A Comparison of Receiver Strategies in STBC MIMO Systems in a Challenging Environment

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Abstract—The subject of this paper is a comparison of the different criteria and strategies of signal reception in spacetime coding MIMO systems operating under dynamic flat fading and channel cross-correlation. The dynamic fading means that a Doppler shift is comparable with the transmission rate. The criterion of comparison is a mean bit error rate. The BER characteristics are obtained via Monte Carlo simulation. Many strategies and antennas configurations are analyzed, including space-time maximum likelihood, and zero forcing, as well as 2x1, 2x2 and 2x3 antenna setup. Simulation demonstrates that the receiver with 3 antennas and simple maximum likelihood decoder usually acts better than receiver with a fewer number of antennas and a more sophisticated reception strategy.

Keywords- STBC MIMO; ML; ZF; ML-ST

I. INTRODUCTION

Potential performance benefits and a remarkable capacity promised by MIMO (multiple input - multiple output) systems attracted a lot of interest in the recent years. However, to keep this promise, many strict conditions have to be satisfied. This refers, inter alia, to the assumption of a quasi-static fading where the channel gains h(t)=h(t+T) [1] [2]. Under real conditions, the transfer function h(t) changes itself over the code word and this leads to interference, supposing the Maximum Likelihood (ML) criterion is used [7][8]. At the same time, the interference suppression methods, e.g. Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) techniques are applied [4][5][6], as well as the QR decomposition [10] and the Maximum Likelihood Space-Time decoding (ML-ST) [8]. These strategies are usually examined in MISO (multiple input - single output) configuration and in the absence of a cross-correlation. Those approaches outperform linear combiner with the ML detector. However, they are more complex. The influence of the number of receiving antennas on this benefit was not taken into account, as well as the effect of cross-correlation.

In the present paper, several antennas architectures and popular detector schemes - ML, ML-ST, ZF, MMSE - are analyzed, both for channel cross-correlation and for dynamic flat fading. The comparison of performance is carried out on the basis of BER characteristics.

The rest of the paper is organized as follows. Section II describes the system model. Section III deals with

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interference suppression techniques. In Section IV, the Monte Carlo simulations are carried out and the obtained results are analyzed. Section V contains a concise conclusion.

II. SYSTEM MODEL

The considered system consists of two transmitting and one, two or three receiving antennas, Fig 1. All antennas are potentially mobile and omni-directional. It is assumed that the system exploits the space-time coding strategy [3]. In principle, the maximum likelihood decision rule is applied assuming that the interference suppression (IS) is performed first. The channel undergoes frequency flat and timeselective Rayleigh fading. The channel gains are identical Gaussian random variables with zero means and autocorrelation function $1/2E[h_i(t)h_i^*(t+\tau)] = \sigma^2 R(l)$ where R(l) follows Jakes' model [12].

$$R(l) = J_0(2\pi F_d T_s l) \tag{1}$$

 $J_0(\)$ is the zero-order Bessel function of the first kind, F_d stands for maximum Doppler shift and T_s is the symbol duration time.



Figure 1. Considered MIMO System

We assume that the links between different antennas are identically distributed, however they can be correlated. The perfect estimation of the channel transfer functions h(t) is presupposed.

The considered scheme exploits the code word of Alamouti [3]. For the 2x2 MIMO system, received signals in antennas Rx1, Rx2 at some point of time *t* are r_{1t} , r_{2t} respectively, while at the next moment $t+T - r_{1T}$, r_{2T} , where T – signal symbol duration and n_i are the independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and some variance N₀.

$$r_{1t} = h_{11t}S_1 + h_{12t}S_2 + n_1$$

$$r_{2t} = h_{21t}S_1 + h_{22t}S_2 + n_2$$

$$r_{1T} = h_{12T}S_1^* - h_{11T}S_2^* + n_3$$

$$r_{2T} = h_{22T}S_1^* - h_{21T}S_2^* + n_4$$
(2)

