The min-max in LC Versus the max-log MAP in LC Method for Soft-Decision Decoding of MTR Codes

Nikola Djuric, Vojin Senk
Faculty of Technical Sciences, University of Novi Sad
Novi Sad, Serbia
E-mail: ndjuric@uns.ac.rs, vojin_senk@uns.ac.rs

***Abstract* – The approaches for soft-decision decoding of maximum-transition-run (MTR) codes have been extensively researched lately. Two different methods, named as *min-max in LC* and *max-log MAP in LC* have emerged as a fairly suitable for MTR code utilization in magnetic recording systems. Both methods were designed to enable the propagation of the soft-values through the logic circuits of the MTR decoder, allowing the concatenation of MTR code with some powerful error-correcting codes, such as the low-density parity-check (LDPC) codes. In this paper, the *min-max in LC* versus the *max-log MAP in LC* approach is considered in the framework of simple and the straightforward LDPC-MTR concatenation over E^2PR4 magnetic recording channel. Both methods show nearly the same coding gain of 2 dB at BER = 10^{-5}.**

Keywords – soft-decision decoding; constrained coding; MTR codes; LDPC codes.

I. INTRODUCTION

The maximum-transition-run (MTR) codes have been confirmed as very useful constrained codes for E^2PR4 magnetic recording channel, preventing the dominant error-event ± [+1 –1 +1] and consequently improving the distance properties of the channel by increasing the minimal squared Euclidean distance between two distinct sequences in this type of the channel. Their design was primarily oriented towards constrained features, resulting with unsatisfactory error correcting capabilities. Thus, the MTR codes need to be merged with some powerful error-correcting code, such as the well-established low-density parity-check (LDPC) code [2].

There were several attempts to combine/concatenate the MTR and LDPC [3]-[5], requiring that the MTR decoder is able to handle soft-values [6]. Independently, an approach of a simple and straightforward serial concatenation of these codes was considered, providing as a result two additional methods for the soft-decision decoding of MTR codes: the *min-max in LC* and *max-log MAP in LC* methods [7], [8].

In this paper, the *min-max in LC* and *max-log MAP in LC* methods are considered and compared as methods that enable propagation of the soft-values through the logic circuits of the MTR decoder in LDPC-MTR concatenation, where LDPC acts as an outer and MTR as an inner code, over the one-track one-head E^2PR4 magnetic recording channel.

Section II presents a brief overview of the *min-max in LC* and *max-log MAP in LC* approaches, while Section III explains the LDPC-MTR encoding over the E^2PR4 channel. Section IV brings complexity analyses of the two proposed methods and Section V offers simulation results and performance comparison. Finally, Section VI presents the conclusion to this paper.

II. MTR CODES AND SOFT-DECISION DECODING

The MTR codes bring a constraint in the encoding process by eliminating three or more consecutive transitions in the recording sequence of E^2PR4 channel, and thus eliminate the most troubleshooting error-event ± [+1 –1 +1], enhancing E^2PR4 channel distance properties [10].

The MTR codes were originally presented by Moon and Brickner [1]. One of their major benefits is that they can be realized using simple logic circuits, as it is presented in Figure 1, for the well know rate 4/5 (2, k = 8) MTR code.

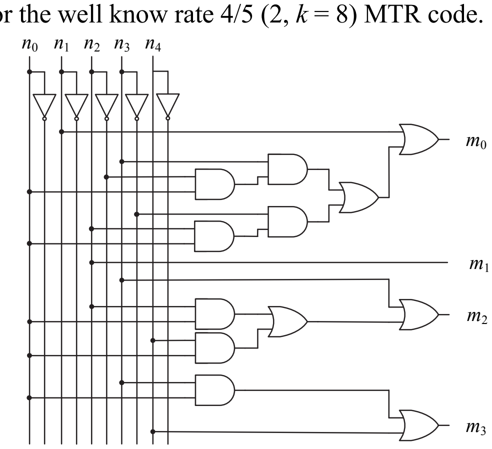

Figure 1. Rate 4/5 (2, k = 8) MTR decoder.

Such feature enables an easy hardware implementation, providing the low-cost realization for the MTR decoder.

