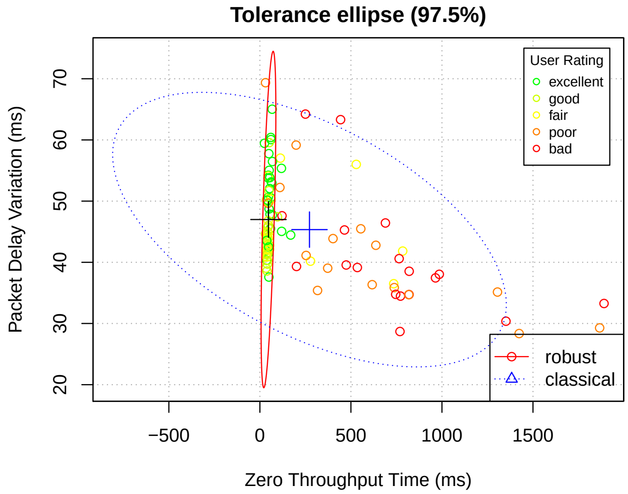

Fig. 2. The Zero Throughput Time (T_Z) - Packet Delay Variation (S_D) data plane and the 97.5 % tolerance ellipse computed with classical and robust statistical methods. The data points are colored by to their User Rating (UR).

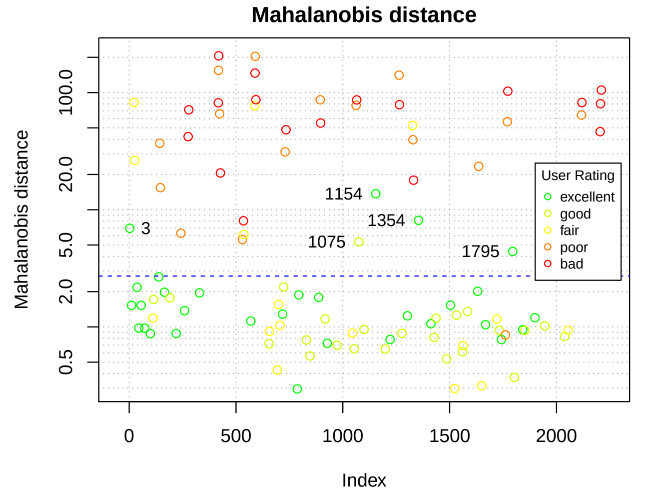

Fig. 3. The Mahalanobis distance per measurement point. The $\sqrt{\mathcal{X}^2_{2,0.975}}$ cutoff is drawn with a dashed blue line. The data points are colored according to their User Rating (UR).

rated data points. This corresponds to the theoretical χ_2 distribution's 97.5 % quantile. The tolerance ellipse is shown in Figure 2. The figure shows the tolerance ellipse obtained with classical statistics and with robust statistical methods. The robust location estimation of the data set is depicted with the black crosshair, the blue crosshair is the center as specified by classical statistics. It is clear that the classical tolerance ellipse is inflated toward the outliers of, mostly, T_Z. The robust ellipse seems to cope well with the outliers. The robust estimation of the center $(47.61, 46.98)$ focuses on the major mass center of the data set, in contrast to the classical center estimation $(260.52, 45.57)$. The estimation of location for S_D is approximately the same for both methods but the T_Z is 547.19 % overestimated by the classical method compared to the robust one.

When we analyze the consistency of the URs inside the tolerance ellipse and outside we observe that inside the ellipse mostly 5 and 4 ratings are located. Outside the ellipse, the lower ratings reside. UR 3 seems to be mostly inside the tolerance ellipse. We do not observe a tendency of the URs distribution in these two areas. After interviewing the humans under investigation we observed that the streamed videos were mostly of satisfying quality, and when poor performance was noticed the satisfaction was very bad. In other words, the users perceived a reasonably good service, or a quite bad service, and rarely something in between. This is translated in a binary satisfaction, where the service perceived is either *good* (contraction of URs *excellent* and *good*) or *bad* (contraction of URs *poor* and *bad*). We can simulate such behavior by the use of the tolerance ellipse, namely inside the ellipse good service is perceived and outside bad one is perceived.

