Fuzzy One-Decision Making Model with Fuzzified Outcomes in the Treatment of Necrotizing Fasciitis

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***Abstract*—By proposing a new approach to fuzzy decision making, we try to support the medical decision, concerning recommendations for the treatment with hyperbaric oxygen (HBO). This treatment can be used for patients, suffering from necrotizing fasciitis. Due to the disease rarity, it sometimes is difficult for a physician to determine, if a single patient needs the treatment with HBO. We thus identify the decision with a linguistic variable, equipped with treatment recommendation levels. The choice of the appropriate level is based on values of clinical symptoms, found in the patient. To extract the optimal recommendation level for the treatment with HBO, we involve fuzzy set techniques in the decision model. In the paper, we mainly concentrate on designs of fuzzy sets, standing for clinical symptoms and recommendation levels. The levels act as the outcomes, dependent on the cumulative input of the patient's clinical markers. Since the focus is laid on a parametric structure of the outcomes, then we can categorize the model as robust approach to algorithmic modeling of outcomes, being part of eHealth data records.**

Keywords-fuzzy one-decision making; fuzzy sets; families of membership functions; s-functions; necrotizing fasciitis; treatment with hyperbaric oxygen.

I. INTRODUCTION

Necrotizing fasciitis (NF) is a rare, but deadly soft tissue infection. The disease is known from Hippocratic times, but has been newly rediscovered in modern times as an "infection with flesh eating bacteria" by Jones in 1871 [1]. More specifically, the illness was described in 1952 by Wilson [2], who also renamed these types of infections as necrotizing fasciitis. The NF group contains various types of infections, usually treated with antibiotics and surgery [3]. In some cases, the treatment with hyperbaric oxygen (HBO) is the adjunct of treatments, mentioned above [4]. Blekinge County City Hospital in Karlskrona, Sweden, has the possibility of providing HBO. Therefore, we serve the treatment to NF patients, who live in the south-eastern part of Sweden.

From the clinical point of view, we want to know, if the patient has a good prognosis of recovery without recommendations for the specialized treatment with HBO or he/she needs the HBO supplement.

To make this prognosis, a physician has to rely on his experience. Nevertheless, the number of patients is not so large, which makes difficult to solve routinely the problems of HBO dosing.

Therefore, we initialize the mathematical model of fuzzy decision making, which considers only one decision (indications of treating the patient with HBO).

From our design, we have excluded a utility matrix, which constitutes the main part in most of fuzzy decision making models [5]-[9]. The entries of the matrix are stated as numerical or verbal utilities, assigned to pairs (decision, state). When using the utility matrix, shown in Section II, the researchers developed different decision methods, like, e.g., unequal objectives or minimization of regret in order to extract an optimal decision [10][11].

The theoretical designs of fuzzy decision making [5]-[11] were benefited in practical applications like, e.g., medical decisions [12], making decisions in nutrition [13] or making decisions in stock market [14].

The authors applied fuzzy decision making with the utility matrix to select the most efficacious treatment having an effect on a collection of clinical symptoms. In our proposals, the pair (decision, state) was interpreted as (treatment, symptom) [15]-[17]. The results of determining the optimal decision-treatment had a general nature, and were not adapted to the health state of a single patient.

The decision, concerning the treatment with HBO, is differentiated in recommendation levels. These create a scale of hints, telling us, if the health condition of the patient agrees with the decision of giving HBO to him/her or not. For one decision "treatment with HBO", we arrange a verbal recommendation range of stages in two families of terms, namely, "stages of non-indication" versus "stages of indication". The conversion of these terms in two families of fuzzy sets with parametric membership functions is planned as a substantial contribution in the model proposed. The procedure of establishing two common formulas of parametric membership functions of fuzzy sets, representing "stages of non-indication" and "stages of indication", should prevent us from determining the boundary values of fuzzy sets in an intuitive manner. Another task to fulfill will concern the introduction of fuzzy sets, assigned to symptoms. By cumulating the symptoms intensities, we wish to find the patient's clinical characteristics. To accept the most convincing recommendation for HBO dosing, the cumulated

 ISBN: 978-1-61208-470-1

characteristics of the patient will be tested in all decision levels.

We recall the classical fuzzy decision making model with the utility matrix in Section II. Our proposition of the one-decision model is sketched in Section III. Section IV contains the descriptions of constructions of clinical entry data. The structure of fuzzified outcomes will be engineered in Section V. The case study, referring to the treatment with HBO, will be tested in Section VI. We will formulate some concluding remarks in Section VII.

II. THE MODEL OF FUZZY DECISION MAKING WITH THE UTILITY MATRIX

Let us recall the definition of a fuzzy set.

If X is a collection of objects denoted generically by x, then the fuzzy set A in X is a set of ordered pairs $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x) \in [0,1]$ [18].

Each element x gets a membership degree $\mu_A(x)$, which expresses the strength of the relationship between x and A. Membership degrees, equal to 1, inform about the total relation between the element and the set. The function $\mu_A : X \to [0,1]$ is called "*the membership function*" of A.

