Calculations of the Packet Loss Ratio in Finite-Buffer Queues

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***Abstract*—In most packet networks and the Internet, packets are queued at routers' output interfaces before being sent to next nodes. As the buffers for these queues are finite and the incoming traffic is unpredictable, with possible bursts, a fraction of arriving packets is lost, due to the buffer overflow. The packet loss ratio is thus one of the most important performance characteristics, widely studied via actual network measurements, simulations and mathematical modeling. The purpose of this paper is to discuss the methods of computing the loss ratio using queueing models and to collect the results obtained via the empty queue probability method. Various queuing models are taken into account, with general service time distribution and different assumptions on the arrival stream (Poisson, group structure, autocorrelated).**

***Index Terms*—Internet, performance evaluation, packet losses, queueing model, loss ratio**

I. Introduction

Contemporary networks are usually organized according to the "best effort" concept, which means that there is no resource reservation on the path from the sender to the destination. Moreover, statistical multiplexing of streams of packets in routers causes occasional bursts of traffic arriving to routers' output interfaces. As a consequence, the buffer of the interface is overflowed for some time, and the newly arriving packets are lost. Obviously, these losses have a deep impact on the performance of the network and the overall quality of communication.

The single, most important characteristic describing packet losses is the loss ratio, denoted herein by L. Obviously, L is defined as the number of packets lost at an interface, divided by the total number of packets arriving to it in a long time interval. The loss ratio and other characteristics of the loss process have been widely studied using mathematical modeling, (e.g., [1]–[9]), discrete-event simulators (like Omnet++ and Ns-3, [10], [11]) and actual network measurement (e.g., [12]–[18]).

In this paper, we deal with computations of the loss ratio using analytical approach. As queues of packets at routers' output interfaces are simple single-server queues, the appropriate analytical model is the classic queueing model with the single service station and a finite buffer, possibly with various assumptions on the arrival stream. In particular, we are interested in M/G/1N, M^X/G/1/N, MMPP/G/1/N and BMAP/G/1/N queueing models, in Kendall's notation.

The loss ratio in a queueing model can be computed using direct and indirect methods. In the direct method, $M(t)$ is computed first. It denotes the average number of losses in an interval $(0, t)$. Then, the loss ratio is obtained as a limit of $M(t)$ divided by the average number of packets arriving in $(0, t)$, which is equal to λt. We have:

$$L = \lim_{t\to\infty} L(t) = \lim_{t\to\infty} \frac{M(t)}{\lambda t}, \tag{1}$$

where λ is the intensity of the arrival process. For examples of computations of the loss ratio using the direct method, the reader is referred to [6]–[8]. The important advantage of this method is that it gives not only the stationary characteristic, L, but also its transient counterpart, $L(t)$. Thus, the evolution of the loss ratio in short time periods can be studied. However, obtaining the distribution of $M(t)$ may be not easy. This is an important drawback of the method.

As an alternative to the direct method, the indirect method based on the empty queue probability can be used. Let p_0 denote the stationary probability that the queue is empty. It is well-known that he following relation holds true (see, e.g., [1]):

$$L = \frac{\rho - \rho'}{\rho}, \tag{2}$$

where ρ is the offered load of the queue and ρ' is the carried load of the queue. Moreover, we have:

$$\rho' = 1 - p_0. \tag{3}$$

Thus, from (2) and (3) it follows that L is simply a function of p_0 and ρ:

$$L = \frac{1}{\rho}(\rho - 1 + p_0). \tag{4}$$

Formula (4) is more friendly than (1), as it does not involve any transient sub-characteristics. In addition to that, the stationary distribution of the queue size, which contains p_0, is one of the most commonly studied and derived characteristics. Therefore, several ready-to-use results for p_0 are available in the literature.

Moreover, formula (4) is general in the sense that it holds for any type of the arrival process and service time distribution. It holds also for more complicated packets dropping schemes, like those based on the dropping function, [19], used in active queue management. (For more information about active queue management, see [20]–[22]) and the references given there). Finally, it holds even in the case of the queue with vacations (see e.g., [23]).

In the remaining sections, it will be shown how the loss ratio can be calculated using known results on the empty queue probability.

