

A Method for Measuring Similarity of Simulation Time-Series Data Based on Dynamic Time Warping

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Abstract—Against the measuring problem of similarity between simulation time-series data and reference time-series data with different lengths or distorted timeline, a similarity measuring method based on DTW is proposed. Considering the trends of time-series data, a new windowing algorithm for Dynamic Time Warping (DTW) is presented. The correctness of the algorithm is proved. On this basis, simulation time-series data and reference time-series data are segmented according to event information, and the overall similarity between them is obtained by calculating DTW distance in different segments. Finally, an example is given to validate the reasonability and validity of the proposed method.

Keywords—simulation; time-series data; similarity measure; DTW; segmentation

I. INTRODUCTION

As one of the significant means of learning and changing the objective world, the simulation technique is widely used in many fields, such as spaceflight, aviation, electronics and communication [1][2][3]. Simulation system is the application form of simulation technique. Its credibility directly determines the success or failure of the simulation application. Through investigating the difference of simulation system state variables and output variables from actual system state variables and output variables, the simulation credibility can be measured. In other words, we need to investigate whether the similarity between simulation data and reference data is good enough. According to the difference of data arrangement, simulation data can be divided into simulation time-series data and non-time-series data. In the same way, reference data can also be divided to reference time-series data and non-time-series data. This paper studies how to measure the similarity between simulation time-series data and reference time-series data. Without special instructions, simulation data and reference data respectively represent simulation time-series data and reference time-series data in the following content.

Generally speaking, simulation data and reference data are considered a whole separately when investigating the similarity between them, and then their similarity is calculated with an appropriate measuring method. However, various events may occur in the running of actual system, such as “stage-1/stage-2 separation”, “fairing jettison”, “satellite-rocket separation” during rocket flight. The occurrence of an event usually brings about violent changes in actual system

state. Therefore, simulation data and reference data should be divided into segments, and the similarity between two corresponding segments is investigated alone. Combined with all the investigative results, the overall similarity between simulation data and reference data could be obtained. However, the occurrence time of an event is usually advanced or delayed owing to disturbance caused by outside factors. So, the same segments of simulation data and reference data may be distorted on the timeline, which puts forward special requirements on time series similarity measure.

At present, the commonly used time series similarity measures are Euclidean distance [4][5], Edit distance [6] and DTW distance [7][8][9]. The advantages of Euclidean distance include high calculation speed, simple principle and orthogonal transformation invariance. However, it simply regards time series as a point of multidimensional Euclidean space ignoring the time factor of data, and is very sensitive to abnormal points. The most crucial problem is Euclidean distance only applies to measure similarity between time series with synchronization on the timeline. Edit distance is applicable to string sequence. If measuring object is not string sequence, it should be transformed into string sequence by some rule, and then use Edit distance to measure the similarity between two string sequences. But the algorithm applied to transforming non-string sequence into string sequence is not accurate enough, which seriously affect the application effect of Edit distance. Besides, Edit distance also requires two string sequences keep a strict synchronization on the timeline. DTW distance does not require the sampling time of two time series are synchronous, not be sensitive to abnormal points, furthermore, it is able to measure the similarity of time series with different lengths or distorted timeline. Therefore, we introduce DTW into system simulation field, and propose a method for simulation time-series data similarity measure based on DTW. The layout of the paper is as follows: In Section 2, the basic concepts of DTW is introduced. In Section 3, a method for measuring similarity between simulation data and reference data is proposed. In Section 4, the effectiveness of the proposed method is validated through an application example. In Section 5, conclusion is drawn.

II. DTW

DTW is a nonlinear warping technique put forward by Japanese scholar Itakura in the 1960s, and was originally applied in speech recognition. In 1994, DTW was applied to

time-series analysis by Berndt and Clifford [10], and achieved great success. The basic concepts of DTW will be introduced briefly as following.