In classical fuzzy decision making, we introduce the notions of a space of states (e.g., symptoms) $X = \{x_1,...,x_n\}$ and a decision space (e.g., treatments) $D = \{d_1,...,d_d\}$. The utility matrix U, given by

$$U = \begin{array}{c} \\ d_1 \\ \vdots \\ d_d \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & \cdots & x_n \end{array} \\ \begin{bmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & \ddots & \vdots \\ u_{d1} & \dots & u_{dn} \end{bmatrix} \end{array}, \tag{1}$$

has the entries u_{bj}, $b = 1,\dots,d$, $j = 1,\dots,n$ [5]-[9]. Each u_{bj} is the fuzzy utility of applying decision d_b to state x_j. In most of applications, u_{bj} are evaluated intuitively as values belonging to interval [0, 1], e.g., utility of $(d_1,x_1) = 0.7$. Some users prefer determining the utilities as fuzzy sets, e.g., utility of (d_1,x_1) = "large".

The aggregated utility U_{d_b} of d_b was estimated as $U_{d_b} = \sum_{j=1}^{n} u_{bj}$ in the early trials of adapting fuzzy decision making to practical solutions.

The operation $\max(U_{d_1},...,U_{d_d})$ allowed selecting the optimal d_b, satisfying the maximum criterion. Later on, utilities U_{d_b} have been calculated with a more complicated precision.

III. THE OUTLINE OF FUZZY ONE-DECISION MAKING

Before discussing our conception of fuzzy decision making, let us add other useful definitions.

The support of a fuzzy set A, supp(A), is not a fuzzy set (it is a crisp set) of all $x \in X$, such that $\mu_A(x) > 0$ [18].

The α-cut of a fuzzy set A, A_α, is a non-fuzzy set of all $x \in X$ such that $\mu_A(x) \geq \alpha$ [18].

The Euclidean distance $d(P_1, P_2)$ between points $P_1 = (x_1,\mu(x_1))$ and $P_2 = (x_2,\mu(x_2))$, in the two dimensional system with x- and $\mu(x)$-axes, is estimated as $d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (\mu(x_2) - \mu(x_1))^2}$.

In the current trial of fuzzy decision making, we suppose that set $D = \{d\}$ consists of only one decision, i.e., the treatment with HBO. Decision d, made for patient P_i, $i = 1,\dots,p$, is constructed as a linguistic variable, whose verbal values are term-sets L_l, $l = 1,\dots,m$. These terms are edited as recommendation levels of the treatment with HBO. The levels graduate indications of the treatment, scaled from the most contraindicated to the most advised by a physician.

We still keep the set of symptoms $X = \{x_1,...,x_n\}$.

For patient P_i, $i = 1,\dots,p$, we need to sample the characteristics s_i, informing about presence or absence of symptoms X_j, $j = 1,\dots,n$, in the patient. Symptoms X_j are typical of necrotizing fasciitis. We test the behavior of s_i on all levels L_l to select this L_l, for which the patient characteristics matches best.

Let us suppose that each clinical marker X_j, $j = 1,\dots,n$, is replaced by a fuzzy set, also named X_j. If a marker value $x_{i,j}$ for symptom X_j is found in P_i, then the membership degree $\mu_{X_j}(x_{i,j})$ will be assigned to $x_{i,j}$. The way of designing membership functions $\mu_{X_j} : X_j \to [0,1]$ will be evolved in Section IV.

The importance weights w_j of symptoms X_j are added to the formula of s_i to emphasize X_j's harmful influence on the disease course. Due to the professional experience, the physician suggests the placement of X_j in the sequence $X_1 \succ \ldots \succ X_n$, where "$\succ$" means "$X_j$ emerges more dangerous impact on the patient health state than X_h, j, $h = 1,\dots,n$. We state $w_1 > \ldots > w_n$ and want $\sum_{j=1}^{n} w_j = 1$.

The collected patient characteristics s_i (the numerical knowledge about the symptoms), made via all $x_{i,j}$ and w_j, will be derived for patient P_i as

$$s_i = \sum_{j=1}^{n} \mu_{X_j}(x_{i,j}) \cdot w_j, \; i = 1, \dots, p. \tag{2}$$

We note that the minimal value of s_i is 0 since, for all minimal $\mu_{X_j}(x_{i,j}) = 0$, we obtain $s_i = \sum_{j=1}^{n} 0 \cdot w_j = 0$, $i = 1, \dots, p$.

The maximal value of s_i will reach 1 if, for all maximal $\mu_{X_j}(x_{i,j}) = 1$, $s_i = \sum_{j=1}^{n} 1 \cdot w_j = 1 \cdot \sum_{j=1}^{n} w_j = 1 \cdot 1 = 1$, $i = 1, \dots, p$.

Hence, $s_i \in [0, 1]$, $i = 1,\dots,p$.

The term-sets L_l, $l = 1,\dots,m$, are designed as a collection of fuzzy sets, assisting recommendation levels of decision d. Sets L_l have their supports allocated in a common non-fuzzy reference set $L = [0, 1]$ in compliance with the domain of s_i ($s_i \in [0, 1]$).

We prove the action of s_i, found in P_i, in each L_l by computing $\mu_{L_l}(s_i)$, $l = 1,\ldots,m$. We adopt the optimal decision level L_l of d as level L^*, satisfying the condition $\mu_{L^*}(s_i) = \max_{1\le l\le m}(\mu_{L_l}(s_i))$.

Equation 2 shows the decision process for patient P_i as a procedure

$$\begin{bmatrix} \mu_{X_1}(x_{i,1})\cdot w_1 \\ \vdots \\ \mu_{X_n}(x_{i,n})\cdot w_n \end{bmatrix} \to s_i = \sum_{j=1}^{n}\mu_{X_j}(x_{i,j})\cdot w_j \to \begin{pmatrix} \mu_{L_1}(s_i) \\ \vdots \\ \mu_{L_m}(s_i) \end{pmatrix} \tag{3}$$

$$\to d = L^* \text{ for which } \mu_{L^*}(s_i) = \max_{1\le l\le m}\mu_{L_l}(s_i).$$

We emphasize that the decision level, selected for P_i, is patient-tailored.