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Namely, in Section II, the M/G/1/N model with Poisson arrivals and general service times will be considered, while in Section III – the M^X/G/1/N model, with batch Poisson arrivals and general service times. Then, Section IV will be devoted to MMPP/G/1/N model, with autocorrelated arrivals of Markov-modulated Poisson type. In Section V, the BMAP/G/1/N model will be considered, with the most general arrival process, incorporating both the autocorrelation and the batch structure. In Section VI, two numerical examples will be presented. Finally, in Section VII remarks concluding the paper will be gathered.

II. Loss ratio in the M/G/1/N model

In this model, the arrival stream is Poisson with intensity λ. The service time has distribution function $F(t)$, which is not further specified, and the buffer size is N, including the service position. The offered load of the system is:

$$\rho = \lambda \int_0^\infty t dF(t).$$

The formula for p_0 in this model can be taken, for instance, from chapter 5 of [1]. Namely, we have:

$$p_0 = \frac{1}{\rho \sum_{k=0}^{N-1} \xi_k + 1}, \tag{5}$$

where

$$\xi_0 = 1, \qquad \xi_1 = \frac{1-a_0}{a_0}, \tag{6}$$

$$\xi_{k+1} = \frac{1}{a_0}\left[\xi_k - \sum_{i=0}^{k-1} a_{k-i+1}\xi_i - a_k\right], \quad k \geq 1, \tag{7}$$

$$a_k = \int_0^\infty \frac{e^{-\lambda u}(\lambda u)^k}{k!} dF(u), \quad k \geq 0. \tag{8}$$

Therefore, from (4) we obtain:

$$L = \frac{1}{\rho}\left[\rho - 1 + \frac{1}{\rho \sum_{k=0}^{N-1} \xi_k + 1}\right], \tag{9}$$

which is easy to deal with in numerical calculations.

III. Loss ratio in the M^x/G/1/N model

In this model, the arrival stream is batch Poisson with intensity of batches equal to λ. The size of an arriving batch has distribution $\{b_1, b_2, \ldots, \}$, where b_i denotes the probability of a batch of size i. The service time has distribution function $F(t)$, which is not further specified, and the buffer size is N, including the service position.

The offered load of the system is now:

$$\rho = \lambda \bar{b} \int_0^\infty t dF(t).$$

where $\bar{b}$ is the average batch size:

$$\bar{b} = \sum_{k=1}^\infty k b_k. \tag{10}$$

In section 5.6 of [1], it is shown that:

$$p_0 = \frac{\pi_0}{\pi_0 + \rho/\bar{b}}, \tag{11}$$

where π_0 is the stationary probability that the queue is empty just after a packet departure.

The value of π_0 can be calculated in the following way. Let π_k^∞ denote the probability that the system is empty just after a packet departure in the M^X/G/1 model, i.e., with infinite buffer. In [24], section III.2.3, it was shown that:

$$q(z) = \sum_{k=0}^\infty \pi_k^\infty z^k = \frac{1-\rho}{\bar{b}} \cdot \frac{(1-b(z))f(\lambda - \lambda b(z))}{f(\lambda - \lambda b(z)) - z}, \tag{12}$$

where

$$f(s) = \int_0^\infty e^{-st} dF(t), \tag{13}$$

$$b(z) = \sum_{k=1}^\infty b_k z^k. \tag{14}$$

From (12), the values of π_k^∞ can be easily extracted using one of the available methods for generating function inversion.

For instance, using the method of [25], we obtain:

$$\pi_k^\infty \approx \frac{1}{2klr^k}\Big[a_0(k,l,r) + (-1)^k a_k(k,l,r) + 2\sum_{j=1}^{k-1}(-1)^j Re(a_j(k,l,r))\Big], \tag{15}$$

with

$$a_j(k,l,r) = \sum_{n=0}^{l-1} e^{-\pi i n/l} q(re^{\pi i(n+lj)/lk}),$$

and the following values of method parameters: $l = 1$, $r = 10^{-4/k}$.

Finally, the value of π_0 can be obtained from π_k^∞ values, namely:

$$\pi_0 = \frac{\pi_0^\infty}{\sum_{j=0}^{N-1} \pi_j^\infty}, \tag{16}$$

(see section 5.6 of [1]).