D. Spread of the Mahalanobis Distance

Figure 3 shows the Mahalanobis distance per measurement point. The points are colored according to their UR, and the horizontal dashed line is the $\sqrt{\mathcal{X}^2_{p,0.975}}$ cutoff at 2.72. The *good* and *excellent*-rated measurement points above the cutoff are numbered. We observe that 91 % of the URs above 3 are under the cutoff and 97 % of the URs under 3 are captured by the $\sqrt{\mathcal{X}^2_{p,0.975}}$ cutoff. As a result 9 % of good URs are located in the outlier area, i.e., points 3, 1075, 1154, 1354, and 1795. 68.75 % of the *fair* ratings are below the cutoff line. Ideally we want a clear separation between *good* and *bad* ratings. Possible causes why this is not the case is improper rating of the objects of concern or caused by the smoothing effect of the EWMA. The latter effect can be diminished by using a smaller α in equation 4. Yet, altering α also affects other measurement points. In the case of our measurements, $\alpha = 0.75$ yields optimum results. Only one value rated under 3 falls under the cutoff. Reasons for this are similar to the previous case.

E. Contamination of T_Z & S_D

The $\sqrt{\mathcal{X}^2_{p,0.975}}$ cutoff divides our data set in two parts. The data points representing the network conditions yielding good user perceptions lie below the cutoff; above the cutoff are the outliners, indicators of unstable network conditions. With this information, we can model the T_Z and S_D as contaminated time series. The conventional model of contaminated data is given by

$$F = (1 - \varepsilon)\, G_0 + \varepsilon\, H, \qquad (6)$$

where $\varepsilon \in [0, 0.5]$ is the degree of contamination, G_0 is the model distribution and H is the contaminating distribution. ε must be smaller than 0.5 because this is the limit where the contamination would become the model distribution and visa versa. The model in equation 6 can be applied to our QoS metrics where the data points below the cutoff are of distribution G_0 and the data point above the cutoff of distribution H. ε is defined to be the ratio of data points above the cutoff over the total number of data points, in our experiment $\varepsilon = 0.137$.

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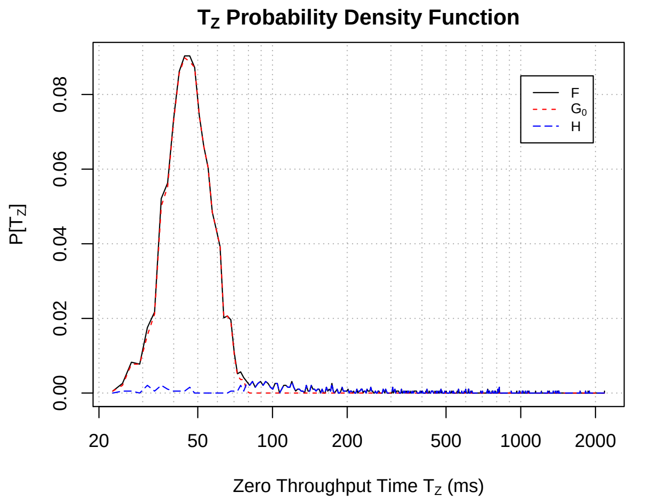

Fig. 4. Histogram of the Zero Throughput Time (T_Z), the model and the contamination distribution with a bin size of 2.15 ms.