Definition 1 Suppose time series $X=(x_1, x_2, \dots, x_m)$, $Y=(y_1, y_2, \dots, y_n)$, $m \geq 1$, $n \geq 1$. If matrix D satisfies

$$D = \begin{pmatrix} d(x_1, y_1) & d(x_1, y_2) & \dots & d(x_1, y_n) \\ d(x_2, y_1) & d(x_2, y_2) & \dots & d(x_2, y_n) \\ \vdots & \vdots & \ddots & \vdots \\ d(x_m, y_1) & d(x_m, y_2) & \dots & d(x_m, y_n) \end{pmatrix} \quad (1)$$

where $d(x_i, y_j) = |x_i - y_j|$, $i=1, 2, \dots, m$, $j=1, 2, \dots, n$, then we say D is the distance matrix of X and Y .

Definition 2 Suppose time series $X=(x_1, x_2, \dots, x_m)$, $Y=(y_1, y_2, \dots, y_n)$, $m \geq 1$, $n \geq 1$. If sequences $P = \langle p_1, p_2, \dots, p_K \rangle$, $p_k \in \{(a, b) | a=1, 2, \dots, m, b=1, 2, \dots, n\}$, $k=1, 2, \dots, K$, $\max(m, n) \leq K \leq m+n+1$, satisfies

(1) Boundary conditions: $p_1=(1, 1)$, $p_K=(m, n)$;

(2) Continuity: Given $p_k=(a, b)$, then $p_{k+1}=(a', b')$,

where $|a' - a| \leq 1$ and $|b' - b| \leq 1$;

(3) Monotonicity: Given $p_k=(a, b)$, then $p_{k+1}=(a', b')$, where $a' - a \geq 0$ and $b' - b \geq 0$, or $a' - a > 0$ and $b' - b \geq 0$, then we say P is a warping path between X and Y .

There are exponentially many warping paths that satisfy the above conditions, as shown in Figure 1. However, we are only interested in the path which has the minimum cumulative distance, and regard it as the basis of measuring similarity degree between two time series.

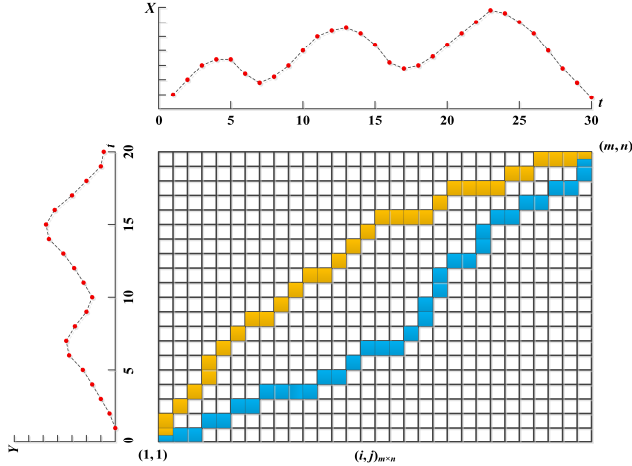


Figure 1. Warping paths between two time series

Definition 3 Suppose time series $X=(x_1, x_2, \dots, x_m)$, $Y=(y_1, y_2, \dots, y_n)$, $m \geq 1$, $n \geq 1$, then the DTW distance between X and Y is

$$DTW(X, Y) = \min \left\{ \sum_{k=1}^K d(x_{a_k}, y_{b_k}) \mid \forall P = \langle (a_1, b_1), \dots, (a_K, b_K) \rangle \right\} \quad (2)$$

The smaller $DTW(X, Y)$ is, the higher degree of similarity between X and Y is. Otherwise it is lower.

III. SIMILARITY MEASURING METHOD BASED ON DTW

This section discusses the warping window of DTW first, and constructs a new windowing algorithm. On the basis of the new windowing algorithm, a method for measuring similarity between simulation data and reference data is proposed.

A. Warping Window of DTW

In the application of DTW, swing scope of warping path is generally limited. On the one hand, this can narrow the scope to search optimal warping path, and improve calculation speed of DTW distance. On the other hand, even more important, this can avoid a morbid warping path, and ensure the reasonableness of calculation result of DTW distance. At present, the most common windowing algorithm is: for any point $p_k=(a_k, b_k)$ in the warping path, it is required to satisfy $|a_k - b_k| \leq L$. It is obvious that the value of L directly affects the reasonableness of calculation result of DTW distance. However, the value of L is generally given by experts in practice, which has strong subjectivity. In addition, warping window of the same width cannot precisely describe the distortion of time series in timeline.

