The Potential of Support-Rich Environment for Teaching Meaningful Mathematics to Low-Achieving Students

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***Abstract*— We have designed a collaborative, computer-supported, rich environment to promote meaningful mathematics among low-achieving students (LAS). Fifth-grade students interchangeably solved decimal subtraction tasks with peers in the context of a computer game and simulations, and in discussion sessions, led by their teachers, in foursomes. We describe the results of the first round of our design-based research, where we traced three such groups, using observations and interviews. We found that the computer context was both constructive and destructive, in terms of students' learning. The group discussions did not yield the rich discussions we had hoped for. Yet, overall, the environment was successful because students gained meaningful mathematical knowledge and practiced active, thoughtful, and collaborative socio-mathematical behavior, which is dramatically different from what they were used to.**

Keywords-low-achieving students; support-rich environment; computer games; scaffoldings; computer-supported collaborative learning.

I. Introduction

The question of how students' construction of meaningful knowledge can be supported presents an important challenge to researchers and teachers alike. Teaching the complex topic of mathematics to low-achieving students (LAS) poses a special challenge, owing to LAS's unique cognitive and behavioral characteristics [12]. The teaching and learning processes of LAS have been studied by examining different teaching methods, strategies, and tactics (e.g., [2]). However, we found sparse work on the effectiveness of rich environments, let alone environments of computer-supported collaborative-learning (CSCL), on the learning processes and outcomes of LAS.

In fact, LAS characteristics, which we describe next, might bring one to suspect the feasibility of teaching LAS basic mathematics, let alone in (Computer Supported) Collaborative Learning (CS)CL settings. Nonetheless, we hypothesized that a rich CSCL environment, involving a computer game, real context mathematics, peer discussions, and teacher mediation may be the key for addressing the LAS's unique and diversified needs. Here, we describe the results of the first round of a design-based research we have conducted to examine these hypotheses. We first describe the characteristics of LAS. Then, we review the literature and how it influenced our hypotheses and design. Next, we describe a study, the first round of a design-based research in which we examined our hypotheses. We traced the participation of 3 groups of four students each in the activities we had designed, using various data sources, such as the videotapes and audiotapes of the classes, interviews, and ad-hoc conversations with the students and the teachers, along with observations. We discuss our findings and the practical implications on our design framework and the broader community. Our main conclusion is that CSCL, when carefully designed, can promote LAS learning of meaningful mathematics as well as the development of socio-mathematical skills.

The rest of the paper is structured as follows. In Section II, we review the literature on LAS as well as on successful interventions in terms of meaningful learning. Then, we describe our pedagogical design (Section IV), and the literature that inspired us in the design, such as the decision to involve a computer-game session in which students work in pairs, and small-group discussions led by the teacher (Section III). We then describe the study (Section V). We examined how the rich environment either hinders or supports students' construction of mathematical meaning. Our focus was on the mutual interplay between the two contexts in which students worked (on the computer and in group discussions). We present the findings (Section VI) and discuss them (Section VII).

II. LAS and Meaningful Mathematics

There is no single, definitive profile for LAS [7][16]. In fact, most of studies have not focused on the methodological criteria used to identify those students with learning disabilities [16]. LAS are commonly identified based on two factors: teacher reports and their performance on standardized or informal tests (students' score below the 50th percentile on standardized tests; however, they are not diagnosed as having learning disabilities) [2]. In attempting to explain LAS's poor performance, the literature focuses on cognitive deficiencies and on behavioral manifestations of their failures. LAS find it difficult to retrieve basic

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mathematics knowledge from their memory [10]. Craik [5] terms this difficulty as 'fragile memory', a product of superficial data processing. They also lack meta-cognitive skills [9], and are sensitive to the learning contexts. They thus find it much harder than others to solve simple and complex addition and subtraction problems. These difficulties may lead them to use less sophisticated strategies and to make more errors.