Therefore, from (4) we have:

$$L = \frac{1}{\rho}\left[\rho - 1 + \frac{\pi_0^\infty}{\pi_0^\infty + \sum_{j=0}^{N-1} \pi_j^\infty \rho/\bar{b}}\right]. \tag{17}$$

IV. Loss ratio in the MMPP/G/1/N model

In this model, the arrival stream is the Markov-Modulated Poisson Process (MMPP), [26]. The service time has distribution function $F(t)$, which is not further specified, and the buffer size is N, including the service position.

The MMPP process is usually parametrized by two $m \times m$ matrices: Q, which is an infinitesimal generator of a continuous-time Markov chain, and

$$\Lambda = diag(\lambda_1, \ldots, \lambda_m), \tag{18}$$

where λ_i is a temporary arrival rate.

The important advantage of the MMPP process is that it enables mimicking the shape of the autocorrelation function of interarrival times, together with their marginal distribution (see, e.g., [27]).

The average arrival intensity for MMPP is:

$$\lambda = \pi\Lambda\mathbf{1}, \tag{19}$$

where

$$\mathbf{1} = (1, \dots, 1)^T \tag{20}$$

and π is the stationary vector for matrix Q, i.e., such that:

$$\pi Q = (0, \dots, 0), \quad \pi\mathbf{1} = 1. \tag{21}$$

Therefore, the load of the system equals:

$$\rho = \pi\Lambda\mathbf{1}\int_0^\infty t dF(t). \tag{22}$$

The MMPP traffic has the following characteristics. The k-th moment of the interarrival time is equal to:

$$m_k(T_i) = k!\ p\ (\Lambda - Q)^{-(k+1)}\Lambda \cdot \mathbf{1}, \tag{23}$$

while the variance of the interarrival time:

$$Var = m_2(T_i) - \frac{1}{\lambda^2}.$$

The k-lag autocorrelation of interarrival times equals:

$$Corr(k) = \frac{Cov(k)}{Var},$$

with

$$\begin{aligned} Cov(k) =& p\ (\Lambda - Q)^{-2}\Lambda\big[\big((\Lambda - Q)^{-1}\Lambda\big)^{k-1} \\ & - \mathbf{1}\ p\big](\Lambda - Q)^{-2}\Lambda\ \mathbf{1}, \end{aligned}$$

where

$$p = \frac{1}{\lambda}\pi\Lambda.$$

Finally, the generating function for the counting function is equal to:

$$P^*(z, t) = e^{(Q-(1-z)\Lambda)t}, \quad |z| \le 1, \tag{24}$$

where

$$P^*(z, t) = \sum_{n=0}^\infty P(n, t)z^n,$$

and $P_{i,j}(n, t) = \mathbb{P}(\overline{N}(t) = n, J(t) = j|\overline{N}(0) = 0, J(0) = i)$ is the counting function for the MMPP, $\overline{N}(t)$ is the number of arrivals in interval $(0, t]$ and $J(t)$ is the state of the modulating chain at time t.

In what follows, we will use the following matrices of size $m \times m$:

$$\begin{aligned}
\mathbf{0} &= [0]_{i,j}\,, \\
I &= \text{ identity matrix}, \\
A_k(s) &= \left[\int_0^\infty e^{-st}P_{i,j}(k, t)dF(t)\right]_{i,j}, \\
\overline{A}_n(s) &= \sum_{k=n}^\infty A_k(s), \\
B_n(s) &= A_{n+1}(s) - \overline{A}_{n+1}(s)(\overline{A}_0(s))^{-1}, \\
Z(s) &= \left[\frac{Q_{ij}(1-\delta_{ij})}{s + \lambda_i - Q_{ii}}\right]_{i,j}, \\
E(s) &= \left[\frac{\Lambda_{ij}}{s + \lambda_i - Q_{ii}}\right]_{i,j}, \\
R_0(s) &= \mathbf{0}, \quad R_1(s) = A_0^{-1}(s), \\
R_{k+1}(s) &= R_1(s)(R_k(s) - \sum_{i=0}^k A_{i+1}(s)R_{k-i}(s)), \ \ k \ge 1,
\end{aligned}$$

where δ_{ij} is the Kronecker symbol.