Against the above problems, a new windowing algorithm called TS algorithm is presented in this section. Its basic idea is through analyzing the trend of time series, data matching is limited to segments in which two time series have the same trend. So, it can effectively prevent the excessive distortion of warping path. The steps of TS algorithm are as follows:

Step 1 Calculate the trend sequence of time series $X=(x_1, x_2, \dots, x_m)$ and $Y=(y_1, y_2, \dots, y_n)$, and get $X'=(x'_1, x'_2, \dots, x'_{m-1})$, $Y'=(y'_1, y'_2, \dots, y'_{n-1})$, where

$$x'_i = \begin{cases} 0, & x_i = x_{i+1}; \\ 2(x_{i+1} - x_i) / (|x_i| + |x_{i+1}|), & \text{otherwise;} \end{cases} \quad (3)$$

$$y'_j = \begin{cases} 0, & y_j = y_{j+1}; \\ 2(y_{j+1} - y_j) / (|y_j| + |y_{j+1}|), & \text{otherwise;} \end{cases} \quad (4)$$

$i=1, 2, \dots, m-1, j=1, 2, \dots, n-1$.

Step 2 Calculate the DTW distance between X' and Y' , and get optimal warping path $P' = \langle p'_1, p'_2, \dots, p'_K \rangle$;

Step 3 On the basis of warping path P' , we get warping window $W = \{(a, b) | a=1, 2, \dots, m, b=1, 2, \dots, n, \{(a-1, b-1), (a-1, b), (a, b-1), (a, b)\} \cap \{p'_1, p'_2, \dots, p'_K\} \neq \emptyset\}$, and then calculate the DTW distance between X and Y .

TS algorithm has great objectiveness due to no need of expert knowledge, and can precisely describe the distortion of time series in timeline.

Theorem 1 Suppose time series $X=(x_1, x_2, \dots, x_m)$ and $Y=(y_1, y_2, \dots, y_n)$. W is the warping window calculated with TS algorithm, then there exists at least one warping path between X and Y in W .

Proof For proving that there exists a warping path between X and Y in W , we only need to prove the following two conditions:

(1) $(1, 1) \in W$, $(m, n) \in W$;

Suppose P' is optimal warping path between X' and Y' , so, $p'_1 = (1,1)$ and $p'_K = (m-1, n-1)$, and then

$$\begin{aligned} & \{(0,0), (0,1), (1,0), (1,1)\} \cap \{p'_1, p'_2, \dots, p'_K\} = (1,1) \neq \emptyset \\ & \{(m-1, n-1), (m-1, n), (m, n-1), (m, m)\} \cap \{p'_1, p'_2, \dots, p'_K\} \\ & = (m-1, n-1) \neq \emptyset \end{aligned}$$

Therefore, $(1,1) \in W$, $(m,n) \in W$.

(2) If $(a,b) \in W$ and $(a,b) \neq (m,n)$, then $\{(a,b+1), (a+1,b), (a+1,b+1)\} \in W \neq \emptyset$;

Suppose P' is optimal warping path between X' and Y' , so for $\forall (a',b') \in \{p'_1, p'_2, \dots, p'_K\}$, $(a',b') \neq (m-1, n-1)$, we all have

$$\{(a',b'+1), (a'+1,b'), (a'+1,b'+1)\} \cap \{p'_1, p'_2, \dots, p'_K\} \neq \emptyset$$

Then

$$\begin{aligned} & \left. \begin{aligned} & (a,b) \in W \\ & (a,b) \neq (m,n) \end{aligned} \right\} \Rightarrow \{(a-1,b-1), (a-1,b), (a,b-1), (a,b)\} \cap \\ & \{p'_1, p'_2, \dots, p'_K\} \neq \emptyset \\ & \Rightarrow \{(a-1,b), (a,b-1), (a,b)\} \cup \\ & \{(a-1,b+1), (a,b), (a,b+1)\} \cup \\ & \{(a,b), (a+1,b-1), (a+1,b)\} \cup \\ & \{(a,b+1), (a+1,b), (a+1,b+1)\} \cap \\ & \{p'_1, p'_2, \dots, p'_K\} \neq \emptyset \\ & \Rightarrow \{(a-1,b), (a-1,b+1), (a,b), (a,b+1)\} \cap \{p'_1, p'_2, \dots, p'_K\} \cup \\ & \{(a,b-1), (a,b), (a+1,b-1), (a+1,b)\} \cap \{p'_1, p'_2, \dots, p'_K\} \cup \\ & \{(a,b), (a,b+1), (a+1,b), (a+1,b+1)\} \cap \{p'_1, p'_2, \dots, p'_K\} \neq \emptyset \\ & \Rightarrow \{(a,b+1), (a+1,b), (a+1,b+1)\} \cap W \neq \emptyset \end{aligned}$$