In Section IV, we construct the entries of the model.

IV. THE CONSTRUCTION OF ENTRY DATA

Symptoms X_j are recognized as quantitative and qualitative features. We assign fuzzy sets X_j, $j = 1,\ldots,n$, to both types. As the rising order of symptom values (real values or codes) is associated with the growing states of the disease threat then, as a consequence, the membership functions of X_j will be constructed as ascending functions.

For the measurable symptoms X_j, taking values $x_{i,j}$ in interval [α, γ] continuously, we have prepared the membership function $\mu_{X_j}(x_{i,j})$ as a parametric *s*-function $s(x_{i,j}, \alpha, \beta, \gamma)$, yielded by [18]

$$\mu_{X_j}(x_{i,j}) = s(x_{i,j},\alpha,\beta,\gamma) =$$

$$\begin{cases} 0 & \text{for} \quad x_{i,j} \le \alpha, \\ 2\left(\frac{x_{i,j}-\alpha}{\gamma-\alpha}\right)^2 & \text{for} \quad \alpha < x_{i,j} \le \beta, \\ 1-2\left(\frac{x_{i,j}-\gamma}{\gamma-\alpha}\right)^2 & \text{for} \quad \beta < x_{i,j} \le \gamma, \\ 1 & \text{for} \quad x_{i,j} > \gamma, \end{cases} \tag{4}$$

where $\beta = \frac{\alpha+\gamma}{2}$, $j = 1,\ldots,n$, $i = 1,\ldots,p$, $x_{i,j} \in \text{supp}(X_j)$.

Example 1

Symptom "*age*"= X_2 is a fuzzy set, constrained by the membership function $s(x_{i,2}, 18, 59, 100)$. For, e.g., $x_{i,2} = 76$, we estimate $\mu_{X_2}(76) = 1-2\left(\frac{76-100}{100-18}\right)^2 = 0.828$ in accordance with the condition 59<76<100.

We adopt the own procedure [19] to calculate the membership degrees for compound qualitative symptoms X_j, characterized by a list of codes $C_{X_j} = \{0,\ldots,k,\ldots,z\}$, where $k = 0,\ldots,z$, are non-negative integers. Let us assume that z is an even integer. The codes k mark alternative answers to a question, investigating the intensity of symptom X_j in P_i. We suppose that answer 0 denies the presence of X_j, whereas value z confirms X_j's critical stage. Code value $\frac{0+z}{2}$ indicates the uncertain symptom status as "*medium intensity*", "*difficult to say*", and the like.

Let us first set up a function $g(k)$, which starts with $g(0) = -1$ and terminates with $g(z) = 1$. In general,

$$g(k) = g(0) + k\cdot\frac{g(z)-g(0)}{z} = -1 + k\cdot\frac{2}{z} \tag{5}$$

for $k = 0,\ldots,z$.

Interval [–1, 1], containing discrete values $g(k)$, constitutes a support of fuzzy set X_j, assisting the compound qualitative symptom. In order to estimate membership degrees of $g(k)$, where $k = 0,\ldots,z$, we use, as the membership function of X_j, the *s* function

$$\mu_{X_j}(g(k)) = s(g(k),-1,0,1) =$$

$$\begin{cases} 2\left(\frac{g(k)+1}{2}\right)^2 & \text{for} \quad -1 \le g(k) \le 0, \\ 1-2\left(\frac{g(k)-1}{2}\right)^2 & \text{for} \quad 0 \le g(k) \le 1. \end{cases} \tag{6}$$

After examining in detail the properties of (6), we note that: the lack of the symptom $g(0) = -1$ is characterized by membership 0, and the critical condition of the symptom $g(z) = 1$ is tied to membership 1. The value $\frac{g(0)+g(z)}{2} = \frac{-1+1}{2} = 0$, assigned to an uncertain appearance of X_j, is furnished with membership 0.5. These features of (6) logically agree with medical expectations for symptoms coded.

Example 2

The levels of symptom "*medical state*" = S_1 are coded as: "*comfortable*" = 0, "*satisfactory*" = 1, "*stable*" = 2, "*critical but stable*" = 3, and "*critical*" = 4. In accordance with (5), for $k = 0,\ldots,4$, $g(0) = -1 + 0\cdot\frac{2}{4} = -1$, $g(1) = -0.5$ $g(2) = 0$, $g(3) = 0.5$, and $g(4) = 1$. The membership degrees, found for $g(k)$, $k = 0,\ldots,4$, are, by (6), numbers:
$\mu_{X_1}(g(0)) = 0$, $\mu_{X_1}(g(1)) = 0.125$, $\mu_{X_1}(g(2)) = 0.5$, $\mu_{X_1}(g(3)) = 0.875$, and $\mu_{X_1}(g(4)) = 1$.

In the last part of Section IV, let us solve the problem of assigning the importance weights w_j to symptoms X_j. By "importance" we mean the strength of X_j's adverse and harmful power in the running process of the illness diagnosed. We bring into light another own mathematical algorithm, allowing the estimation of weights [19].

Generally, if we consider n symptoms X_j to find importance weights for them, we will wish to arrange them in the sequence $X_1 \succ ... \succ X_n$ in accordance with the expert's opinion. We want the sum of all weights w_j, assisting X_j, $j = 1,...,n$, to be 1. Therefore,

$$n \cdot r + (n-1) \cdot r + ... + 2 \cdot r + 1 \cdot r = 1 \quad (7)$$

where r is a quotient dependent on n.