The key idea of the *min-max in LC* and the *max-log MAP in LC* methods was to implement the soft-decision approach in existing logic circuits and to assemble the novel MTR decoder with soft-decision capabilities. The soft-value of the variable x is defined as

$$L(x) = \log[P(x=1)/P(x=0)], \tag{1}$$

where *log*() function is a natural logarithm. The sign of $L(x)$ represents the binary decision of a variable, the so called hard-decision, while the magnitude represents the confidence or reliability of a binary decision [6].

Using the soft-decision approach, the MTR decoder will be able to offer the binary decision, as well as the reliability of that decision for all its outputs, increasing information redundancy that will be delivered to the next step decoder. Such new MTR decoder that is capable to handle input soft-values and produce subsequent soft-values on its outputs is necessary in iterative decoding schemes.

A. *Brief overview of the min-max in LC method*

The *min-max in LC* method suggests enhancing the existing Boolean logic circuits, so that they are able to produce the output soft-values according to the following expressions [7], [11]:

$$\begin{aligned} L_{out}^{NOT}(x) &= -L_{in}(x), \\ L_{out}^{AND}(x_1, x_2) &= \max[L_{in}(x_1), L_{in}(x_2)], \\ L_{out}^{OR}(x_1, x_2) &= \min[L_{in}(x_1), L_{in}(x_2)], \\ L_{out}^{XOR}(x_1, x_2) &= sign[L_{in}(x_2)] \cdot sign[L_{in}(x_2)] \\ &\quad \cdot \min[L_{in}(x_1), L_{in}(x_2)], \end{aligned} \tag{2}$$

where *max*() and *min*() function returns the higher and lower value of two variables, respectively, while the *sign*() function returns the sign of the variable.

B. *Brief overview of the max-log MAP in LC method*

In this method, the logic circuits create the output soft-value according to the following rules [8], [12]:

$$L_{out}^{\max-\log MAP\,in\,LC} = (R_1 - R_0)/2\,, \tag{3}$$

where the coefficients R_1 and R_0 are obtained, for *AND* logic circuits, as:

$$\begin{aligned} R_1 &= L_{in}(\mathrm{x}_1) + L_{in}(\mathrm{x}_2), \\ R_0 &= \max[-L_{in}(\mathrm{x}_1) - L_{in}(\mathrm{x}_2), -L_{in}(\mathrm{x}_1) + L_{in}(\mathrm{x}_2), \\ &\qquad + L_{in}(\mathrm{x}_1) - L_{in}(\mathrm{x}_2)], \end{aligned} \tag{4}$$

while for *OR* logic circuits as:

$$\begin{aligned} R_1 &= \max[-L_{in}(\mathrm{x}_1) + L_{in}(\mathrm{x}_2), + L_{in}(\mathrm{x}_1) - L_{in}(\mathrm{x}_2), \\ &\qquad + L_{in}(\mathrm{x}_1) + L_{in}(\mathrm{x}_2)], \\ R_0 &= -L_{in}(\mathrm{x}_1) - L_{in}(\mathrm{x}_2), \end{aligned} \tag{5}$$

and finally, for *XOR* logic circuits, using expressions:

$$\begin{aligned} R_1 &= \max[-L_{in}(\mathrm{x}_1) + L_{in}(\mathrm{x}_2), + L_{in}(\mathrm{x}_1) - L_{in}(\mathrm{x}_2)], \\ R_0 &= \max[-L_{in}(\mathrm{x}_1) - L_{in}(\mathrm{x}_2), + L_{in}(\mathrm{x}_1) + L_{in}(\mathrm{x}_2)]. \end{aligned} \tag{6}$$

The *NOT* logic circuits work the same as for the *min-max in LC* method.

Several additional details on the *min-max in LC* and *max-log MAP in LC* methods can be found in several previously published papers [7]-[9].

C. *The soft-decision MTR decoder*

The soft-decision MTR decoder will be realized using the improved log circuits, with the implementation of one of soft-decision methods, the *min-max in LC* and *max-log MAP in LC* methods, as depicted in Figure 2.

