Enhanced Robust Convex Relaxation Framework for Optimal Controllability of Certain Large Complex Networked Systems

An Accelerant Amalgam and Bespoke Numerical Stability Paradigm for a Decoupled and Sequenced Control Strategy on Dense and Homogeneous Temporal Networks

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Abstract— **Efficient Controllability Problems (ECP) for Large Complex Networked System (LCNS) often involve solving a succession of convex optimization problems, with varied approaches to optimally resolve each problem. In various cases, even when the input set is specifically designed/architected to segue to a convex paradigm, the resultant output set may still turn out to be nonconvex. Further processing is necessary to reach the desired convex paradigm, such as via certain relaxation techniques. However, the involved transformation, during the processing, may result in further nonconvex optimization problems, thereby highlighting the need/opportunity to utilize an Enhanced Robust Convex Relaxation (ERCR) framework. In this paper, we illuminate how leveraging such an ERCR framework, to discern how the involved LCNS's topological structure, facilitates or prevents the diffusion of control signals and/or augmented control signals, which in turn informs the computations related to an accelerant amalgam and numerical stability paradigm for effectively leveraging a set of control/driver nodes to influence yet another set of control/driver nodes so as to steer the LCNS to a target state, if a decoupled and sequenced control strategy is utilized. The numerical stability paradigm employed by the ERCR framework is, potentially, of scientific gain and shows promise in contending with certain round-off errors, thereby better facilitating the transformation of certain uncontrollable cases into controllable cases, if temporal networks are considered. For those paradigms, wherein the Bak–Tang–Wiesenfeld (BTW) sandpile cascading effect is a potentiality, this facilitation may be quite significant.**

Keywords-Cyber-physical systems; cyber-physical power system; large complex networked systems; temporal networks; supply chain vulnerability; efficient controllability; strong controllability; control signal energy cost; robust convex relaxation; accelerant amalgam; numerical stability; neural network; controllability Gramian; Gramian submatrices.

I. INTRODUCTION

Interest in the controllability problem of complex networks is burgeoning. Some studies have posited that while control of a substantive portion of the nodes may be ideal in the cases of some smaller networks, controlling a smaller subset of nodes may be more practical for larger, more complex networks. Accordingly, various studies have examined the problem set of influencing or controlling Large Complex Networked Systems (LCNS) with limited external Control Signals (CS) [1], which is often referred to as the Network Controllability Problem (NCP) [2]. Along this vein, other works have tackled the problem of selecting the smallest number of CS to ensure controllability of such LCNS [3]. Yet, the solving of such Minimum Controllability Problems (MCP) is just one step [4]. A further step involves solving related Efficient Controllability Problems (ECP), which focus on minimizing both the number of control nodes needed, as well as minimizing the control signal energy needed. However, these ECP have been shown to exhibit Non-deterministic Polynomial-time Hardness (NP-Hardness). Various approximation algorithms and heuristical approaches have been utilized to achieve sub-optimal solutions to these NP-Hard ECPs [1]. To aggravate matters, these sub-optimal approaches tend to falter further when elevated notions of specific (e.g., output) controllability [5][6] are contended with, and, practically speaking, actual controllability is difficult to achieve (as contrasted to merely mathematical controllability [7]).

More robust approaches have been proposed for tackling the NP-Hard ECPs as well as the issue of actual controllability. Various works have focused on augmenting the set of input CS on "properly chosen" control or "driver nodes" [6], which connotes the paradigm of certain nodes within the network having the potential of control authority to drive [8]. Yet, even if the control/driver nodes are "properly chosen" — and even if the LCNS is controllable (putting aside the issue of mathematical versus actual controllability) — via the chosen control/driver nodes, the Control Signal Energy Cost (CSEC) that those nodes require may be "unrealistically large" [8]; in other words, "if the number of control signals is small, the energy cost demanded … could be prohibitively high" [9]. There is yet another issue; a substantive portion of the studies are focused upon linear systems because, at least over short time scales, continuous nonlinear systems are approximated as linear [10]; for this reason, the involved networks are approximated and assumed to have n-dimensional Linear Time-Invariant (LTI) dynamics [2]. To further the discussion regarding practicality, just as a prohibitively high CSEC would not be practical, controllability over only short time scales would be comparably impractical (e.g., the inability to exert control at a desired time, as the window of control may have already passed). This further extends the problem, as temporal considerations are at play, into the realm of Temporal Problems (TPs); moreover, as the temporal duration is uncertain, the problem is that of TP with uncertainty (TPU) [11]. Not only does the controllability need to persist over a sufficiently long time scale or reasonable extended period of time, the actual ability to control, when desired, needs to occur in a finite period of time (i.e., immediately or As Soon As Possible — ASAP). Hence, it seems that the revised optimality problem becomes one of ascertaining the sufficient number of input CS to steer a minimal number of control/driver nodes at a reasonable energy cost ($CSEC_{OPT}$), over an extended period of time (TPU_{OPT}) (as contrasted to TPU_{max}), but which can be activated and effectuated within a finite period of time (e.g., ASAP). Accordingly, the main contribution of the paper is to introduce a strategy for transforming optimization problems to convex form so as to reduce the complexity class from NP-Hard to polynomial time, such as for the ECP-related computations, using an Enhanced Robust Convex Relaxation (ERCR) framework equipped with a bespoke numerical stability paradigm.