Further,

$$w_j = (n - j + 1) \cdot r \quad (8)$$

for $j = 1,...,n$.

Example 3

The decisive symptoms for the recognition of necrotizing fasciitis are listed in the importance order, decided by the physician, as "*medical state*" = $X_1 \succ$"*age*" = $X_2 \succ$"*risk factors*" = $X_3 \succ$"*crp*" = $X_4 \succ$*wbc* = $X_5 \succ$"*temperature*" = X_6. The abbreviation "*crp*" stands for C-reactive proteins and "*wbc*" – for white blood cells. In conformity with (7), equation $6r+5r+4r+3r+2r+r = 1$ provides $r = 0.0476$. After employing (8), we receive, in turn for $j = 1,…,6$, the weights: $w_1 = (6–1+1)\cdot 0.0476= 0.2856$, $w_2 = 0.238$, $w_3 = 0.1904$, $w_4 = 0.1428$, $w_5 = 0.0952$, and $w_6= 0.0476$.

V. THE STRUCTURE OF FUZZIFIED OUTCOMES

As (3) recommends, we should now generate a sample of output recommendation fuzzy levels L_l of decision d, $l = 1,…,m$. The supports of L_l cover parts of [0, 1], as proved in Section III ($s_i \in [0, 1]$). To calculate the membership degrees of signal s_i in L_l, $i = 1,…,p$, we need to derive a formula of the membership function of each L_l. The largest value $\mu_{L_l}(s_i)$, $l = 1,..,m$, points out the optimal recommendation level of decision d, advised for P_i.

Theoretically, m can be either an even or an odd positive arbitrary integer. An own procedure [20], expanded in this paper, helps us to derive membership functions of L_l. These are dependent only on two parameters, namely, a number m of term-sets in a list of decision d and a width E of the common reference set L, containing all supports of L_l.

In the medical problem discussed, m is supposed to be the even number, as the differentiation of non-indication and indication levels of the HBO treatment is bipartite. Our intention is to derive two common formulas of membership functions of sets L_l, separated in two families. Then, we do not need to predetermine the boundary values of supports of fuzzy sets in an intuitive or a random way. By the way, it is important to emphasize that the procedure can be easily computerized.

Due to the definition of the α-cut set of a fuzzy set (introduced in Section III), we denote by $L_{l,\alpha}$ a set of $s_i \in L = [0, 1]$, for which $\mu_{L_l}(s_i) \geq \alpha, l = 1, ..., m$.

As a pattern of membership function of L_l, the s-function $s(s_i, \alpha_{L_l}, \beta_{L_l}, \gamma_{L_l}) = \mu_{L_l}(s_i)$ is arranged in accord with (4).

If we wish to narrow domains of functions $s(s_i, \alpha_{L_l}, \beta_{L_l}, \gamma_{L_l}) = \mu_{L_l}(s_i)$ and, consequently, to narrow supports of fuzzy sets L_l, then we will modify $s(s_i, \alpha_{L_l}, \beta_{L_l}, \gamma_{L_l})$ as $s(s_i, \alpha_{L_l} \cdot \delta, \beta_{L_l} \cdot \delta, \gamma_{L_l} \cdot \delta)$.

Function $s(s_i, \alpha_{L_l} \cdot \delta, \beta_{L_l} \cdot \delta, \gamma_{L_l} \cdot \delta)$ is expanded by

$$s(s_i, \alpha_{L_l} \cdot \delta, \beta_{L_l} \cdot \delta, \gamma_{L_l} \cdot \delta) = \begin{cases} 0 & \text{for } s_i \leq \alpha_{L_l} \cdot \delta, \\ 2\left(\frac{s_i - \alpha_{L_l} \cdot \delta}{(\gamma_{L_l} - \alpha_{L_l})\delta}\right)^2 & \text{for } \alpha_{L_l} \cdot \delta \leq s_i \leq \beta_{L_l} \cdot \delta, \\ 1 - 2\left(\frac{s_i - \gamma_{L_l} \cdot \delta}{(\gamma_{L_l} - \alpha_{L_l})\delta}\right)^2 & \text{for } \beta_{L_l} \cdot \delta \leq s_i \leq \gamma_{L_l} \cdot \delta, \\ 1 & \text{for } s_i \geq \gamma_{L_l} \cdot \delta. \end{cases} \quad (9)$$

Values $0 < \delta < 1$ have an effect of narrowing domains in (9). Value $\delta = 1$ allows returning to (4).

Theorem 1

Let us suppose that term-sets L_l, $l = 1,…,m$, have supports included in the common non-fuzzy reference set L, where $\min(L) = 0$. Patient characteristics s_i belongs to L and the width of L is E.

If m is even, then we divide all fuzzy sets L_l in two families. A family of "*left*" sets $L_1,…, L_{\frac{m}{2}}$ contains L_t sets, where $t = 1,…, \frac{m}{2}$. A family of "*right*" sets $L_{\frac{m+2}{2}},…,L_m$ is composed of $L_{\frac{m+2}{2}+t-1}$ sets for $t = 1,… \frac{m}{2}$.