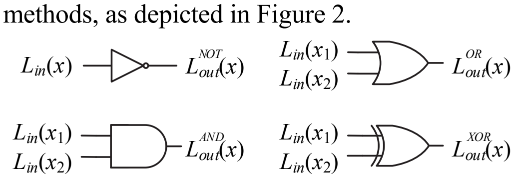

Figure 2. Redesigned Boolean logic circuits.

Using this approach, the idea is to preserve the simplicity of the hardware realization of the decoder and at the same time to design circuits capable of working with soft-values.

III. MTR ENCODING/DECODING OVER E²PR4 CHANNEL

Performances of the *min-max in LC* and *max-log MAP in LC* were considered over the high density E^2PR4 magnetic recording channel, encoded by the simple LDPC-MTR serial concatenation. In this early stage of investigation, the simulation schemes do not employ any specially designed codes.

A. *LDPC–MTR encoding process*

The LDPC-MTR encoding was performed using R = 0.96 LDPC as an outer code and a well-known, rate 4/5 (2, k = 8) MTR, as an inner code, in the simulation scheme depicted in Figure 3.

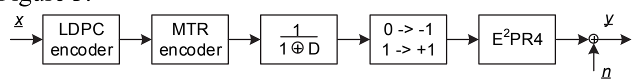

Figure 3. LDPC–MTR encoding over E^2PR4 channel.

The LDPC code of the length N = 4732, with M = 169 parity bits and column-weight 3 is based on Kirkman triple systems [13].

Unfortunately, the resulting code rate of concatenation is only R = 0.771. This is not quite suitable for high density magnetic recording; however, it is not limiting for such initial investigation of MTR soft-decision decoding methods.

It was assumed that the read-back signal y is distorted with the additive white Gaussian (AWGN) noise n and that the signal-to-noise ratio (SNR) is defined as

$$SNR = 10\log(E_b / N_o) = 10\log(E_c / 2R\sigma^2)\,, \tag{7}$$

where $E_c = RE_b$ is the symbol bit energy at channel output, N_o is one-sided power spectral density and σ^2 is noise variance.

B. *LDPC–MTR decoding process*

The channel detection was done using the Soft-Output Viterbi Algorithm (SOVA), with the detection window of 20 symbols, while the LDPC code was decoded by the message-passing algorithm [14].

During the decoding process, the soft information was exchanged between SOVA decoder and message-passing, as depicted in Figure 4.

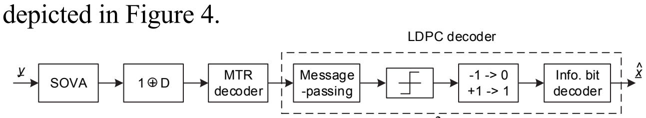

Figure 4. LDPC–MTR decoding over E^2PR4 channel.

Intention of this paper was to investigate the performance of the proposed methods for the MTR soft-decision decoding. Thus, the simulation scheme implements the reverse and straightforward concatenation of LDPC-MTR, without any iterative exchange between them, focusing on the ability of soft-values propagation through the MTR decoder.

IV. THE COMPLEXITY OF THE PROPOSED METHODS

Both methods suggest the redesign and enhancement of the conventional logic circuits, though they determine the

 ISBN: 978-1-61208-402-2

output soft-values for particular logic circuits in a different way. This section is intended to emphasize the difference between them, as well as their complexity.

A. AND logic circuits

The analysis of difference between the output soft-values of methods, for *AND* circuits, was performed considering that both input soft-values range between -8 and +8, with the step of 0.1. The major difference can be noticed when inputs have the same value but different sign, as depicted in Figure 5, part *a*.

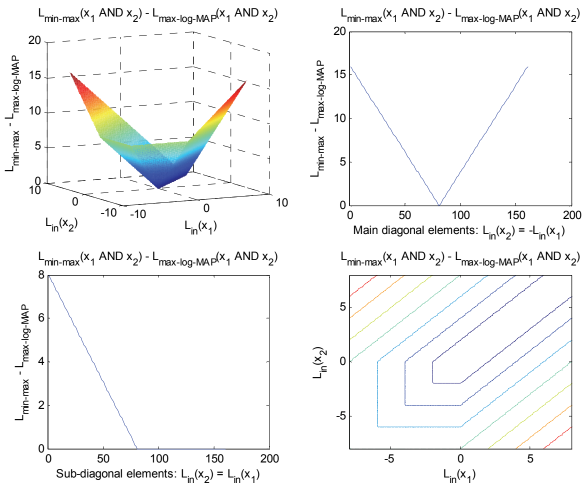

Figure 5. Difference of the output soft-value for AND circuits.

