Stability and Control of Power Grids with Diluted Network Topology

Liudmila Tumash
CNRS, Grenoble INP, GIPSA-lab
Grenoble, France
email: liudmilatumash@gmx.de
ORCID 0000-0002-8144-5356

Simona Olmi
Istituto dei Sistemi Complessi
CNR - Consiglio Nazionale delle Ricerche
Sesto Fiorentino, Italy
email: simona.olmi@fi.isc.cnr.it
ORCID 0000-0002-8272-3493

Eckehard Schöll
Institut für Theoretische Physik
Technische Universität Berlin
Berlin, Germany
Potsdam Institute for Climate Impact Research
Potsdam, Germany
email: schoell@physik.tu-berlin.de
ORCID 0000-0002-7318-2672

***Abstract*—We consider sparse random networks of Kuramoto phase oscillators with inertia in order to investigate the dynamics emerging in high-voltage power grids. The corresponding natural frequencies are assumed to be bimodally Gaussian distributed, thus modeling the distribution of both power generators and consumers, which must be in balance. Our main focus is on the theoretical analysis of the linear stability of the frequency-synchronized state, which is necessary for the stable operation of power grids, and the control of unstable synchronous states. We demonstrate by numerical simulations that unstable frequency-synchronized states can be stabilized by feedback control. Further, we extend our study to include stochastic temporal power fluctuations and discuss the interplay of topological disorder and Gaussian white noise for various model configurations. Our results are compared with those obtained for the real power grid topology of Italy.**

***Index Terms*—power grids, sychronization, stability, control, diluted network.**

I. Introduction

The traditional way to generate power by using fossile energy has induced the risk of global warming caused by large emission of carbon dioxide gases. Nowadays, we are witnessing a drastic regime shift in the operation of power grids towards renewable energy sources, since more and more energy generating units become supplied by natural sources, such as wind parks and photovoltaic arrays, see [14], [15]. This regime shift has triggered three major issues that have to be considered to provide the sustainable operation of power grids. The first issue is *decentralization* meaning that the power system based on renewable energy sources represents a distributed network carrying many small units of energy to the consumers (unlike conventional power grids), see [2]. The second issue is a strong *spatial separation* between power generators and consumers [4], e.g., wind energy is usually produced near the sea, and solar energy is harvested in sunny areas, while power consumption is highest in industrial agglomerations in other parts of the country. Finally, the third issue is related to the strong dependence on weather conditions, which leads to the increasing fraction of *strongly fluctuating power output* [8]. Thus, the focus of this study is on power grids based on renewable energy sources characterized by sparse networks. For this reason, we have considered random Erdös-Renyi networks with low average connectivity to model the network topology underlying high voltage transmission grids [11], [13].

The Kuramoto model with inertia presented in [1] is a standard mathematical model used to study the dynamical behavior of power generators and consumers [5]–[7], [11], [12], [16]. Thus, we consider sparse random networks of Kuramoto phase oscillators with inertia to investigate the dynamics emerging in high-voltage power grids. In general, power grids tend to synchronize their frequencies to the standard ac power frequency $\Omega = 50$ Hz (or 60 Hz in some countries). We distinguish the power generated by power sources ($P^i_{source} > 0$) from the power consumed by passive machines or loads ($P^i_{cons} < 0$) by assuming the bimodal Gaussian distribution of natural frequencies of power generators and consumers with opposite picks as in [5]. Although the bimodal frequency distribution is a very important feature of the model, most of the previous studies consider either a unimodal frequency distribution [6] or δ-function shaped bimodal distributions [7]. In our work we use the bimodal Gaussian distribution of frequencies, which models consumed and generated power in a more realistic way.

For the stable operation of power grids the produced power must be equal to the consumed power, which implies maintaining a synchronous state of the entire network. We provide the theoretical analysis of the linear stability of the frequency-synchronized state, and the control of unstable synchronous states that are usually characterized by large differences of initial phases. The stability criteria were derived based on the properties of the initial phase differences of the oscillators. We demonstrate by numerical simulations that unstable frequency-synchronized states can be stabilized by feedback control if the coupling between oscillators is strong enough. Additionally, we include stochastic temporal power fluctuations by adding Gaussian white noise to the Kuramoto model and discuss how does it influence the frequency synchronization.