We assume that sets $L_{1,0.5}$, $L_{t,0.5}–L_{t-1,0.5}$, $t = 2,..., \frac{m}{2}$, $L_{\frac{m+2}{2}+t-1,0.5} - L_{\frac{m+2}{2}+t,0.5}$, $t = 1,..., \frac{m-2}{2}$, and $L_{m,0.5}$, established by α-cuts of $L_1,…, L_{\frac{m}{2}}$ and $L_{\frac{m+2}{2}},…,L_m$ for $\alpha = 0.5$, have the same width. Suppose further that the membership functions of the last "*left*" set $L_{\frac{m}{2}}$ and the first "*right*" set $L_{\frac{m+2}{2}}$ have the intersection point on membership level 0.5. Hence, the common formulas for membership functions of L_l in their families are given by

$$\mu_{L_t}(s_i) = \begin{cases} 1 & \text{for } s_i \le \frac{(m-2)E}{2(m-1)}\delta(t), \\ 1-2\left(\frac{s_i - \frac{(m-2)E}{2(m-1)}\delta(t)}{\frac{E}{(m-1)}\delta(t)}\right)^2 & \text{for } \frac{(m-2)E}{2(m-1)}\delta(t) \le s_i \le \frac{E}{2}\delta(t), \\ 2\left(\frac{s_i - \frac{mE}{2(m-1)}\delta(t)}{\frac{E}{(m-1)}\delta(t)}\right)^2 & \text{for } \frac{E}{2}\delta(t) \le s_i \le \frac{mE}{2(m-1)}\delta(t), \\ 0 & \text{for } s_i \ge \frac{mE}{2(m-1)}\delta(t), \end{cases} \quad (10)$$

where $\delta(t) = \frac{2}{m}\cdot t$, $t = 1,\ldots,\frac{m}{2}$, for the "*left*" family and, for the "*right*" family,

$$\mu_{L_{\frac{m+2}{2}+t-1}}(s_i) = \begin{cases} 0 \text{ for } s_i \le E - \frac{mE}{2(m-1)}\varepsilon(t), \\ 2\left(\frac{s_i - \left(E - \frac{mE}{2(m-1)}\varepsilon(t)\right)}{\frac{E}{(m-1)}\varepsilon(t)}\right)^2 \\ \text{for } E - \frac{mE}{2(m-1)}\varepsilon(t) \le s_i \le E - \frac{E}{2}\varepsilon(t), \\ 1-2\left(\frac{s_i - \left(E - \frac{(m-2)E}{2(m-1)}\varepsilon(t)\right)}{\frac{E}{m-1}\varepsilon(t)}\right)^2 \\ \text{for } E - \frac{E}{2}\varepsilon(t) \le s_i \le E - \frac{(m-2)E}{2(m-1)}\varepsilon(t), \\ 1 \text{ for } s_i \ge E - \frac{(m-2)E}{2(m-1)}\varepsilon(t), \end{cases} \quad (11)$$

if $\varepsilon(t) = 1 - \frac{2}{m}\cdot(t-1)$, $t = 1,\ldots,\frac{m}{2}$. The membership functions are derived on the basis of (4) and (9).

Proof:

We start with the assumption: $L_{1,0.5}$, $L_{t,0.5}$–$L_{t-1,0.5}$, $t = 2,\ldots,\frac{m}{2}$, $L_{\frac{m+2}{2}+t-1,0.5} - L_{\frac{m+2}{2}+t,0.5}$, $t = 1,\ldots,\frac{m-2}{2}$, and $L_{m,0.5}$ have the same width. It results in making the partition of reference set L in m–1 subintervals with the same width equal to $\frac{E}{m-1}$.

We estimate *Euclidean distance* between points ($\alpha_{L_{\frac{m}{2}}}$,1) and ($\frac{E}{2}$,1) as $\frac{E}{2(m-1)}$ (half a width of the middle subinterval lying along set L).

We compute $\alpha_{L_{\frac{m}{2}}} = \frac{E}{2} - \frac{E}{2(m-1)} = \frac{(m-2)E}{2(m-1)}$ and $\gamma_{L_{\frac{m}{2}}} = \frac{E}{2} + \frac{E}{2(m-1)} = \frac{mE}{2(m-1)}$ to assure that the membership function of $L_{\frac{m}{2}}$ intersects the membership function of $L_{\frac{m+2}{2}}$ in point ($\frac{E}{2}$,0.5).

If $\beta_{L_{\frac{m}{2}}} = \frac{E}{2}$, then $\mu_{L_{\frac{m}{2}}}(s_i) = 1 - s(s_i, \frac{(m-2)E}{2(m-1)}, \frac{E}{2}, \frac{mE}{2(m-1)})$.

We employ (4) to get the membership function of $L_{\frac{m}{2}}$ as a formula

$$\mu_{L_{\frac{m}{2}}}(s_i) = \begin{cases} 1 & \text{for } s_i \le \frac{(m-2)E}{2(m-1)}, \\ 1-2\left(\frac{s_i - \frac{(m-2)E}{2(m-1)}}{\frac{E}{(m-1)}}\right)^2 & \text{for } \frac{(m-2)E}{2(m-1)} \le s_i \le \frac{E}{2}, \\ 2\left(\frac{s_i - \frac{mE}{2(m-1)}}{\frac{E}{(m-1)}}\right)^2 & \text{for } \frac{E}{2} \le s_i \le \frac{mE}{2(m-1)}, \\ 0 & \text{for } s_i \ge \frac{mE}{2(m-1)}\delta(t). \end{cases} \quad (12)$$

The "*left*" family of fuzzy sets from (10) will be generated, when we add function $\delta(t) = \frac{2}{m}\cdot t$, $t = 1,\ldots,\frac{m}{2}$, to (12) in accordance with (9).