Moreover, analyzing the two-dimensional matrix, whose columns and rows represent $L_{in}(x_1)$ and $L_{in}(x_2)$, it can be noticed that over the main diagonal, where $L_{in}(x_2) = -L_{in}(x_1)$, the difference between methods is very high and symmetrically distributed over the point where $L_{in}(x_1)$ and $L_{in}(x_2)$ are equal to zero, as depicted in Figure 5, part *b*.

Furthermore, over the sub-diagonal of the matrix, where $L_{in}(x_2) = +L_{in}(x_1)$, the difference between methods is lower and exists only when input values are negative, as depicted in Figure 5, part *c*. In case that both input soft-values are positive, the *min-max in LC* and *max-log MAP in LC* identically assess the output soft-values.

Analyzing other points in $[L_{in}(x_1), L_{in}(x_2)]$ matrix, which are off-diagonal elements, it can be noticed that certain positive differences exist, as depicted in Figure 5, part *d*. This difference is not extreme as those on the main diagonal and thus it can be concluded that both of these two methods asses correctly the output soft-value, but with different confidence.

Even both proposed methods use the *min*() and *max*() functions, it seems that the *min-max in LC* approach produces some higher soft-values on the output of the *AND* logic circuit than the *max-log MAP in LC* method. Only in a case when both input values $L_{in}(x_1)$, $L_{in}(x_2)$ are equal and positive, these two methods produce the same output soft-value.

B. OR logic circuits

The similar analysis of difference between assessment in the suggested methods can be performed for *OR* logic circuits, as depicted in Figure 6.

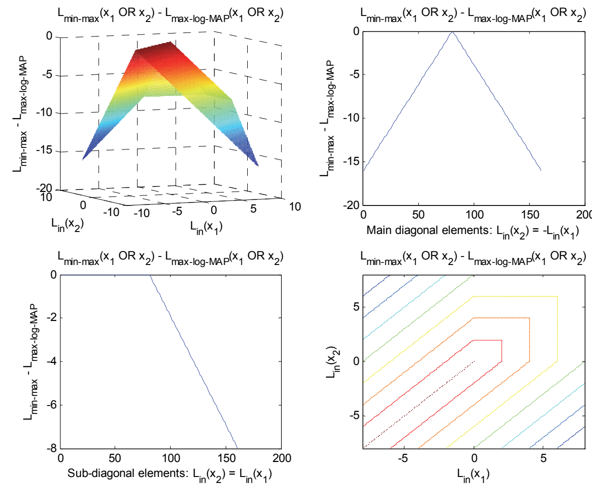

Figure 6. Difference of the output soft-value for OR circuits.

In this case, the *min-max in LC* assesses lower output soft-values, compared to *max-log MAP in LC*, with the similar situation that the major difference lies on the main diagonal of $[L_{in}(x_1), L_{in}(x_2)]$ matrix, where $L_{in}(x_2) = -L_{in}(x_1)$. Also, we have symmetric distribution over the point where $L_{in}(x_1)$ and $L_{in}(x_2)$ are equal to zero, as depicted in Figure 6, part *b*.

Additionally, on the sub-diagonal of $[L_{in}(x_1), L_{in}(x_2)]$ matrix, both methods equally assess output values, when input values are equal and negative, as depicted in Figure 6, part *c*.

Moreover, analyzing the shape of the difference function, it can be concluded, again, that both methods similarly assess soft-values on the circuit output, unfortunately, with quite different confidence when $L_{in}(x_2) = -L_{in}(x_1)$.

