Modelling of Cascading Effect in a System with Dependent Components via Bivariate Distribution

Hyunju Lee[1] and Ji Hwan Cha[2]
Department of Statistics
Ewha Womans University
Seoul, Rep. of KOREA
e-mail[1]: hyunjlee@ewhain.net
e-mail[2]: jhcha@ewha.ac.kr

***Abstract*— A cascading failure is a failure in a system of interconnected parts, in which the breakdown of one element can lead to the subsequent collapse of the others. Cascading effect is quite common in power grids and can also frequently occur in computer networks (such as the Internet). In this paper, we consider systems composed of components having interdependence and cascading effect. For this, based on the notion of conditional failure rate, a new bivariate distribution for modelling the lifetimes of dependent components is constructed. A comparison of systems having interdependence and cascading effect with those having independent components is also performed.**

Keywords-modelling of cascading effect; bivariate distribution; dependence; load-sharing components; parallel system

I. INTRODUCTION

Dependent random quantities can frequently be encountered in practice and they have been modelled by using bivariate distributions. In the literature, various specific parametric models for bivariate distributions have been suggested and studied [1]-[4]. A good review on the modelling of multivariate survival models can be found in [5]. An excellent encyclopedic survey of various bivariate distributions can be found in [6].

In this paper, we propose a new general class of dependent distributions, which is different from the previous models in [1]-[6]. The structure of this paper is as follows. In Section 2, taking into account the physics of failure of items and the interrelationship between them, we propose and discuss a general methodology for constructing a general class of bivariate distributions. In Section 3, based on the developed class, we study the lifetimes of systems having interdependence and cascading effect. Finally, in Section 4, concluding remarks are given.

II. MODELLING BIVARIATE DISTRIBUTION

Suppose that the system is composed of two components: component 1 and component 2 and they start to operate at time $t=0$. The original lifetimes of components 1 and 2, when they start to operate, are described by the corresponding failure rates $\lambda_1(t)$ and $\lambda_2(t)$, respectively. These original lifetimes of components 1 and 2 are denoted by X_1^* and X_2^*, respectively, assuming that X_1^* and X_2^* are stochastically independent.

Similar to [1]'s model, we consider the practical situation when the failure of one component increases the stress of the other component, which results in the shortened residual lifetime of the remaining component. Under this type of dependency, we denote the corresponding eventual lifetimes of components 1 and 2 by X_1 and X_2, respectively.

Define $\Psi_1(t)=1$ ($\Psi_2(t)=1$) if component 1 (component 2) is functioning at time t, whereas $\Psi_1(t)=0$ ($\Psi_2(t)=0$) if component 1 (component 2) is at failed state at time t. For notational convenience, let $\tilde{i}=2$ when $i=1$; whereas $\tilde{i}=1$ when $i=2$. Then, we assume the conditional failure rate of component i is given by:

$$r_i(t\,|\,\Psi_{\tilde{i}}(s)=1, 0\le s\le t)$$

$$\equiv \lim_{\Delta t\to 0}\frac{1}{\Delta t}P(t<X_i\le t+\Delta t\,|\,\Psi_{\tilde{i}}(s)=1, 0\le s\le t, X_i>t)$$

$$=\lambda_i(t),\quad i=1,2,\quad (1)$$

and

$$r_i(t\,|\,\Psi_{\tilde{i}}(s)=1, 0\le s<u; \Psi_{\tilde{i}}(s)=0, u\le s\le t)$$

$$\equiv \lim_{\Delta t\to 0}\frac{1}{\Delta t}P(t<X_i\le t+\Delta t\,|$$

$$\Psi_{\tilde{i}}(s)=1, 0\le s<u; \Psi_{\tilde{i}}(s)=0, u\le s\le t, X_i>t)$$

$$=\alpha_i(t-u)\lambda_i(t),\ t\ge u,\ i=1,2,\quad (2)$$

where $\alpha_i(w)\ge 1$, for all $w\ge 0$, $i=1,2$.

In the following discussions, the notations $S(x_1,x_2)$, $f(x_1,x_2)$ will be used to denote the joint survival function and the joint probability density function (pdf) of X_1 and X_2, respectively. We will now suggest the joint distribution of X_1 and X_2 under the assumed model. The proof can be found in [7].