In [28], it was shown that:

$$p_0 = \lim_{s\to 0+} s\phi_N(s), \tag{25}$$

where

$$\phi_N(s) = M_N^{-1}(s)l_N(s), \tag{26}$$

and

$$\begin{aligned}
& M_N(s) \\
& = (I - Z(s))[R_{N+1}(s)A_0(s) + \sum_{k=0}^N R_{N-k}(s)B_k(s)] \\
& - E(s)[R_N(s)A_0(s) + \sum_{k=0}^{N-1} R_{N-1-k}(s)B_k(s)],
\end{aligned} \tag{27}$$

$$\begin{aligned}
l_N(s) &= E(s)\sum_{k=0}^{N-1} R_{N-1-k}(s)g_k(s) \\
& - (I - Z(s))\sum_{k=0}^N R_{N-k}(s)g_k(s) + z(s),
\end{aligned} \tag{28}$$

$$\begin{aligned}
g_k(s) &= \overline{A}_{k+1}(s)(\overline{A}_0(s))^{-1}r_N(s) - r_{N-k}(s), \\
r_n(s) &= (r_{n,1}(s), \dots, r_{n,m}(s))^T, \\
r_{n,i}(s) &= \begin{cases} 0 \text{ if } & n > 0, \\ \sum_{j=1}^m d_{0,i,j}(s) \text{ if } n = 0, \end{cases} \\
d_{k,i,j}(s) &= \int_0^\infty e^{-st}P_{i,j}(k, t)(1 - F(t))dt, \\
z(s) &= ((s + \lambda_1 - Q_{11})^{-1}, \dots, (s + \lambda_m - Q_{mm})^{-1})^T.
\end{aligned}$$

It must be stressed that all the aforementioned functions and matrices can be computed numerically. In particular, the most demanding $A_k(s)$ and $d_{k,i,j}(s)$ can be obtained using the well-known uniformization method (see, e.g., [29]). In particular, we can obtain the following formulas for $A_k(s)$ and $\overline{D}_k = [d_{k,i,j}(s)]_{i,j}$:

$$A_k(s) = \sum_{j=0}^{\infty} K_{k,j} \int_0^{\infty} \frac{e^{-(\theta+s)t}(\theta t)^j}{j!} dF(t),$$

$$\overline{D}_k(s) = \sum_{j=0}^{\infty} K_{k,j} \int_0^{\infty} \frac{e^{-(\theta+s)t}(\theta t)^j}{j!} (1-F(t))dt,$$

where

$$\begin{aligned} K_{0,0} &= I, \\ K_{n,0} &= \mathbf{0}, \qquad n \geq 1, \\ K_{0,j+1} &= K_{0,j}(I+\theta^{-1}(Q-\Lambda)), \\ K_{n,j+1} &= \theta^{-1}\Lambda K_{n-1,j} + K_{n,j}(I+\theta^{-1}(Q-\Lambda)), \end{aligned}$$

$$\theta = \max_i \{(\Lambda - Q)_{ii}\}.$$

All the remaining quantities are simple functions of $A_k(s)$, $d_{k,i,j}(s)$ and system parameters. Thus, (4) and (25) yield:

$$L = \frac{1}{\rho}\left[\rho - 1 + \lim_{s\to 0+} sM_N^{-1}(s)l_N(s)\right]. \tag{29}$$

where $M_N^{-1}(s)$ and $l_N(s)$ can be effectively computed from (27) and (28).

V. LOSS RATIO IN THE BMAP/G/1/N MODEL

In this model, the arrival stream is the Batch Markovian Arrival Process (BMAP), [29]–[31]. The service time has distribution function $F(t)$, which is not further specified, and the buffer size is N, including the service position.

The BMAP process is a 2-dimensional Markov process $(\overline{N}(t), J(t))$ on the state space $\{(i,j) : i \geq 0, 1 \leq j \leq m\}$ with an infinitesimal generator Q in the form:

$$Q = \begin{bmatrix} D_0 & D_1 & D_2 & D_3 & \cdot\cdot \\ & D_0 & D_1 & D_2 & \cdot\cdot \\ & & D_0 & D_1 & \cdot\cdot \\ & & & \cdot & \cdot\cdot \end{bmatrix},$$

where D_k, $k \geq 0$, are matrices of size $m \times m$, D_0 has nonnegative off-diagonal elements and negative diagonal elements, D_k, $k \geq 1$, are nonnegative, and the sum

$$D = \sum_{k=0}^{\infty} D_k \tag{30}$$

is an irreducible infinitesimal generator.