The changes in the third and fourth row of (2) follow from the fact that signals S_{l} , S_{2} in the t+T period are conjugated and S_{1} is multiplied by (-1). By means of manipulating the receive vector $r = [r_{1t}, r_{1T}, r_{2t}, r_{2T}]^{T}$ the matrix notation of (2) takes the form.

$$\begin{bmatrix} r_{1t} \\ r_{1T}^{*} \\ r_{2t}^{*} \\ r_{2T}^{*} \end{bmatrix} = \begin{bmatrix} h_{11t} & h_{12t} \\ h_{12T}^{*} & -h_{11T}^{*} \\ h_{21t} & h_{22t} \\ h_{22T}^{*} & -h_{21T}^{*} \end{bmatrix} \begin{bmatrix} S_{1} \\ S_{2} \end{bmatrix} + \begin{bmatrix} n_{1} \\ n_{2}^{*} \\ n_{3} \\ n_{4}^{*} \end{bmatrix} \Leftrightarrow r = HS + n \quad (3)$$

The decoder proposed by Alamouti acts as follows [3]

$$\widetilde{S} = H^H r \Longrightarrow \widetilde{S} = H^H H S + H^H n \tag{4}$$

where \widetilde{S} is an estimate vector of transmitted symbols. Ignoring in (4) the noise component $H^H n$ for a quasi-static channel, h(t)=h(t+T), the estimates of signals are

$$\begin{bmatrix} \widetilde{S}_{1} \\ \widetilde{S}_{2} \end{bmatrix} = \begin{bmatrix} |h_{11}|^{2} + |h_{12}|^{2} + |h_{21}|^{2} + |h_{22}|^{2} & 0 \\ 0 & |h_{11}|^{2} + |h_{12}|^{2} + |h_{21}|^{2} + |h_{22}|^{2} \end{bmatrix} \begin{bmatrix} S_{1} \\ S_{2} \end{bmatrix}$$
(5)

In real channels, however, the off-diagonal elements of $H^H H$ are no longer zero and this introduces an interference.

III. INTERFERENCE SUPPRESSION

The main idea leading to interference suppression techniques is to built a matrix W to fulfill condition

$$WH = I$$
 (6)

where I stands for diagonal matrix of real elements. In the ZF technique it takes a form [4]

$$W = (H^{H}H)^{-1}H^{H}$$
(7)

As a consequence, elements causing the interference are reduced to zero but the variance of system noise is at the same time increased, what obviously deteriorates system performance. The way to reduce this effect is to built a modified matrix (8) and minimize the noise via the MMSE technique [13].

$$W = (H^{H}H + \sigma^{2}I)^{-1}H^{H}$$
(8)

where I -identity matrix and σ^2 - noise variance.

Most publications deal with systems with one receiving antenna. This assumption provides an opportunity to use a simple matrix inversion. However, for a few receiving antennas the problem of matrix inversion is more complex. Such a matrix is no longer the square one and the advanced pseudo-inversion has to be carried out [9]

$$A^{+} = (A^{T}A)^{-1}A^{T}$$
(9)

Another way to perform such an operation is a singular value decomposition (SVD):

$$A = USV^T \Longrightarrow A^+ = V(S^T S)^{-1} S^T U^T$$
(10)

Still another technique employs a space-time decoder, ML-ST [8]. It chooses the most probable sequence of the transmitted pair of symbols. The ML detector minimizes the product

$$\left|r - H\tilde{x}\right|^2 \tag{11}$$

where \tilde{x} stands for pair of symbol, in BPSK case $\tilde{x} \in \{(11), (1,0), (0,1), (0,0)\}$.

IV. SIMULATION RESULTS

In order to compare the different strategies the Monte Carlo simulation method was applied. As a measure of channel variations, the normalized fading rate F_dT_s was used, where F_d stands for a maximum Doppler spread and T_s for a symbol duration time (at times, this parameter is called the normalized fading bandwidth, BT).

Three values of F_dT_s were used: 0.01 for quasi-static fading, 0.05 for moderate fading and 0.1 for fast fading. The first results, Fig.2, refer to ZF and MMSE techniques. The benefit of using MMSE instead of ZF becomes more and more negligible as the number of receiving antennas exceeds one. For the 2x2 setup, both detectors provide comparable results even for fast fading, and for the 2x3 setup, the difference disappears. Such results were expected.