From the above, we know that there exists at least one warping path between X and Y in W .

B. Steps of Algorithm

Suppose reference data $R=(r(t_1), r(t_2), \dots, r(t_n))$, simulation data $S=(s(t_1), s(t_2), \dots, s(t_n))$, $t_1 < t_2 < \dots < t_n$. Event sequence $E=(e_1, e_2, \dots, e_K)$ is happened to actual system during $t_1 \sim t_n$, and its happening time is $T_1=(t_{1,1}, t_{1,2}, \dots, t_{1,K})$; The same event sequence $E=(e_1, e_2, \dots, e_K)$ is happened to simulation system during $t_1 \sim t_n$, and its happening time is $T_2=(t_{2,1}, t_{2,2}, \dots, t_{2,K})$.

To measure the similarity between reference data R and simulation data S , a similarity measuring method is proposed,

$$R_k = \begin{cases} (r(t_1), r(t_2), \dots, r(t_i)), & k=1, t_i \leq t_{1,k} < t_{i+1} \\ (r(t_i), r(t_{i+1}), \dots, r(t_{i+j})), & 2 \leq k \leq K, t_{i-1} \leq t_{1,k-1} < t_i, t_{i+j} \leq t_{1,k} < t_{i+j+1} \\ (r(t_i), r(t_{i+1}), \dots, r(t_n)), & k=K+1, t_{i-1} \leq t_{1,k-1} < t_i \end{cases} \quad (5)$$

$$S_k = \begin{cases} (s(t_1), s(t_2), \dots, s(t_i)), & k=1, t_i \leq t_{2,k} < t_{i+1} \\ (s(t_i), s(t_{i+1}), \dots, s(t_{i+j})), & 2 \leq k \leq K, t_{i-1} \leq t_{2,k-1} < t_i, t_{i+j} \leq t_{2,k} < t_{i+j+1} \\ (s(t_i), s(t_{i+1}), \dots, s(t_n)), & k=K+1, t_{i-1} \leq t_{2,k-1} < t_i \end{cases} \quad (6)$$

in combination with research achievement in the DTW theory. Its steps are as follows:

Step 1 According to the event sequence E , R is segmented into $R=\langle R_1, R_2, \dots, R_{K+1} \rangle$, and S is segmented into $S=\langle S_1, S_2, \dots, S_{K+1} \rangle$, as shown in (5) and (6).

Step 2 Using group AHP to determine the weight of each segment. This step includes the following four substeps:

Step 2.1 According to the actual situation of simulation object, a number of experts with relevant background and different knowledge structure are invited to set up an evaluation team $P=\{p_1, p_2, \dots, p_m\}$. The subjective weight of expert p_i is obtained from professional title, evaluation mode and the degree of familiarity with simulation object. The formula is as follows:

$$\lambda_{p_i}^{(s)} = \frac{l_{i,1} \times l_{i,2} \times l_{i,3}}{\sum_{j=1}^m l_{j,1} \times l_{j,2} \times l_{j,3}} \quad (7)$$

where $l_{i,1}$, $l_{i,2}$, $l_{i,3}$ are valued as shown in Table 1;

TABLE I. THE FACTOR VALUE OF EXPERT SUBJECTIVE WEIGHT

Professional title	$l_{i,1}$	Evaluation mode	$l_{i,2}$	The degree of familiarity with simulation object	$l_{i,3}$
professor/researcher/professor senior engineer	4	two-way anonymous	3	very familiar	4
associate professor/associate researcher/ senior engineer	3	one-way anonymous	2	familiar	3
lecturer/assistant researcher/engineer	2	two-way open	1	general understanding	2
ta/intern researcher/assistant engineer	1			not familiar	1