Recently, Karagiannakis et al. [14] developed a model that can be used to sketch students' mathematical profiles for four domains (numbers, memory, number line, and reasoning); they empirically examined it to determine whether and how it can differentiate students with and without difficulties in learning mathematics. According to their analysis, students, both the normal/high achievers and the underachievers, do not all share the same sets of strong or weak mathematical skills. In addition, under achievement in mathematics is not related to weaknesses in a single domain (e.g., numbers, memory, number line, and reasoning). They also suggest that for LAS students, just like for the other students, cognitive strengths or weaknesses may rely on any of the four domains (mentioned above) of their model. Their findings empirically strengthen the heterogeneity of this population group.

Experiencing repeated failures and difficulties in keeping up with the class might in turn, decrease their motivation and sense of internal responsibility and make them more passive learners. It might also lead them to act impulsively, rely on the judgment and feedback of an external authority [12], and avoid collaborative work with peers [1]. Their schooling-purposed interaction in class is, for the most part, with the teacher.

These characteristics probably underlie many teachers' beliefs that LAS are unable to deal with tasks involving high-order thinking skills and that the most effective way of promoting mathematical performance in LAS is to 'drill and kill', that is, to focus more on the mathematical algorithms than on the mathematical meaning [15]. However, despite their difficulties, there is empirical evidence that in certain environments LAS are capable of enhancing their mathematical understanding. There is empirical evidence that LAS can exhibit mathematical reasoning orally when placed in intimate and supportive learning environments, such as in small groups where they are tutored [3][15]. Peltenburg et al. [20] show that, in a familiar context with the help of technological tools, LAS can succeed in solving subtraction problems by using an indirect addition strategy spontaneously, rather than the conventional direct subtraction strategy. Karagiannakis and Cooreman [13] suggest that these interventions should be designed for repeated success by building on a student's strengths, while avoiding use of repetitive tasks that cause repetitive failure experiences, thereby maximizing the learning opportunities of all students.

This led us to assume that a rich environment that includes technological tools, small groups, and teacher's support building on LAS' strengths might be the key for their success.

III. The Literature Inspiring the Design and Hypotheses

Our design was inspired by the socio-cultural theoretical perspective on learning, especially the notion of distributed scaffolding. Scaffolding is "titrated support that helps learners learn through activity. It helps learners perform tasks that are outside their independent reach and consequently develop the skills necessary for completing such tasks independently" [24, p.306] . Because LAS vary in their behavior, in our design we sought to design distributed scaffoldings [22], i.e., to integrate and sequence multiple forms of support via various means. Different scaffolds interact with each other; sometimes they produce a robust form of support, a synergy [24], and other times they might sabotage the learning processes and the outcome.

We were inspired by the Learning in Context approach, namely, the idea of presenting mathematical concepts and procedures in a context relevant to the child's day-to-day life [11], and in particular, the Realistic Mathematics Education (RME) theoretical framework. According to the RME framework, students should advance from contextual problems using significant models that are situation related, to mathematical activity at a higher level (e.g., engaging in more formal mathematical reasoning). As students progress from informal to more formal mathematics, their "*model of*" the situation is transformed into a "*model for*" reasoning. We hypothesized that RME could be the key to promote meaningful learning for LAS, because the subtraction tasks, the mathematics to be mastered, will be associated with real-life experiences, which might mitigate their fragile memory and tendency for superficial processing of new knowledge.

We aimed at transforming students' social and socio-mathematical norms, from passive to active, from isolated to social collaboration, from impulsive to thoughtful. We were motivated by the premise that digital games, by the nature of their design, have the potential to motivate students in becoming active rather than passive, by enabling experimentation and exploration without fear of failing in front of the entire class [8][23]. The use of games for teaching may be particularly beneficial for LAS because of their tendency to remain passive and to comply with authoritative voices. We were aware of the possibility that a hands-on, minds-off strategy might emerge, especially because of the tendency for impulsivity. This is one of the reasons students were asked to work with peers in front of the computer. We assumed that collaborative settings would trigger twofold interactions: with the system and with the co-learner. Peers would explain their calculations to each other, and question other actions, which would bring about reflection and thoughtfulness [6].