The arrival intensity is equal to:

$$\lambda = \pi \sum_{k=1}^{\infty} kD_k\mathbf{1}, \tag{31}$$

where π is the stationary vector fulfilling equations:

$$\pi D = (0,\ldots,0), \quad \pi\mathbf{1} = 1. \tag{32}$$

Therefore, the system load is now:

$$\rho = \pi \sum_{k=1}^{\infty} kD_k\mathbf{1} \int_0^{\infty} tdF(t). \tag{33}$$

The BMAP process has very powerful modeling capabilities. It not only enables mimicking the shape of the autocorrelation function of interarrival times together with their marginal distribution (as in MMPP), but also many other properties, like the batch structure and the correlation between the current intensity and the batch size.

The BMAP process has the following characteristics. The intensity of arrivals of batches is:

$$\lambda_g = \pi(-D_0)\mathbf{1},$$

while the average batch size:

$$\eta = \frac{\lambda}{\lambda_g}.$$

The variance of the time between consecutive batches is:

$$Var = -\frac{2}{\lambda_g}\pi D_0^{-1}\mathbf{1} - \frac{1}{\lambda_g^2};$$

while the k-lag autocorrelation of interarrival times (for batches):

$$Corr(k) = p\ D_0^{-1}C(C^{k-1} - \mathbf{1}\ p)D_0^{-1}C\ \mathbf{1}/Var,$$

where

$$C = -D_0^{-1}(D - D_0),$$

and p is a vector fulfilling the system:

$$p(C-I) = (0,\ldots,0), \quad p\ \mathbf{1} = 1.$$

Finally, the generating function for the counting function is:

$$P^*(z,t) = \sum_{n=0}^{\infty} P(n,t)z^n = e^{D(z)t},$$

with

$$D(z) = \sum_{k=0}^{\infty} z^k D_k, \quad |z| \leq 1.$$

For the BMAP/G/1/N queueing model, it was proven in [32] that:

$$p_0 = \lim_{s\to 0+} s\varphi_N(s), \tag{34}$$

where

$$\varphi_N(s) = H_N^{-1}(s)m_N(s,l), \tag{35}$$

and

$$\begin{aligned} H_N(s) =& R_{N+1}(s)A_0(s) + \sum_{k=0}^{N} R_{N-k}(s)B_k(s) \\ &- \sum_{k=N+1}^{\infty} Y_k(s), \end{aligned} \tag{36}$$

$$m_N(s) = \sum_{k=0}^{N} Y_{N-k}(s) \sum_{i=0}^{k} R_{k-i}(s) g_i(s) - \sum_{k=0}^{N} R_{N-k}(s) g_k(s) + y(s), \qquad (37)$$

$$Y_k(s) = \left[\frac{\lambda_i p_i(k,j)}{s+\lambda_i} \right]_{i,j},$$

$$\lambda_i = -(D_0)_{ii}, \qquad 1 \le i \le m,$$

$$p_i(0,i) = 0, \qquad 0 \le i \le m,$$

$$p_i(0,k) = \frac{1}{\lambda_i}(D_0)_{ik}, \qquad 1 \le i,k \le m, \quad k \ne i,$$

$$p_i(j,k) = \frac{1}{\lambda_i}(D_j)_{ik}, \qquad 1 \le i,k \le m, \quad j \ge 1,$$

$$y(s)=((s+\lambda_1)^{-1}, \ldots, (s+\lambda_m)^{-1})^T,$$

and the remaining functions, i.e., A_k, $\overline{A}_k$, B_k, R_k, g_k and r_k are defined in the same way as in the previous section.