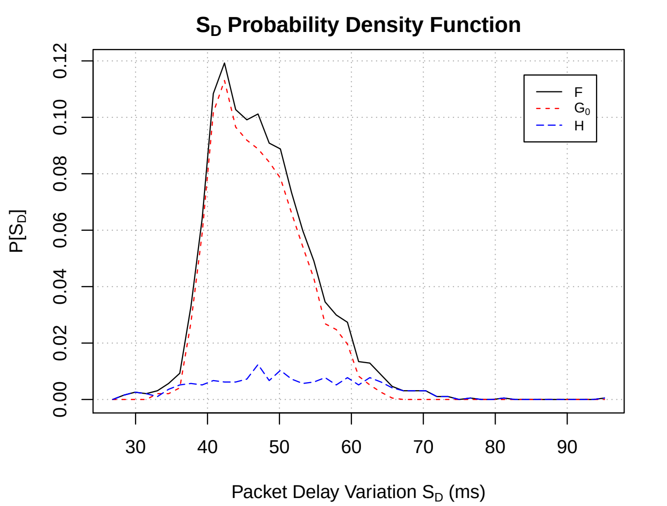

Fig. 5. Histogram of the Packet Delay Variation (S_D), the model and the contamination distribution with a bin size of 1.55 ms.

Figures 4 and 5 show the Probability Distribution Function (PDF) of T_Z and S_D, respectively. The black line is the PDF of the measured metrics, F in equation 6. The dashed red line corresponds to G_0 (the model distribution) and the blue dashed line is the identified contamination, i.e., H.

The contamination of T_Z in Figure 4 covers the tail of the F distribution. The model distribution G_0 fits well the body of F as only minor contamination is present. This suggests that the tail of T_Z can be an estimator for the QoE. The contamination H of S_D at the other hand is mixed in the whole distribution. Similar to T_Z, the contamination accounts for most of the tail and in the case of S_D also the head of F. H is considerable more present in the body of S_D than compared to T_Z. For the S_D it is impossible to identify the contamination merely on the basis of F.

When applying the $\sqrt{\mathcal{X}^2_{p,0.975}}$ cutoff we assumed approximate multinormal distribution. The *Anderson-Darling* normality test on G_0 of T_Z and S_D does not yield results in favor of the normality hypothesis. Both data sets reveal positive skewness, which is larger for S_D than for T_Z. A larger data set helps to clarify the normality hypothesis. Yet, we showed that the Mahalanobis distance is an effective estimate for QoE.

V. QoE Analysis During Runtime

The Mahalanobis distance in the above analysis was computed offline after the experiments took place. Employing the Mahalanobis distance analysis during runtime implies the computation of the S matrix and the center of the data during runtime. For performance reasons this is not desirable, especially on devices with scarce resources such as hand-held devices. The Mahalanobis distance computation can be simplified by using a predefined S matrix and μ vector. The computation of the Mahalanobis distance is then reduced to the multiplication of a matrix, and two vectors. The accuracy of the Mahalanobis distance estimates are consequently dependent on the selected S and μ. These values must be computed on a network with a QoS that yields good QoE most of the time. S and μ might differ for different Internet access technologies and should be studied separately before merging.

VI. Conclusions and Future Work

In this paper, we focused on the use of the Mahalanobis distance for the user satisfaction estimation of streaming video services. We showed with the help of Mahalanobis distance that there is an approximate binary relationship between QoS and QoE in 3G networks. We also showed that we can reduce the dimensions of the QoS metrics to two, without loosing much information. Of the QoS metrics measured in our experiments, T_Z and S_D are the metrics that best describe the data set with regards to its variation.

We also showed by example that robust statistical methods yield far better and reliable results than classical statistical methods. Robust statistical methods are able to handle outliers better than classical methods. In our data set, we are particularly interested in outliers, thus appropriate statistical methods are of great importance.

Future work includes the generalization of the Mahalanobis distance method in QoE modeling. We have shown that for the case of streaming over 3G networks, the Mahalanobis distance seems to be a good indicator of user satisfaction. Studies of different Internet access technologies and services will shed light on the *Mahalanobis' distance* generalized applicability.

Also, an optimized cutoff might yield better results and it is subject for future work. A larger data set than used in our paper is necessary to obtain more significant statistical results. A hysteresis approach to the binary modeling is also a good solution to prevent oscillatory behavior in binary modeling.

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