Step 2.2 The difference between R and S in each segment has different degree of influence on simulation credibility. So, expert e_i is invited to make comparison between the influence degrees, then we get judgement matrix

$$A_i = (a_{jk}^{(i)})_{(K+1) \times (K+1)} \quad (8)$$

$i=1, 2, \dots, m$;

Step 2.3 The consistency ratio of judgement matrix reflects the thinking logicity of expert when he makes a judgement. The greater the consistency ratio, the more serious the logic conflict of judgement matrix, the less objective weight of relevant expert; On the contrary, the more objective weight of relevant expert. Hence, the objective weight of

expert p_i is obtained from the consistency ratio. The formula is as follows:

$$\lambda_{p_i}^{(c)} = \frac{1/e^{CR(A_i)}}{\sum_{j=1}^m (1/e^{CR(A_j)})} \quad (8)$$

where $CR(A_i)$ is the consistency ratio of A_i , whose solving process is referred to literature [11];

Step 2.4 According to expert subjective weight and objective weight, expert synthesis weight is calculated by the following formula:

$$\lambda_{p_i} = \tau \lambda_{p_i}^{(s)} + (1-\tau) \lambda_{p_i}^{(c)} \quad (9)$$

where τ is an adjusting coefficient, $0 \leq \tau \leq 1$. The smaller τ is, the more attention pay to the actual performance of expert in evaluation work; The greater τ is, the more attention pay to the qualification of expert;

Step 2.5 Combined with expert synthesis weight, synthesis judgement matrix is obtained by using Hadamard convex combination to aggregate individual judgement matrices as follows:

$$\bar{A} = (\bar{a}_{jk})_{(K+1) \times (K+1)} = \left(\prod_{i=1}^m (a_{jk}^{(i)})^{\lambda_{p_i}} \right)_{(K+1) \times (K+1)} \quad (10)$$

At last, the weight of each segment is calculated by \bar{A} , then we get $\lambda_1, \lambda_2, \dots, \lambda_{K+1}$;

Step 3 Calculate the average DTW distance R_i and S_i based on TS algorithm, the formula is as follows:

$$\overline{DTW}(R_i, S_i) = \frac{DTW(R_i, S_i)}{l_i} \quad (11)$$

where $DTW(R_i, S_i)$ is the length of optimal warping path, l_i is the number of steps in optimal warping path, $i=1, 2, \dots, K+1$;

Step 4 Calculate the similarity between R and S , the formula is as follows:

$$\Psi(R, S) = \frac{1}{1+\delta} \quad (12)$$

$$\text{where } \delta = \frac{\sum_{i=1}^{K+1} \lambda_i \cdot \overline{DTW}(R_i, S_i)}{\sqrt{\frac{1}{n} \cdot \sum_{i=1}^n \min(r^2(t_i), s^2(t_i))}}, \quad 0 \leq \Psi(R, S) \leq 1.$$

IV. APPLICATION EXAMPLE

To avoid interception by our anti-aircraft missile, enemy attack missiles always tend to be changing orbit dramatically. Guidance law needs to be designed for our missile to intercept enemy missile effectively. A guidance simulation model is set up to validate guidance law. Take lateral acceleration output of this model for example, the effectiveness of the proposed method in this paper is validated.

Assume the motion of the target is as follows:

1) Before orbital maneuver: keep 8000m height, do horizontal straight line motion with a speed in 600m/s;

2) After orbital maneuver: keep 8000m height, do sine motion around the straight line, the sine law: $150\sin(0.3t)\text{m}$;

3) The maneuver time $t(s)$ obeys the normal distribution: $N(6, 0.2^2)$.

Data sampling interval: 0.02s, Sampling period: 0s~10s, 500 samples in total, as shown in Fig.2, where R is reference data of lateral acceleration and S is simulation data of lateral acceleration.