C. XOR logic circuits

In the case of *XOR* logic circuits, the *min-max in LC* and the *max-log MAP in LC* methods assess output soft-values with almost negligible difference, which ranges around 10^{-15}, as depicted in Figure 7, part *a*. Thus, both the proposed methods are equally confident and offer a quite suitable soft-value for *XOR* circuits.

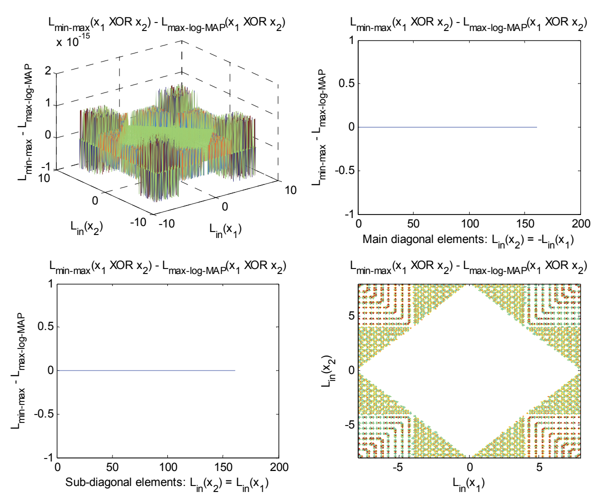

Figure 7. Difference of the output soft-value for XOR circuits.

Even more, there is no any difference on the main and sub-diagonal of $[L_{in}(x_1), L_{in}(x_2)]$ matrix, as depicted in Figure 7, parts *b* and *c*, while on a wide range of the off-diagonal elements this difference is zero, as shown in Figure 7, part *d*.

These two proposed methods have suitable assessments of the soft-values on the circuit outputs. However, during their implementation into the new MTR decoder, the attention has to be paid to the complexity and the total number of the required operations in the decoder.

D. *Analysis on the required number of operations*

The *min-max in LC* and *max-log MAP in LC* methods are based on the *min*() and *max*() functions in order to assess the soft-values. Regarding the number of operations, the *min-max in LC* is in slight advantage, as depicted in Figure 8.

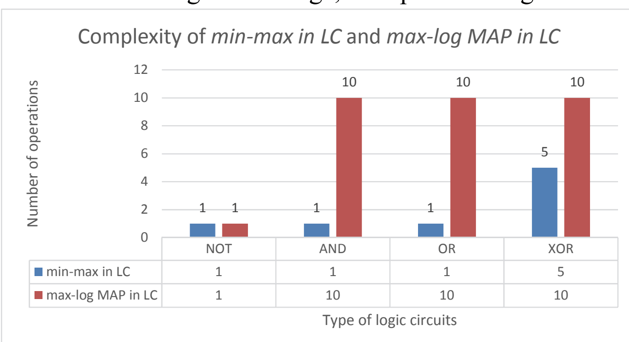

Figure 8. Required number of operations.

Unfortunately, the several required operations with input soft-values, in the *max-log MAP in LC*, increase the number of overall operations, even though the complexity of both methods is still of the same degree. However, regarding the assembly of the new MTR decoder that is based on the soft-decision approach, the total number of the required operations by both proposed methods in all logic circuits should be considered, since that can have a substantial impact on the power consumption of the newly created decoder.

Analyzing the decoder for the well know rate 4/5 (2, k = 8) MTR code, depicted in Figure 1, it can be noticed that such realization requires 5 *NOT*, 8 *AND* and 5 *OR* logic circuits, with a total number of decoder operations as depicted in Figure 9.

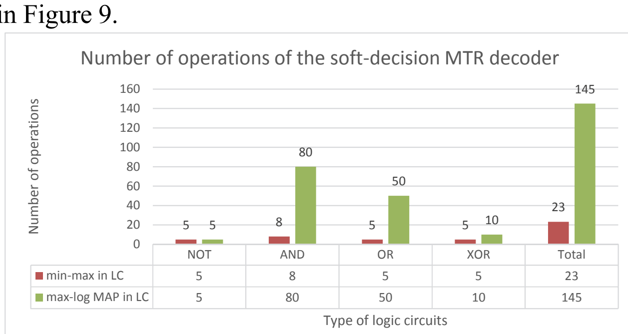

Figure 9. Total number of operations for the MTR decoder.