The present extended abstract is structured as follows. In Sec. II, the Kuramoto model with inertia, the network topology and natural frequency distribution are presented. In Sec. III we characterize frequency synchronization and discuss the conditions under which this state occurs, as well as we

 ISBN: 978-1-61208-855-6

apply a control method to stabilize the unstable frequency-synchronized solutions. The results and main contributions of this study are discussed in Sect. IV.

II. Model

We consider a system consisting of a population of $i = 1, ..., N$ coupled Kuramoto oscillators with inertia that reads

$$m\ddot{\theta}_i + \dot{\theta}_i = \Omega_i + \frac{K}{N_i}\sum_{j=1}^{N} A_{ij}\sin\left(\theta_j - \theta_i\right), \qquad (1)$$

where θ_i and $\dot{\theta}_i$ are the instantaneous phase and frequency, respectively, of the oscillator i. Parameters $m > 0$ and $K > 0$ indicate the inertial mass of generators and the coupling constant of the network equivalent to the transmission line capacities between loads and generators, respectively. A is the connectivity matrix: its entries A_{ij} are one if nodes i and j are connected, and zero otherwise. N_i is the node degree of the i-th element, thus denoting the number of the links outcoming from this node. Finally, Ω_i represents the natural frequency of the oscillator i, whose value is chosen in accordance with the bimodal Gaussian distribution

$$g(\Omega) = \left[\frac{p_g}{\sqrt{2\pi}}e^{-\frac{(\Omega-\Omega_{0+})^2}{2}} + \frac{1-p_g}{\sqrt{2\pi}}e^{-\frac{(\Omega+\Omega_{0-})^2}{2}}\right]. \qquad (2)$$

III. Frequency Synchronization and Control

In particular, we aim to investigate the stability of the synchronous solution emerging in a power grid network by linearizing the state around the frequency-synchronized solution, which gives a Jacobian matrix with constant coefficients, and analyzing the eigenvalues of this linearized system. The calculation of the eigenvalue with the largest real part λ_{max} will be the main criterion for determining the synchronization stability. Stability means that the sign of the largest real part is negative.

We analyse the frequency synchronized solution for the network characterized by coupling beyond some critical value (for $K < K_c$ no frequency synchronized solution is possible). We have obtained that stable frequency synchronization occurs in systems whose initial phases are close enough, i.e., $\left|\theta_j^0 - \theta_i^0\right| < \frac{\pi}{2}$. If this condition on phases is violated, we deal with unstable solutions, which we stabilize with a control loop.

Further, we search for sets of initial phases that satisfy the frequency-synchronized solution numerically using Levenberg-Marquardt algorithm, which might find stable and unstable solutions. For example, Fig.1a),b) presents a stable frequency-synchronized solution that is obtained for almost identical initial phases, while for the case of largely varying initial phases we obtain an unstable frequency-synchronized solution as illustrated in Fig.1c),d).

We further stabilize such unstable frequency-synchronized solutions obtained for $\left|\theta_j^0 - \theta_i^0\right| \geq \frac{\pi}{2}$ by introducing a control term u_i into the original system (1)

$$\dot{\theta}_i = \omega_i$$

$$\dot{\omega}_i = \alpha\Omega_i - \alpha\omega_i + \frac{K}{N_i}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i) + u_i,$$

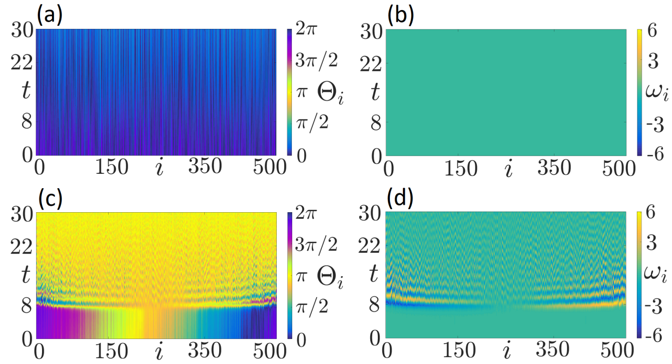

Fig. 1. Spatio-temporal evolution of phases θ_i and frequencies ω_i, which satisfy the condition for frequency-synchronized solution. Stable solution: (a) phases; (b) frequencies; parameters: $Re(\lambda_{max}) = -0.083$, $K = 10$. Unstable solution: (c) phases; (d) frequencies; parameters: $\lambda_{max} = 2.41$, $K = 70$. Other parameters: $m = 6$, $p = 0.20$ (connectivity ratio), $\Omega_0 = 2$, $N = 500$.