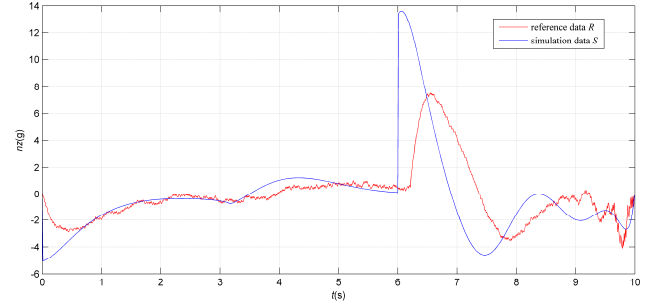


Figure 2. Comparison between reference and simulation lateral acceleration

Using the proposed method to calculate similarity between R and S as follows:

First, according to maneuver time of target, R is segmented into $R = \langle R_1, R_2 \rangle$, and S is segmented into $S = \langle S_1, S_2 \rangle$, where $R_1 = (r(t_1), r(t_2), \dots, r(t_{310}))$, $R_2 = (r(t_{310}), r(t_{311}), \dots, r(t_{500}))$, $S_1 = (s(t_1), s(t_2), \dots, s(t_{300}))$, $S_2 = (s(t_{300}), s(t_{301}), \dots, s(t_{500}))$.

Then, invite three experts to constitute an evaluation team $P = \{p_1, p_2, p_3\}$, and use (7) to calculate subjective weight of expert as follows:

$$\lambda_{p_1}^{(s)} = \frac{4 \times 3 \times 4}{4 \times 3 \times 4 + 4 \times 3 \times 2 + 3 \times 3 \times 3} = 0.485$$

$$\lambda_{p_2}^{(s)} = 0.242, \quad \lambda_{p_3}^{(s)} = 0.273.$$

The difference between R and S has influence on simulation credibility, and the influence degree is different before and after target orbital maneuver. In the light of this, each expert gives an individual judgement matrix as follows:

$$A_1 = \begin{pmatrix} 1 & 1/3 \\ 3 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 1 & 1/4 \\ 4 & 1 \end{pmatrix}$$

In combination with A_1 , A_2 , and A_3 , use (9) to calculate objective weight of expert: $\lambda_{p_1}^{(c)} = \lambda_{p_2}^{(c)} = \lambda_{p_3}^{(c)} = 0.333$; On this basis, use (10) to calculate synthesis weight of expert: $\lambda_{p_1} = \tau \lambda_{p_1}^{(s)} + (1-\tau) \lambda_{p_1}^{(c)} = 0.447$, $\lambda_{p_2} = 0.265$, $\lambda_{p_3} = 0.288$, where $\tau = 0.75$.

In combination with synthesis weight of expert, use (11) to calculate comprehensive judgement matrix which is

$$\bar{A} = \begin{pmatrix} 1 & 0.411 \\ 2.436 & 1 \end{pmatrix}$$

And then we get $\lambda_1 = 0.291$, $\lambda_2 = 0.709$.

Next, the average DTW distances between R_1 and S_1 , R_2 and S_2 are calculated respectively based on TS algorithm: $\overline{DTW}(R_1, S_1) = 0.572$, $\overline{DTW}(R_2, S_2) = 1.437$, their warping paths are shown in Fig.3.

Finally, calculate the overall similarity between R and S

$$\Psi(R, S) = \frac{1}{1 + \frac{0.291 \times 0.572 + 0.709 \times 1.437}{1.655}} = 0.583$$

As contrast, use Euclidean distance to calculate the similarity between R and S , and get $\Psi'(R, S) = 0.380$. In calculation process of $\Psi(R, S)$ and $\Psi'(R, S)$, the alignment way of time series is shown in Fig.4.

It is clearly observed that the proposed method can measure similarity between simulation data and reference data with different length, and it has a strong robustness on the distortion of timeline. $\Psi(R, S) > \Psi'(R, S)$ also shows that the proposed method make up for the deficiency of traditional simulation data similarity measures, and the measurement result is more objective and reasonable.