The modifier $\delta(t)$, $0 < \delta(t) \le 1$, is inserted in (12) to cause narrowing effects of supports of L_t, $t = 1,\ldots,\frac{m}{2}$. Function $\delta(t)$ reveals the properties: $\delta(\frac{m}{2}) = 1$ (no impact on the support of the last "*left*" set $L_{\frac{m}{2}}$) and $\delta(1) = \frac{1}{m/2} = \frac{2}{m}$ (the largest scale value 1 is divided by the number of left sets). If we suppose that $\delta(t) = a\cdot t$, then the solution of equation $a\cdot\frac{m}{2} = 1$ will provide $a = \frac{2}{m}$. Hence, $\delta(t) = \frac{2}{m}\cdot t$.

We now construct the membership function of the first right fuzzy set $L_{\frac{m+2}{2}}$ as a reverse membership function of $L_{\frac{m}{2}}$. We find

$$\mu_{L_{\frac{m+2}{2}}}(s_i) = \begin{cases} 0 & \text{for } s_i \le E - \frac{mE}{2(m-1)}, \\ 2\left(\frac{s_i - \left(E - \frac{mE}{2(m-1)}\right)}{\frac{E}{m-1}}\right)^2 & \text{for } E - \frac{mE}{2(m-1)} \le s_i \le E - \frac{E}{2}, \\ 1-2\left(\frac{s_i - \left(E - \frac{(m-2)E}{2(m-1)}\right)}{\frac{E}{m-1}}\right)^2 & \text{for } E - \frac{E}{2} \le s_i \le E - \frac{(m-2)E}{2(m-1)}, \\ 1 & \text{for } s_i \ge E - \frac{(m-2)E}{2(m-1)}. \end{cases} \quad (13)$$

The membership functions of sets $L_{\frac{m+2}{2}}$,...,L_m are initialized after inserting a new modifier $\varepsilon(t) = 1 - \frac{2}{m}\cdot(t-1)$, $t = 1,\ldots,\frac{m}{2}$, $0 < \varepsilon(t) \le 1$, in (13). The insertion matches the model provided by (9) and proves (11). For $t = 1$, we get $\varepsilon(1) = 1$, while $t = \frac{m}{2}$ follows $\varepsilon(\frac{m}{2}) = \frac{1}{m/2} = \frac{2}{m}$. We derive $\varepsilon(t) = 1 - a\cdot(t-1)$ to ensure the equality $\varepsilon(1) = 1$.

Equation $1 - a\cdot(\frac{m}{2} - 1) = \frac{2}{m}$ has solution $a = \frac{2}{m}$.

Example 4

The term list of decision d = "*recommendation for treating with HBO for patient* P_i" is stated as $d = \{L_1$ = *strong non-indication for treating with HBO*", L_2 = *moderate non-indication for treating with HBO*", L_3 =" *moderate indication for treating with HBO*", L_4 = "*strong indication for treating with HBO*". L_1 and L_2 belong to the "*left*" family of fuzzy sets, whereas L_3 and L_4 build the "*right*" family of fuzzy sets. Sets L_l have the supports included in interval [0, 1], due to the statement $s_i \in [0, 1]$. For $m = 4$ and $E = 1$, we get

$$\mu_{L_1}(s_i) = \begin{cases} 1 & \text{for } 0 \le s_i \le 0.166, \\ 1 - 2\left(\frac{s_i - 0.166)}{0.166}\right)^2 & \text{for } 0.166 \le s_i \le 0.25, \\ 2\left(\frac{s_i - 0.333}{0.166}\right)^2 & \text{for } 0.25 \le s_i \le 0.333, \\ 0 & \text{for } s_i \ge 0.333, \end{cases} \quad (14)$$

and

$$\mu_{L_2}(s_i) = \begin{cases} 1 & \text{for } 0 \le s_i \le 0.333, \\ 1 - 2\left(\frac{s_i - 0.333)}{0.333}\right)^2 & \text{for } 0.333 \le s_i \le 0.5, \\ 2\left(\frac{s_i - 0.666}{0.333}\right)^2 & \text{for } 0.5 \le s_i \le 0.666, \\ 0 & \text{for } s_i \ge 0.666, \end{cases} \quad (15)$$

when setting $t = 1$ ($\delta(1) = 0.5$) and $t = 2$ ($\delta(2) = 1$) in (10), respectively.

The action of placing $t = 1$ ($\varepsilon(1) = 1$) and $t = 2$ ($\varepsilon(2) = 0.5$), in (11), yields

$$\mu_{L_3}(s_i) = \begin{cases} 0 & \text{for } 0 \le s_i \le 0.333, \\ 2\left(\frac{s_i - 0.333}{0.333}\right)^2 & \text{for } 0.333 \le s_i \le 0.5, \\ 1 - 2\left(\frac{s_i - 0.666}{0.333}\right)^2 & \text{for } 0.5 \le s_i \le 0.666, \\ 1 & \text{for } s_i \ge 0.666, \end{cases} \quad (16)$$

and

$$\mu_{L_4}(s_i) = \begin{cases} 0 & \text{for } 0 \le s_i \le 0.667, \\ 2\left(\frac{s_i - 0.667}{0.167}\right)^2 & \text{for } 0.667 \le s_i \le 0.75, \\ 1 - 2\left(\frac{s_i - 0.833}{0.167}\right)^2 & \text{for } 0.75 \le s_i \le 0.833, \\ 1 & \text{for } s_i \ge 0.833. \end{cases} \quad (17)$$