Every session was designed to include interchangeable students' work in front of the computer with their peers, along with group discussions, led by the teacher. Teachers' interactions with students can create zones of opportunities that can be directed to scaffold students' social and emotional development [19]. The teacher can mediate the use of tools (e.g., computer games, online units), orchestrate the students' activities, and reframe them conceptually [17].

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The students, hence, experienced two different collaborative settings. When students worked (in pairs) in front of the computer (game or online units), the teachers were asked to observe them and to offer help when necessary (for instance, if students maintain trial and error strategies or are stuck in their calculation process). In the group discussions, the teachers were asked to focus the discussion on various strategies that can be used to solve subtraction tasks, encourage students to verbalize their thoughts, and encourage them to rely on each other's past experience, thereby facilitating students in learning the meaning of how to participate in the community, i.e., support the transformation of their sociomathematical norms [4]. In these discussions, the teachers also introduced students to new tasks and encouraged them to employ the strategies previously used in a supposedly new context. As we will explain in the next section, in our design we presented tasks sometimes as stories and sometimes as formal subtraction exercises, and gradually increased the difficulty of calculating the numbers whose decimals are half, to numbers, whose decimals include individual units. We assumed that students' sense of security when expressing themselves publicly would increase, since they are in a group of equals, and will experience active (and successful) work with their peers in front of the computer.

IV. The Instructional Design

We developed an extracurricular program for fifth grade LAS. It consisted of ten weekly sessions that focus on subtraction with decimal numbers, a topic that students had not yet learned in their regular classes. Students were categorized into groups of four, according to their regular class, and each group worked with a teacher trained by the second author.

We utilized a *real life context* simulated by an ice-cream shop computer game. Specifically, during the sessions, students played a computer game in which they received orders from random customers, prepared the orders, calculated the price to be paid, and gave change as needed (Figure 1). Because of the heterogeneity of the LAS and their individual needs, we sought to provide a variety of support types. Therefore, students also worked on supplementary online study units concerned with the transition between money and formal representations, as well as change calculations. Students also enacted game-like situations with play money in Israeli bills and coins: New Israeli Shekels (NIS) and agorot (1NIS = 100 agorot, and the smallest coin is 10 agorot). In order to support the transition from the concrete to the abstract, real-paper worksheets were designed, which included exercises in concrete, graphic, and abstract forms.

In order to facilitate a delicate transition from the realistic environment (shop simulation) to formal mathematics, subtraction was first presented through monetary simulations and calculations only, and formal representations were interwoven at a later stage. The program progresses in a *spiral-like manner*. With the help of the teacher, students are expected to progress from one level to the next. The tasks at each level maintain an overall forward trend of increasing complexity, and students are able to revisit earlier levels and solve simpler exercises on the computer on their own. The teachers had the flexibility to attune the program, in response to students' emerging needs.

Figure 1. A screenshot of an online learning unit, where the task at hand is 50-38.6.

In each session, students spent almost half of their time in front of the computer, working in pairs. They were first introduced through online activity to two avatars, a girl and a boy, each of whom described a strategy for calculating the required change. Then they played or worked in pairs on the computer. The other half was devoted to class discussions, as described above. Specifically, in order to address LAS's tendency to passively rely on external authority and to encourage them to take personal responsibility, the teachers were not supposed to correct students' strategies directly, but rather, to ask questions to encourage them to talk aloud about their thinking processes, thus, making diagnosis easier and potentially leading them to correct their own mistakes, re-voicing when needed, and referring them to suitable tools in the environment when necessary. The teacher generally followed these instructions well.

V. The Study

Our goal was to examine our design's hypotheses, i.e., to examine how the rich environment either hinders or supports students' construction of mathematical meaning, especially the mutual interplay between the two contexts in which students worked (on the computer and in group discussions).

A. *Participants*

We traced 12 LAS (4 male, 8 female) from 3 fifth grade classes in suburban schools within the same city, who participated in the program. All participants were chosen based on the recommendation of their mathematics teachers. They all performed under the 50^{th} percentile on standardized tests, yet were not diagnosed as having learning disabilities.