In particular, the control term u can be chosen as a feedback control loop such that

$$u = -\mathbf{C}\begin{pmatrix}\delta\theta \\ \delta\omega\end{pmatrix},$$

where $\mathbf{C} \in R^{N\times 2N}$ is chosen to minimize the following cost functional

$$J(u) = \int_0^\infty \left|\left|\begin{pmatrix}\delta\theta(t) \\ \delta\omega(t)\end{pmatrix}\right|\right|^2 + ||u(t)||^2 dt.$$

This problem is solved via the application of a *linear quadratic regulator* for each set of phases θ_i^*. Basically, the regulator chooses the time-independent matrix $\mathbf{C}$ such that the eigenvalues for the closed-loop system are non-positive when solving the eigenvalue problem for the Jacobian. Thus, the frequency-synchronized solution is stabilized for each particular set of chosen phases θ_i^*, and, regardless of the initial phase differences $\left|\theta_j^0 - \theta_i^0\right|$, we are always able to obtain a stable solution if $K > K_c$.

An example is illustrated in Fig.2, where plots a) and b) represent phase evolution without control. We see that these phases lose synchrony with respect to frequencies, since their initial phases do not have close values. However, if a control action is performed, the frequency-synchronization is stabilized as it is shown in Fig.2c) and d).

IV. Conclusions

In conclusion, we have considered sparse networks of Kuramoto oscillators with inertia to investigate the optimal conditions for the emergence of synchronization in power grids. Going beyond previous work [11] devoted to this type of networks, we have provided a linear stability analysis of the frequency-synchronized solution that is necessary for the stable operation of power grids. We have derived the stability criteria, based on the initial phase differences of the oscillators, and have estimated the critical coupling strength K_c above which a frequency-synchronized solution is possible in the deterministic system. For sufficiently large coupling we have

 ISBN: 978-1-61208-855-6

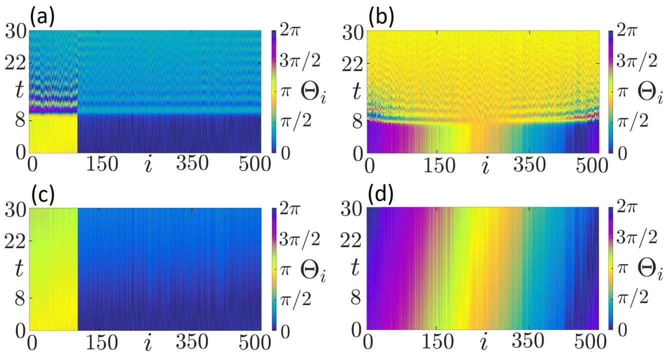

Fig. 2. Spatio-temporal evolution of phases θ_i without (top panel) and with control (bottom panel). Left column: $K = 50$ and initial phases $\theta_1^* = ... = \theta_{150}^* = \pi$, $\theta_{151}^* = ... = \theta_{500}^* = 0$, (a) control off, $\lambda_{max} = 2.802$; (c) control on, $\lambda_{ctrl} = -0.759$. Right column: $K = 70$ and uniformly distributed initial phases, (b) control off, $\lambda_{max} = 2.41$; (d) control on, $\lambda_{ctrl} = -0.823$. Other parameters: as on Fig. 1.

also found unstable solutions that are usually characterized by large differences of initial phases. A similar stability analysis was performed by Mirollo et al. [3] and by Delabays et al. [9] for networks of classical Kuramoto oscillators (without inertia) with different topologies (fully coupled networks and planar graphs respectively). Here we used the stability analysis to characterize unstable synchronous states in diluted networks, which we subsequently stabilize by a control loop. It turns out that our linear feedback control scheme is very efficient in stabilizing unstable frequency-synchronized solutions for arbitrary initial phases, and all $K > K_c$.