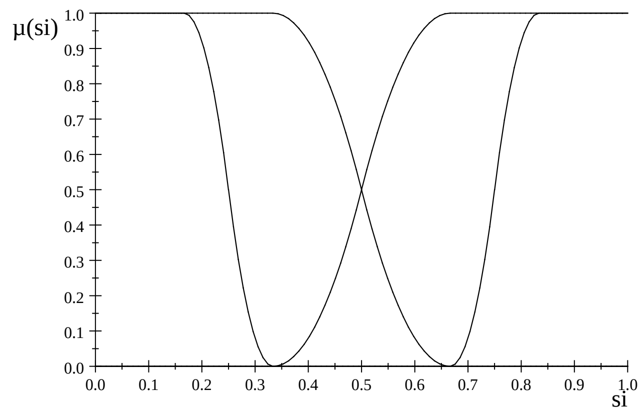

Figure 1. Fuzzy sets L_1-L_4.

Fuzzy sets L_1-L_4 are sketched in Figure 1.

Section VI is devoted to tracking the theoretical proposal by a solution of the medical query, formulated as the recommendation of the treatment with HBO. The decision is made for a single patient.

VI. THE RECOMMENDATION FOR TREATING WITH HBO

It has already been mentioned in Section I that the mathematical apparatus, built in Sections III-V, will be applied to select either a non-indication level or an indication level of decision d.

The data, including the values of crucial clinical markers, have been sampled for 13 patients (12 men and 1 woman) treated in the Blekinge County City Hospital in Karlskrona, Sweden, between 2006 and 2010.

The clinical symptoms, essential in NF, have been introduced in Example 3. For quantitative symptoms we adapt (4) as follows:

$\mu_{X_2=\text{"age"}}(x_{i,2}) = s(x_{i,2}, 18, 59, 100)$,

$\mu_{X_4=\text{"crp"}}(x_{i,4}) = s(x_{i,4}, 0, 250, 500)$,

$\mu_{X_5=\text{"wbc"}}(x_{i,5}) = s(x_{i,5}, 0, 15, 30)$,

and

$\mu_{X_6=\text{"temp."}}(x_{i,6}) = s(x_{i,6}, 36, 38.5, 41)$.

In Example 2, we have already determined the membership degrees for the coded symptom X_1 = "*medical state*" as:

$\mu_{X_1}(g(0)) = 0$, $\mu_{X_1}(g(1)) = 0.125$, $\mu_{X_1}(g(2)) = 0.5$,

$\mu_{X_1}(g(3)) = 0.875$, and $\mu_{X_1}(g(4)) = 1$.

We repeat the algorithm for symptom X_3 = "*risk factors*", coded between 0 and 6, to find $g(0) = -1$, $g(1) = -0.666$, $g(2) = -0.333$, $g(3) = 0$, $g(4) = 0.333$, $g(5) = 0.666$ and $g(6) = 1$. When applying $\mu_{X_3=\text{"risk factors"}}(g(k)) = s(g(k), -1, 0, 1)$, $k = 0,\dots,6$, we list:

$\mu_{X_3}(g(0)) = 0$, $\mu_{X_3}(g(1)) = 0.056$, $\mu_{X_3}(g(2)) = 0.221$,

 ISBN: 978-1-61208-470-1

TABLE I. PATIENT SYMPTOM VALUES AND MEMBERSHIP DEGREES IN FUZZY SETS DESIGNED FOR SYMPTOMS X_J, $J = 1,\dots,6$

P_i	X_1	X_2	X_3	X_4	X_5	X_6
P_1	0.13/1	0.06/32	0/0	0.83/352	0.52/15.3	0/36.2
P_2	0/36.2	0.83/76	0.5/3	0.56/267	0.49/14.9	0.39/38.2
P_3	0.88/3	0.30/50	0.06/1	0.43/232	0.22/10	0.14/37.3
P_4	0.5/2	0.66/66	0.22/2	0.7/305	0.99/28.2	0.29/37.9
P_5	0.88/3	0.75/71	0/0	0.29/189	0.99/27.8	0.14/37.3
P_6	0.5/2	0.45/57	0/0	0.64/281	0.53/15.5	0.42/38.3
P_7	1/4	0.29/49	0.06/1	0.85/363	0.76/19.5	0.03/36.6
P_8	0.88/3	0.89/81	0.5/3	0.91/394	0.36/12.7	0.20/37.6
P_9	1/4	0.48/58	1/6	0.94/413	0.68/18	0.32/38
P_{10}	0.88/3	0.45/57	0.06/1	0.48/246	0.02/3.1	0/35.8
P_{11}	0.5/2	0.52/60	0.22/2	0.06/85	0.62/16.9	0.29/36.5
P_{12}	0.88/3	0.73/70	0.78/4	0.92/403	0.99/28.5	0.32/38
P_{13}	1/4	0.88/80	0.22/2	0.05/76	0.73/18.9	0.98/40.5

$\mu_{X_3}(g(3)) = 0.5$, $\mu_{X_3}(g(4)) = 0.779$, $\mu_{X_3}(g(5)) = 0.944$, and $\mu_{X_3}(g(6)) = 1$.

TABLE I contains the clinical data and assigned to them membership degrees, computed in compliance with the membership functions of X_j. The membership degree of $x_{i,j}$ in X_j appears before the dash, and the $x_{i,j}$ clinical value is placed after the dash, $j = 1,\dots,6$.