Result 1. Under the conditional failure rate model stated in (1)-(2), the joint survival function $S(x_1,x_2)$, for $0<x_1<x_2$, is given by:

$$S(x_1,x_2)$$

$$= \int_{x_1}^{x_2} \lambda_1(u) \exp\left(- \int_0^{x_2-u} \alpha_2(w)\lambda_2(u+w)dw \right)$$
$$\times \exp\left(- \int_0^{u} \lambda_1(w) + \lambda_2(w)\,dw \right) du$$
$$+ \exp\left(- \int_0^{x_2} \lambda_1(w) + \lambda_2(w)\,dw \right), \text{ for } 0 < x_1 < x_2, \quad (3)$$

and $S(x_1, x_2)$, for $0 < x_2 \le x_1$, can be obtained symmetrically by replacing x_1, x_2, $\lambda_1(\cdot)$, $\lambda_2(\cdot)$, $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ in the right-hand side of (3) with respective opposite components. The corresponding joint pdf, for $0 < x_1 < x_2$, is given by:

$$f(x_1, x_2) = \lambda_1(x_1)\lambda_2(x_2)\alpha_2(x_2 - x_1)$$
$$\times \exp\left(- \int_0^{x_1} \lambda_1(w) + \lambda_2(w)\,dw \right) \exp\left(- \int_0^{x_2-x_1} \alpha_2(w)\lambda_2(x_1+w)\,dw \right)$$
$$,\text{for } 0 < x_1 < x_2,$$

and $f(x_1, x_2)$, for $0 < x_2 \le x_1$, can also be obtained symmetrically.

III. System Reliability

In this section, we study the lifetimes of systems when the baseline distributions are Weibull (see [7]). Let $\lambda_i(t) = \mu_i \gamma_i (\mu_i t)^{\gamma_i - 1}$, $t \ge 0$, $i = 1,2$, and $\alpha_i(t) = \alpha_i t + 1$, $\alpha_i > 0$, $i = 1,2$. In this case, from Result 1, the joint survival function of $S(x_1, x_2)$ and the joint pdf $f(x_1, x_2)$ can be obtained.

Suppose that the system is a series system. Then, the lifetime of the system is $T_S = \min\{X_1, X_2\} = \min\{X_1^*, X_2^*\}$. Thus, in this case, the lifetime of a system having interdependence components is the same as that of a system having independent components. Thus, this case is not relevant to the dependence structure of the proposed model.

We now assume that the system is a parallel system. In this case, the lifetime of a system having interdependence components is $T_S = \max\{X_1, X_2\}$. In order to obtain the distribution of T_S, define $T_1 = \min\{X_1, X_2\}$ and $T_2 = \max\{X_1, X_2\}$. Then, the joint pdf of (T_1, T_2), $g(t_1, t_2)$, is given by:

$$g(t_1, t_2) = f(t_1, t_2) + f(t_2, t_1), \ t_1 \le t_2,$$

and thus, the pdf of T_S is given by:

$$f_{T_S}(t) = \int_0^t g(t_1, t)dt_1, \ t \ge 0.$$

Cleary, the pdf of lifetime of a system having independent components can be obtained by setting $\alpha_i = 0$, $i = 1,2$. The graphs of the survival functions of the system having interdependence components when $\mu_i = 0.5$, $\gamma_i = 3$, $\alpha_i = 2$, $i = 1,2$, and the system having independent components are given in Figure 1.

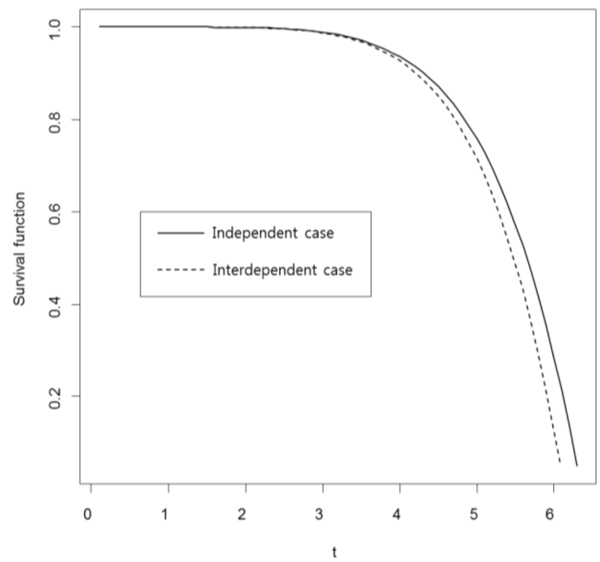

Figure 1. The survival functions.

As can be seen from Figure 1, the survival function for the interdependent case is smaller than that for the independent case. For brevity, we have just considered two simple cases (series and parallel systems). More complex cases could be considered in similar ways.

IV. Concluding Remarks

There have been studies on real systems with several interdependent components and with propagated and cascading effects (see, e.g., Mo et al. [8] and Bobbio et al. [9]). In this paper, a general methodology for constructing new general classes of bivariate distributions has been suggested. Based on the proposed class, the lifetimes of systems having interdependence and cascading effect have been studied. Based on the developed class, numerous bivariate distributions can further be generated and new issues on the estimation and testing of the model parameters should be discussed in the future studies. In this paper, our discussions are mainly focused on generating bivariate models. However, a similar approach could be applied to generate a new class of multivariate distributions. More detailed discussion on this issue is also given in Lee and Cha [7].

Acknowledgment

This work was supported by Priority Research Centers Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2009-0093827). This work was also supported by the National Research Foundation of Korea

(NRF) grant funded by the Korea government (MSIP) (No. 2016R1A2B2014211).

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