Additionally, the *min-max in LC* and *max-log MAP in LC* methods require one *XOR* circuit in the data sequence preprocessing phase, before the sequence enters into the MTR decoder. The *XOR* circuit is not depicted in the MTR decoder, presented in Figure 1, though it should be counted in the total number of the MTR decoding process.

Even though these proposed soft-decision methods have the same degree of complexity, the *min-max in LC* approach requires 6.3 times less operations than the other one. Therefore, the *min-max in LC* outperforms the *max-log MAP in LC* and will be more desirable for the practical implementation into the soft-decision MTR decoder.

V. SIMULATION RESULTS

In order to provide the final conclusion on *min-max in LC* and *max-log MAP in LC* approach for soft-decision decoding of the MTR codes, the comparison was made over the one-track E^2PR4 magnetic recording channel, utilizing serial concatenation of LDPC-MTR codes. Performances of these methods have been summarized in Figure 10.

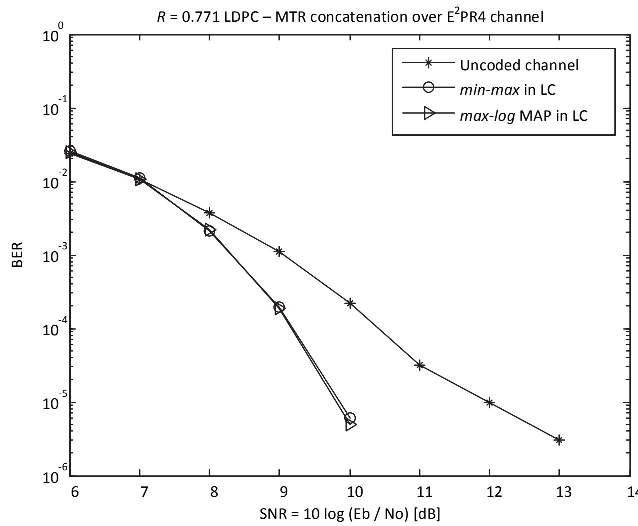

Figure 10. Approaches for MTR soft-decision decoding.

It can be observed that the utilization of both *min-max in LC* and *max-log MAP in LC* approaches in the logic circuits of the new MTR decoder, within the LDPC-MTR serial concatenation, resulted with almost identical and, in addition, the considerable coding gain of 2.2 dB for BER = 10^{-5}, comparing with the uncoded channel.

This result confirms that both methods properly estimate soft-values on the logic circuits outputs, allowing the propagation of the soft-information to the LDPC decoder.

Using both these approaches, the new MTR decoder can be equipped with the soft-decision decoding capability, offering the confidence of binary decisions per output, which is required for the iterative simulation schemes.

Both methods are almost identical with the achieved decoding gain, but a slight advantage is on the side of the *min-max in LC*. This approach resulted with 6.3 time lower number of operations in the MTR decoder, which can be quite valuable in case of extensive utilization of decoder and demanding signal processing. The lower number of arithmetic operations can have strong influence on the power consumption of practical MTR decoder.

VI. Conclusion

This paper presents a comparison of two newly proposed methods, *min-max in LC* and *max-log MAP in LC* methods, for the soft-decision decoding of the MTR code. Both methods suggest the redesign and enhancement of the classical Boolean logic circuits and their implementation into the new MTR decoder.

Both approaches achieved almost identical decoding gain in a serial concatenation of the MTR and LDPC codes, over the E^2PR4 magnetic recording channel. The obtained decoding gain confirms that proposed methods appropriately evaluate soft-values on the outputs of the logic circuits in a soft-decision MTR decoder, enabling their implementation in an iterative decoding scheme.

However, the slight advantage is on the side of the *min-max in LC* method, regarding the required number of operations in a new MTR decoder. The reason for this lies in the fact that the simple *min*() and *max*() decisions are required in the *min-max in LC* method in order to assess the soft-value on the output of logic circuits, compared to the *max-log MAP in LC*, which requires additional operations with the input soft-values before the decision is made.