B. *Data Sources*

In two groups all sessions were videotaped. In one group they were audiotaped. We observed students in their regular class two times before they began participating in our activities. We also observed all the sessions, and documented how the teacher presented the tasks, focusing on the sequence of activities—of both the teacher (e.g., presenting the tasks, intervening during the computer sessions, suggesting a tool, getting students' attention, answering questions) and the students (e.g., how they interact with the computer, with each other, with the teacher, and so forth). We conducted interviews with the CSCL teachers, after the activity as well as ad hoc conversations after every session. We also talked with the parents' class mathematics teachers and to each student after the CSCL activity.

C. *Methods of Analysis*

Our report mainly draws on the analysis of the videotapes. We were inspired by the analysis model of Powell et al. [21] for developing mathematical ideas and reasoning. We fully transcribed one group through videotapes. The transcripts were coded twice by two researchers. We segmented the text into episodes, each beginning with the presentation of a new task and ending with its being accomplished (or the work on it was terminated). For each episode we examined: (1) who participated in it; (2) the knowledge pieces that emerged; (3) the difficulties that arose, including whether they were solved, and if so, how and by whom, especially (d) the support provided by the teacher; and (5) whether the task was successfully accomplished independently or with help from others. We also coded affective utterances, both positive and negative. We compared the results with the video, audio, and notes taken during the observations in the other groups. Interviews were analyzed thematically.

VI. Findings

As we hypothesized, the computerized environment, especially the computer game, encouraged the students to be active as well as engaged in their task. For the most part, they were observed to be very focused on the task in hand. In fact, in 5 sessions, students continued working (or playing) after the class had ended. The students reported in the interviews and ad hoc conversations that they had enjoyed the activity. The following quotes are but two examples of typical phrases heard throughout the entire program: "it was fun…not a regular class", "playing with the computer gives a sense of fun, [vs.] a blackboard, where you just sit and solve exercises".

On the computer the students usually decided to work in turns. In each turn the one on the keyboard gave ice-cream, calculated the price, the change, and returned change. For a few couples, we noticed a different division of labor: the one on the keyboard interacted with the avatar clients and in the meantime, the other did the calculations. In a few cases when one student took over the keyboard the teacher interfered.

During the play, each student solved many subtraction exercises, manifested by the need to give change to customers in the shop.

Failures in this context did not discourage them. On the contrary, this is when we observed collaboration, mathematical discussions with their peers and with the teacher. Usually, when they received a response from a "customer" indicating that the change they gave was incorrect, they were observed pausing to think and sometimes they turned to their peers and verbalized their "solution process". Sometimes this verbalization was performed after their peers asked them how they had worked. The discussion helped them many times to correct themselves. This behavior was dramatically different from the observed passivity (or impulsivity) in the regular classes. Moreover, in this context, the students generally welcomed the teachers' intervention and cooperated with them. Hence, the computer and the peers often generated a synergetic effect on the students.

However, we also observed an appreciable number of situations in which students merely employed trial and error, using the immediate feedback of the computer ("too much" and "too little") to guess the correct answer. Usually the partner became silent in these situations. From the conversations in these situations, we learned that the pressure of time and the wish to gain as many points as possible in the game in a designated time encouraged this behavior. In one extreme example, one student stopped working because the clients became angry, because it took her time to calculate. We also noticed that in the initial lessons the teacher had to compete with students' attention to their computer in these situations. We observed the teacher, in such situations, touching the students' hand or shoulder to get their attention.

We observed many expressions of frustration among the students during group discussions. The teacher borrowed the idea of students taking turns when at the computer and asked them to solve exercises in turns in the group discussions. However, this idea turned out to be less productive. For the most part, the interaction took the form of one student explaining his or her solution process, followed by the teacher's verbalization. The teacher sometimes told the peers to be quiet, in an attempt to assist the individual to think and (re-)calculate. We thus observed almost no rich peer discussions about strategies. In her interview she explained that students' poor discursive habits made her prioritize the individual's learning over building a community and discursive habits.

We expected that during the participation the students' ability and wiliness to provide explanations would increase. During the discussion with the teacher (with or without a computer) the students were constantly asked to describe and explain their strategies. The alienation of this request was prominent in their responses. They became silent, gave vague or non-informative answers (e.g., "I just did so"), and sometimes even said, "I don't remember".