As emerged in (2), the concatenation of membership degrees $\mu_{X_j}(x_{i,j})$ with weights w_j, evaluated in Example 3, $j = 1,\dots,6$, will constitute a basis for the calculation of the cumulated clinical characteristics s_i for patient P_i.

Example 5

Patient P_1 is represented by
$s_1 = 0.125\cdot0.286+0.058\cdot0.238+0\cdot0.19+0.824\cdot0.1428+0.52\cdot0.095+0.003\cdot0.047 = 0.217$.

In order to select one of four decision levels by means of membership degrees in L_l, $l = 1,\dots,4$, we return to (14)-(17).

We choose the decision characterized by the largest membership degree out of $\mu_{L_l}(s_i)$.

Example 6

TABLE II collects s_i, their membership degrees in L_l, $l = 1,\dots,4$, and the physician's assertion already made. The abbreviations mean: PD HBO = the physician's decision, concerning treating the patient with HBO, N = none treating with HBO, and Y = treating with HBO.

For example, for $s_i = 0.217$ (characteristics of P_1), we get: $\mu_{L_1}(0.217) = 1 - 2\left(\frac{0.217-0.166}{0.166}\right)^2 = 0.28$ (0.166 <0.217 <0.25), $\mu_{L_2}(0.217) = 1$ (0.217 < 0.333), $\mu_{L_3}(0.217) = 0$ (0.217 < 0.333), and $\mu_{L_4}(0.217) = 0$ (0.217 < 0.667).

The largest value of the membership degree indicates level L_2.

TABLE II. THE COMPARISON OF FUZZY DECISIONS (UNDERLINED) TO DECISIONS MADE BY THE PHYSICIAN

P_i	s_i	$\mu_{L_1}(s_i)$	$\mu_{L_2}(s_i)$	$\mu_{L_3}(s_i)$	$\mu_{L_4}(s_i)$	PD HBO
P_1	0.217	0.81	<u>1</u>	0	0	N
P_2	0.58	0	0.13	<u>0.87</u>	0	Y
P_3	0.42	0	<u>0.86</u>	0.14	0	N
P_4	0.558	0	0.25	<u>0.75</u>	0	Y
P_5	0.57	0	0.17	<u>0.83</u>	0	Y
P_6	0.41	0	<u>0.89</u>	0.11	0	N
P_7	0.56	0	0.21	<u>0.79</u>	0	Y
P_8	0.73	0	0	<u>1</u>	0.29	Y
P_9	0.80	0	0	<u>1</u>	0.93	Y
P_{10}	0.44	0	<u>0.8</u>	0.2	0	N
P_{11}	0.39	0	<u>0.94</u>	0.06	0	N
P_{12}	0.81	0	0	<u>1</u>	<u>0.97</u>	Y
P_{13}	0.66	0	0	<u>1</u>	0	Y

In the future research, we plan to test the model with an odd number of decisions levels, where the middle level "*wait and see*" will be assigned to values about 0.5.

VII. CONCLUSION AND FUTURE WORK

By suggesting modifications in the classical fuzzy decision making, we have used our model to advise the treatment with hyperbaric oxygen. This treatment can improve the health state in patients, suffering from necrotizing fasciitis.

Instead of designing a utility matrix filled with distinct utilities of pairs (decision, state), we have introduced only one decision, designated by the list of term-sets. These express recommendation levels of the treatment as non-indications and indications. The decision levels are involved in the algorithm in its final phase. This differs the model, proposed in the current paper, from most of fuzzy decision making models, in which decisions are already active in the first stage of designing the utility matrix. It is also worth emphasizing that our decisions are made for individuals, and they have not general characters, as it often happens in other patterns of fuzzy decision making.

The input data and output recommendation levels are fuzzified by designs of own suggestions of membership functions. The membership functions of the outcomes (recommendation levels) are sampled in two common formulas. The formulas depend only on a number of recommendation terms and the width of a reference set, linking all supports of recommendations. The functions are derived in the way, which allows entering an arbitrary number of recommendation levels. This extends the decision scale of linguistic expressions without making changes in formulas.

The own procedures of estimating the importance weights of symptoms and approximating membership degrees of qualitative symptoms have also been added as contributions in imprecise mathematics.

Necrotizing fasciitis is a quite rare entity, and there is no widespread consensus regarding neither treatment nor grading. There were done several attempts of using laboratory results to facilitate grading of the severity of the disease, but as far as we know, they are not used widely. The idea of combining analysis of numerical parameters, such as body temperature, white blood cell count, age etc. with the qualitative estimations, such as, e.g., medical state, is very promising because it will reflect the real decision making progress. The model, tested above, is based on retrospective analysis of data of patients treated with hyperbaric oxygen (HBO) at the surgery department in Karlskrona, Sweden.

We realize that the proposition of making decisions in the case of the HBO dosing has weaknesses, mostly, when the group, used to check the model, has not been very numerous. In spite of this, it seems that we have been successful in selecting essential clinical and biochemical parameters for the correctness of the mathematical model. The decisions have "*softer*" character than two-valued decisions "*yes-no*". This is a result of imprecision, introduced by the overlapping effect of fuzzy sets.

In the further research, we will redefine the ordering of importance weights of symptoms more carefully to refine the results. We also plan to test the model with an odd number of decisions levels, where the middle level "*wait and see*" will be assigned to values about 0.5.

Since an emphasis is laid on the design of recommendation levels, appearing as the output of the mathematical algorithm, then we can classify the model as robust approach to algorithmic modeling of outcomes.

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