The low number of operations recommends the *min-max in LC* method as a fairly suitable for the implementation into the soft-decision MTR decoder, which potentially can have relatively small power consumption.

This paper has indicated also, that *min-max in LC* method is about to produce the less confident output soft-values, for *AND* and *OR* logic circuits, when the input soft-values are equal, though with a different sign, $L_{in}(x_2) = -L_{in}(x_1)$. Thus, this issue can be a foundation for future research on the *min-max in LC* enhancement.

Acknowledgment

This paper has been supported by the Provincial Secretariat for Science and Technological Development of the Autonomous Province of Vojvodina, the Republic of Serbia, through the grant for the project 114-451-2061/2011-01.

References

[1] J. Moon and B. Brickner, “Maximum transition run codes for data storage systems,” IEEE Trans. Magn., vol. 32, no. 5, pp. 3992-3992, September 1996.

[2] D. J. C. MacKay and R. Neal, “Near Shannon limit performance of low density parity check codes,” IEE Electron. Letters, vol. 33, pp. 457-458, March 1997.

[3] R. M. Todd and R. Cruz, "Enforcing maximum-transition-run code constraints and low-density parity-check decoding," IEEE Trans. Magn., vol. 40, no. 6, pp. 3566-3571, November 2004.

[4] T. Kanaoka and T. Morita, "Structured LDPC Codes With Reversed MTR/ECC for Magnetic Recording Channels," IEEE Trans. Magn., vol. 42, no. 10, pp. 2561 - 2563, October 2006.

[5] M. Blaum, R. Galbraith, K. Lakovic and B. Wilson, "Reverse Concatenation with Maximum Transition Run (MTR) Codes for High-Density Perpendicular Recording," IEEE Global Telecommunications Conference – GLOBECOM 2008, November 30 – December 4, 2008, New Orleans, USA, pp. 1-4.

[6] J. Hagenauer, “Source-controlled channel decoding,” IEEE Trans. Comm., vol. 43, No. 9, pp. 2449-2457, September 1995.

[7] N. Djuric and M. Despotovic, “Soft-output decoding in multiple-head MTR encoded magnetic recording Systems,” IEEE International Conference on Communications – ICC 2006, Istanbul, Jun 11 – 15, 2006, vol. 3, pp. 1255-1258.

[8] N. Djuric, “MAP decoding of MTR codes in LDPC-MTR encoded magnetic recording systems,” in Proc. 9th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Services – TELSIKS 2009, Nis, Serbia, 7-9 October, 2009, vol. 34, no. 3, pp. 381- 392.

[9] N. Djuric and V. Senk, “The MAP in LC Decoding of MTR Codes in Two-Track Magnetic Recording Systems”, The 6th International Conference on Communication Theory, Reliability, and Quality of Service – CTRQ 2013, April 21-26, 2013, Venice, Italy, pp. 58-63.

[10] H. K. Thapar and A. M. Patel, “A class of partial-response systems for increasing storage density in magnetic recording”, IEEE Trans. Magn., vol. MAG-25, pp. 3666-3668, Sep. 1987.

[11] N. Djuric and M. Despotovic, “Application of MTR soft-decision decoding in multiple-head magnetic recording systems,” Indian Academy of Sciences, Sadhana – Academy Proceedings in Engineering Science, vol. 34, Part 3, pp. 381–392, June 2009.

[12] N. Djuric and V. Senk, “The MAP in LC As a Soft-decision Method for MTR Decoding in LDPC-MTR Concatenation”, World Symposium On Computer Networks and Information Security International Conference on Computer and Computational Intelligence – ICCCI 2014, Hammamet, Tunisia, June 13-15, 2014, vol. 1, pp. 28, 1-4.

[13] B. Vasic, E. Kurtas and A. Kuznetsov, "Kirkman systems and their application in perpendicular magnetic recording," IEEE Trans. Mag., vol. 38, no. 4, Part: 1, pp. 1705 -1710, July 2002.

[14] F. R. Kschischang, B. J. Frey and H.-A. Loeliger, “Factor graphs and the sum-product algorithm,” IEEE Trans. Inform. Theory, pp. 498-519, February 2001.

 ISBN: 978-1-61208-402-2