In some of the students there was evidence of a change in their discursive manners. In these cases we found that students relied on the money model (especially the fact that 1 nis = 100 agorot) to explain their subtraction strategies even when the subtraction task was phrased in an abstract manner and not in money terms. Real context mathematics, hence, supported students' leaning.

We also expected that the students would develop many strategies for subtraction. Indeed, the teacher posed questions like "in what way would you like to solve this problem?" at least three times in each of the first three sessions. However, we did not observe the emergence of a new strategy. One possible explanation is rooted in our sequencing of students' activities. In the initial lessons, students were introduced by an online unit to two strategies, presented to them by two avatars, who dealt with the task of calculating change. Possibly, this early exposure, together with students' tendencies to rely on external authoritative voices, brought about a fixation in their thoughts. Moreover, sometimes we were not sure that students understood the meaning underlying these strategies.

Nonetheless, in conversations with the teachers in the regular classes after the program ended, the teachers reported that the behavior of most of the participants in their class improved; specifically, that despite their difficulties they were more motivated and less passive.

VII. Discussion and Conclusions

The findings support the premise that RME is valuable in facilitating LAS meaningful learning [11]. Students adapted the real-life money model to resolve the subtraction tasks, even when given in an abstract form.

The computer-peer setting was found to be both supportive and destructive in terms of students' learning. The computer played a major role in making students active and engaged in mathematical discussions about the subtraction task in hand with their peers and the teacher, despite the students' fragile knowledge. We saw moments of synergy [24] when the presence of peers brought about a reflection about a wrong calculation, and a discussion about the strategy applied. The teacher's intervention in this context was welcomed and fruitful. However, we also observed situations in which the computer *game* encouraged trial and error because of the time factor and the competitive nature of games.

The group discussions did not yield the rich discussions we had hoped for. Although we had observed that the ability of most students to provide explanations had developed during their participation, these students did not develop new strategies, but rather, used the strategies they had been introduced to at the beginning. This behavior aligns with the LAS's tendency to focus on a given algorithm, given by an external authority. In addition, in this context, students' discursive acts were mostly in response to the teacher and merely addressed her.

Finally, in our design we had expected a metaphorical diffusion between the two contexts in which students performed and collaborated—that students' activeness, ability, and wiliness to discuss with their peers when failing to solve a task on the computer would diffuse to the group discussion context and that the teacher-led discussions would enrich the mathematical discursive practices, which would then diffuse to the computer context.

Apparently, this diffusion is not straightforward and a fine-tuned design is required to support its occurrence. Therefore, in in the next round we re-designed the group discussions in consultation with the literature on Accountable Talk [18], aiming at better facilitation of establishing the norms of mathematical peer discussions. We minimized the time spent in front of the computer game and instead, added time to the online unit, in which students still simulated the ice-cream shop, but without the pressure of time and gaining points. Finally, we aimed at setting the students' mindset right from the beginning by explaining to them that this class is about *their* strategies. We omitted the introduction to the two strategies, and instead, simulated in class an affair where students brought personal items and had to give money and get change and then conducted a discussion on their calculation strategies.

More work is required to fine tune the design. A larger sample of participants is necessary in order to generalize and further explore LAS learning processes and outcomes in this environment and gain insights as to how to support their learning. Nonetheless, this study shows that overall the rich CSCL environment was successful not only because students gained mathematical knowledge—they also adapted strategies to solve subtraction tasks. These students also practiced socio-mathematical behavior different from what they were used to: from passive reliance on authority, impulsive, and individualistic interactions in class, to active, thoughtful collaboration about mathematical meaning. According to the regular class teachers, to some extent, this behavior has diffused to their regular classes. We thus can conclude that meaningful learning of LAS is feasible and furthermore, that LAS can benefit from CSCL settings, which stands in contrast to their characteristics in the literature as passive or even detached individualists [2]. In this aspect our work makes a modest step towards achieving equity in mathematics education by extending the teaching of mathematical meaning to academically diversified students.

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