Furthermore we have investigated diluted networks with stochastic dynamics due to temporally fluctuating power in order to infer the similarities and differences occurring in the transition to synchronization with respect to the deterministic case. We have added a simple noise term, i.e., Gaussian white noise, rather than correlated noise or intermittent noise, in order to gain insight into the general role played by noise in power systems. Previously, the transition to synchronization has been investigated mainly in deterministic systems [6] or in globally coupled networks [11]. On the other hand, when stochastic systems with Gaussian white noise were considered [10], the focus has not been on the synchronization transition, thus neglecting possible consequences of hysteresis in power systems. In particular, here we have observed that for synthetic diluted networks (independently of the frequency distribution), intermediate noise intensities might play a constructive role in lowering the critical coupling value required to reach (almost complete) frequency synchronization, since noise suppresses intermediate states and reduces the hysteretic region.

Future perspectives of this work might be aimed at a deeper understanding of the applicability of the control scheme within noisy systems, for non-Gaussian noise and realistic topologies.

ACKOWLEDGMENT

We acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) in the framework of the SFB 910, Projektnummer 163436311.

REFERENCES

[1] H. A. Tanaka, A. J. Lichtenberg, and S. Oishi, “First order phase transition resulting from finite inertia in coupled oscillator systems”, Phys. Rev. Lett., vol. 78, pp. 2104-2107, 1997.
[2] T. Ackermann, G. Andersson, and L. Söder, “Distributed generation: a definition”, Elec. Pow. Sys. Res., vol. 57, pp. 195-204, 2001.
[3] R. E. Mirollo and S. H. Strogatz, “The spectrum of the locked state for the Kuramoto model of coupled oscillators”, Phys. D: Nonlin. Phenom., vol. 205, pp. 249-266, 2005.
[4] J. C. Smith, M. R. Milligan, E. A. DeMeo, and B. Parsons, “Utility wind integration and operating impact state of the art”, IEEE transactions on power systems, vol. 22, pp. 900-908, 2007.
[5] G. Filatrella, A. H. Nielsen, and N. F. Pedersen, “Analysis of a power grid using a Kuramoto-like model”, Eur. Phys. J. B, vol. 61, pp. 485-491, 2008.
[6] S. Olmi, A. Navas, S. Boccaletti, and A. Torcini, “Hysteretic transitions in the Kuramoto model with inertia”, Phys. Rev. E, vol. 90, p. 042905, 2014.
[7] M. Rohden, A. Sorge, D. Witthaut, and M. Timme, “Impact of network topology on synchrony of oscillatory power grids”, Chaos, vol. 24, p. 013123, 2014.
[8] M. Anvari, M. Wächter, and J. Peinke, “Phase locking of wind turbines leads to intermittent power production”, Europhys. Lett., vol. 116, p. 60009, 2017.
[9] R. Delabays, T. Coletta, and P. Jacquod, “Multistability of phase-locking in equal-frequency Kuramoto models on planar graphs”, Journal of Mathematical Physics, vol. 58, p. 032703, 2017.
[10] B. Schäfer, M. Matthiae, X. Zhang, M. Rohden, M. Timme, and D. Witthaut, “Escape routes, weak links, and desynchronization in fluctuation-driven networks”, Phys. Rev. E, vol. 95, p. 060203, 2017.
[11] L. Tumash, S. Olmi, and E. Schöll, “Effect of disorder and noise in shaping the dynamics of power grids”, Europhys. Lett., vol. 123, p. 20001, 2018.
[12] H. Taher, S. Olmi, and E. Schöll, “Enhancing power grid synchronization and stability through time delayed feedback control”, Phys. Rev. E, vol. 100, p. 062306, 2019.
[13] L. Tumash, S. Olmi, and E. Schöll, “Stability and control of power grids with diluted network topology”, Chaos, vol. 29, p. 123105, 2019.
[14] D.-N. Ciupageanu, G. Lazaroiu, and L. Barelli, “Wind energy integration: Variability analysis and power system impact assessment”, Energy, vol. 185, pp. 1183-1196, 2019.
[15] L. Barelli et al., “Adaptive voltage control of islanded RES-based residential microgrid with integrated flywheel/battery hybrid energy storage system”, 22nd European Conference on Power Electronics and Applications, pp. 1-10, 2020.
[16] C. H. Totz, S. Olmi, and E. Schöll, “Control of synchronization in two-layer power grids”, Phys. Rev. E, vol. 102, p. 022311, 2020.