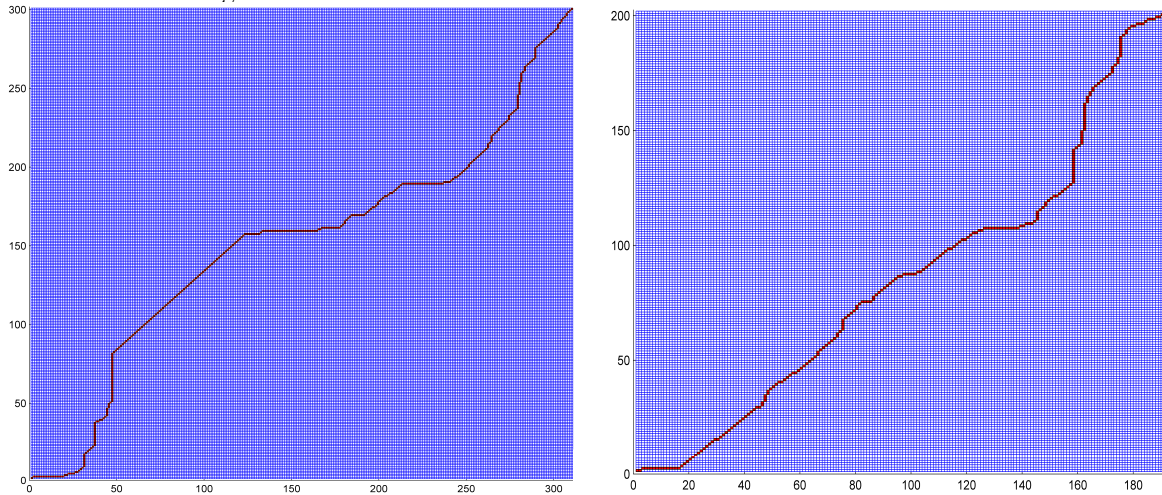


Figure 3. Warping paths of DTW distance

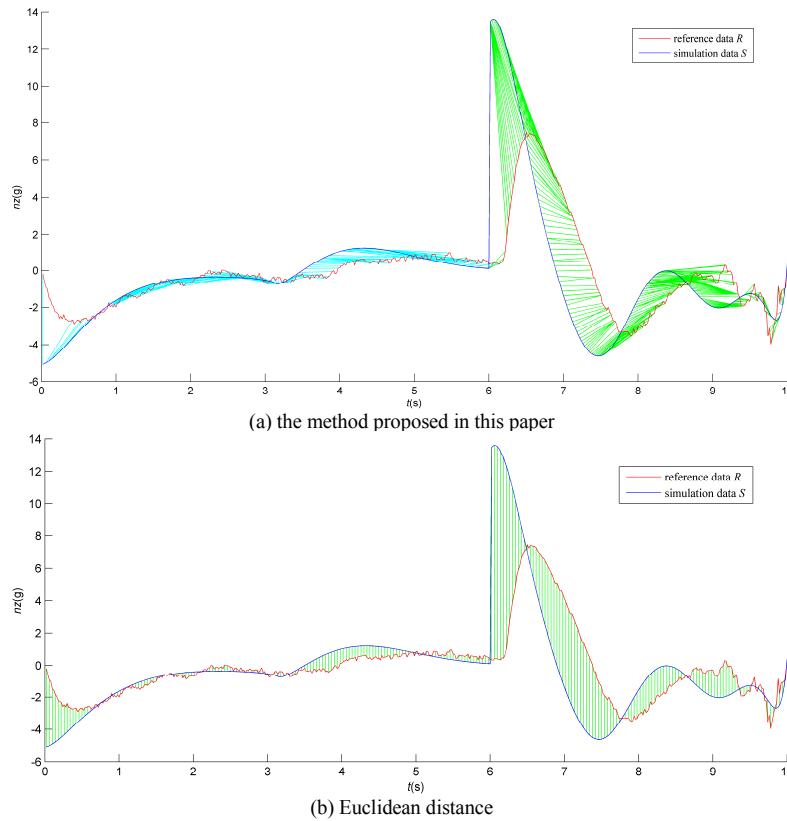


Figure 4. The alignment way of time series

V. CONCLUSION

The similarity between simulation data and reference data is generally investigated when we evaluate the credibility of simulation system. As system states usually change sharply after the occurrence of some event, so, data need to be segmented for analysis. Against simulation time and actual time are not in synch, a similarity measuring method based on DTW is proposed, using a new windowing algorithm for DTW. Application example shows that the proposed method can effectively measure the similarity between simulation data and reference data with different lengths or distorted timeline, and has a good prospect in the field of simulation model validation.

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