It is to be noted that the size of identity matrix I is dictated by the size of the $H^H H$ matrix in (8).



Figure 2. BER performance of ZF/MMSE strategies for 2x1, 2x2, 2x3 antennas setups and $F_dT_s{=}0.01$ - 0.1

For the Alamouti code, this operation always leads to a 2x2 square matrix irrespective of the number of receiving antennas, because there are always 2 transmitting antennas. The noise minimizing component $\sigma^2 I$ have the same value for all considered antennas configurations.

Thus, the benefit of using MMSE instead of ZF becomes smaller with the increase of the number of receiving antennas. One can see from Fig. 2 that the highest impact on the behavior of all characteristics evokes the number of receiving antennas – the higher the number, the lower the BER. The next factor is the type of detector: the ML gives the poorest results, ZF and ML-ST seem better and comparable to each other. The influence of a crosscorrelation is temperate.



Figure 3. . BER performance of ML, ZF, ML-ST strategies for setups 2x1, 2x2, 2x3 and fast fading, negligible cross-correlation

However, it can be noticed that the ML detector in a 2x2 setup gives better performance than all considered detectors in a 2x1 configuration in a range of SNR from 0 to approx. 30dB.

Fig. 5 refers to strong channel cross-correlation and fast fading.

The a, b, c parts of the Fig. 5 refer to ML, ZF and ML-ST respectively. Colors denote the number of receiving antennas and

- solid lines fast fading strong correlated case;
- dashed lines fast fading uncorrelated case;
- dotted bold lines moderate fading uncorrelated case as a reference level.

It can be seen that all considered strategies suffer from fading and cross-correlation. The greatest impact on the behavior of all characteristics again evokes the number of receiving antennas – the higher the number, the lower the BER. The next factor is the type of detector: the ML gives the weakest results, ZF and ML-ST seem better and comparable to each other. The influence of a crosscorrelation is temperate.

The second group of results, Fig.3 and Fig. 4, refer to ML, ZF and ML-ST strategies (MMSE was neglected) for moderate fading and both uncorrelated channel and low channel cross-correlation. The elements of the correlation matrix were set randomly from the values of 0.2 to 0.4. This operation was described in detail in [11].

For the 2x1 setup, deteriorations of performance for all strategies are caused mainly by fading, while cross-correlation can be treated as a nearly small supplement. For 2x2 or 2x3 setups and ML strategy, the influence of both factors - fading and cross-correlation - are comparable, while for ZF and ML-ST the cross-correlation dominates.



Figure 4. BER performance of ML, ZF, ML-ST strategies for setups 2x1, 2x2, 2x3 and fast fading, cross-correlation of p=0.2-0.4



Figure 5. BER performance of ML (a), ZF (b) and ML-ST (c) strategies for strong correlated channels ρ = 0.5 - 0.7 and fast fading FdTS=0.1. Antennas setups 2x1, 2x2 and 2x3

However, it is worth noting that for the 2x3 setup and typical conditions of moderate fading and low cross-correlation, the simple ML detector offers better performance than ZF or ML-ST applied in a 2x1 or 2x2 setup, Fig. 3 and 4.

V. CONCLUSION

We carried out a comparison of ML, ZF, MMSE and ML-ST reception strategies in STBC MIMO systems for 2x1, 2x2 and 2x3 antennas setup. As a criterion of the comparison of the BER, characteristics obtained via Monte Carlo simulation were used. These characteristics show that an additional antenna (setup 2x3) used in a typical moderate fading and low cross-correlation environment gives better results than a sophisticated reception procedure, e.g. ML-ST. The benefits of using MMSE instead of ZF become smaller with an increase of the number of receiving antennas. The greatest impact on characteristics is caused by the number of receiving antennas. It was also shown that a low crosscorrelation acts as an additive error source and is nearly independent on fading. The above conclusions are, however, not valid for a strong cross-correlation case and for fast fading where the ZF/ML-ST strategies bring even better results than ML in a 2x3 setup.

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