Again, all the needed quantities can be computed numerically using the uniformization method of [29]. Namely, the following formulas for $A_k(s)$ and $\overline{D}_k = [d_{k,i,j}(s)]_{i,j}$ can be obtained for the BMAP process:

$$A_k(s) = \sum_{j=0}^{\infty} K_{k,j} \int_0^{\infty} \frac{e^{-(\gamma+s)t}(\gamma t)^j}{j!} dF(t),$$

$$\overline{D}_k(s) = \sum_{j=0}^{\infty} K_{k,j} \int_0^{\infty} \frac{e^{-(\gamma+s)t}(\gamma t)^j}{j!} (1-F(t))dt,$$

where

$$\begin{aligned} K_{0,0} &= I, \\ K_{n,0} &= \mathbf{0}, \qquad n \ge 1, \\ K_{0,j+1} &= K_{0,j}(I+\gamma^{-1}D_0), \\ K_{n,j+1} &= \gamma^{-1} \sum_{i=0}^{n-1} K_{i,j} D_{n-i} + K_{n,j}(I+\gamma^{-1}D_0). \end{aligned}$$

$$\gamma = \max_i \{(-D_0)_{ii}\},$$

Finally, from (4) and (34) we obtain:

$$L = \frac{1}{\rho}\left[\rho - 1 + \lim_{s \to 0+} s H_N^{-1}(s) m_N(s) \right], \qquad (38)$$

where $H_N^{-1}(s)$ and $m_N(s)$ can be effectively computed from (36) and (37).

VI. Examples

We will present now two examples, devoted to MMPP and BMAP traffic, respectively. In both examples the buffer size will be $N = 40$.

A. MMPP example

In this example, we assume constant service time equal to 1 and use MMPP parameters obtained in [33], namely:

$$\Lambda = \begin{bmatrix} 1.0722 & 0 \\ 0 & 0.48976 \end{bmatrix}, \qquad (39)$$

$$Q = \begin{bmatrix} -8.4733 \cdot 10^{-4} & 8.4733 \cdot 10^{-4} \\ 5.0201 \cdot 10^{-6} & -5.0201 \cdot 10^{-6} \end{bmatrix}. \qquad (40)$$

Firstly, by means of (21) we can compute the stationary vector for matrix Q. We obtain:

$$\pi = (0.005889, 0.994110).$$

Then, using (19), we obtain the arrival rate and the load:

$$\lambda = \rho = 0.493190.$$

Exploiting (25) with $s = 10^{-9}$, we can calculate the empty queue probability:

$$p_0 = 0.507129.$$

Finally, from (29) we obtain the loss ratio:

$$L = 0.0006493.$$

B. BMAP example

In this example, we assume contant service time equal to 0.1 and use BMAP parameters from [34], i.e.:

$$D_0 = \begin{bmatrix} -45.5935855 & 1.95261616 & 0.19526161 \\ 0.01952616 & -4.55935855 & 0.19526161 \\ 0.00195261 & 0.01952616 & -0.45593586 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.06508720 & 0.52069762 & 5.20697622 \\ 0.52069762 & 0.00065087 & 0.05792761 \\ 0.05076801 & 0.00650872 & 0.00065087 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0.06508720 & 0.52069762 & 5.20697622 \\ 0.52069762 & 0.00065087 & 0.05792761 \\ 0.05076801 & 0.00650872 & 0.00065087 \end{bmatrix},$$

$$D_8 = \begin{bmatrix} 0.35797962 & 2.86383692 & 28.6383692 \\ 2.86383692 & 0.00357979 & 0.31860186 \\ 0.27922410 & 0.03579796 & 0.00357979 \end{bmatrix}.$$

Firstly, using (32) we can calculate the stationary vector for matrix $D = D_0 + D_2 + D_4 + D_8$. We get:

$$\pi = (0.010598, 0.028056, 0.961345).$$

Then, using (31), we can compute the arrival rate:

$$\lambda = 6.666666.$$

Therefore, from (33) it follows that:

$$\rho = 0.666666.$$

Using (34) with $s = 10^{-9}$, we can obtain the empty queue probability:

$$p_0 = 0.377399.$$

Finally, from (38) we obtain the loss ratio:

$$L = 0.066099.$$

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VII. Conclusions

In this paper, the methods of computing the loss ratio in the single-server queueing model were discussed and the formulas for the loss ratio obtained via the empty queue probability were collected. Various queuing models were taken into account, with general service time distribution and different assumptions on the arrival stream. Namely, the following models were considered: M/G/1N, M^X/G/1/N, MMPP/G/1/N and BMAP/G/1/N. This allows the reader to choose an appropriate model, depending on the assumptions on the traffic in the considered network.

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