The paper is structured as follows. Section I introduces the controllability problem of complex networks. Section II presents relevant background information and discusses the operating environment and the state of the controllability challenge. Section III provides some theoretical foundations and the utilized approach. Section IV delineates a strategy for a sequence of transformations and presents some preliminary experimental findings from using an ERCR framework on dense and homogeneous temporal networks. Section V provides some reflections on potential further heuristical processing, such as by way of LCNS partitioning and the practicality of TN_{Bno} expansion for some real-world applications, such as assessing Supply Chain Vulnerability (SCV). Section VI concludes with some reflections, puts forth some envisioned future work, and the acknowledgements close the paper.

II. BACKGROUND INFORMATION

In accordance with control theory, a system is deemed to be controllable, if it can be driven from an initial state to a desired state with suitable input(s) [4]. It then follows that if the nodes of a LCNS can be steered from an arbitrary initial state vector towards a predefined goal state vector within a finite period of time, then the network is deemed to be controllable [9]. The positing of the actual controllability is another matter; the positing of the accuracy of the controllability is still yet another matter. Among other frameworks, structural controllability had been put forth as a potentially viable analytical framework for ascertaining the controllability of LCNS. However, Cowan et al. have noted the limitations of structural controllability [12] as well as certain of its associated paradoxes; for example, in some cases, the CSEC of a structurally controllable system can be higher than that of a "structurally" uncontrollable system [8]. Alternative frameworks have been proposed, such as by Yuan et al., to include exact controllability (i.e., arbitrary link structures [e.g., directed, undirected] and link weights [e.g., weighted, unweighted] [13]), which better reflects the directed and weighted network configurations found in most real-world systems [35].

With regards to CSEC, Chen et al. asserted that "if the number of control signals is small, the energy cost demanded … could be prohibitively high;" conversely, the energy cost is reduced exponentially as the number of input CS increases [9]. It should, therefore, be axiomatic that the ascertaining of the sufficient number of input CS and their optimal "distribution throughout the complex network" (CS_{opt}) is "vitally important to the feasibility and the efficiency of a Control Action" (CA) [1], which is defined to be the achieving of a predefined goal state vector; along this vein, a "Control Maneuver" (CM) might be comprised of several CAs [1], which at some point might arrive at CA_{OPT} (ascertained over time). Effective CAs and/or their CMs can lead to faster network control/collapse [14. The Target Nodes (TN) involved in the achieving of the predefined goal state vector are deemed to have been subjected to "Targeted Control" (TC) [9]. The computational aims, then, seem to be that of ascertaining a minimum number of optimal control/driver nodes, such as proposed by Gao et al. [9], and their placements, such as proposed by Lindmark et al. [8], that, with sufficiently distributed and available CS, such as proposed by Klickstein et al., would only require a minimum CSEC ($CSEC_{min}$) [8], but the optimal [and practical] CSEC ($CSEC_{OPT}$) would include augmentation CS. In accordance with self-organization theory, a series of small events can cause a chain reaction that can affect any number of components in the system, as delineated by the well-known Bak–Tang–Wiesenfeld (BTW) sandpile effect [15] of non-equilibrium systems in which sand is dropped, one grain at a time, onto the same spot until the addition of one more grain of sand causes an avalanche to slide down the slopes of the growing sandpile; this avalanche also tends to burgeon into a cascading series of avalanches that can grow in size and intensity (i.e., similar to the notion of a cascading effect) [36]. Cascading effects (e.g., cascading failures) have manifested themselves, such as via Northeast Blackout of 2003 and 2012 India Blackouts, wherein the "failure of one or a few components" … triggered the … "successive failures of other components" [16].

The identification of control/driver nodes has been a longstanding goal of many Complex Network Analysis (CNA) efforts [17], such as for Supply Chain Vulnerability (SCV) analysis efforts within the rubric of Supply Chain Risk Management (SCRM). These SCRM efforts have become more complicated, as physical systems and information systems are increasingly being fused into Cyber-Physical Systems (CPS), wherein it is possible to control physical systems, via cyber systems [18]. The implication should be clear; prospective input CS can emanate from either the cyber or physical domains. Both these domains are considered in Guo et al.'s Cyber-Physical Power System (CPPS) model, which touches upon the notion that while current CPPS can provide a modicum of resiliency for high-indexed nodes, they are much less resilient (i.e., vulnerable) to malicious attacks (i.e., targeted attacks) [14]. The implications of the varied attack surfaces of Multi-Domain Operations (MDO) should be axiomatic.

In either case, the previously cited MCP, as applied to the approximated LTI paradigm, can be construed to be a minimum Constrained Input Selection (minCIS) problem [19][20]. With CSEC constraints, the minCIS then becomes a minimum Cost Constrained Input Selection (minCCIS) problem [21][22]. Among other methods, the Projected Gradient Method (PGM) has been used to solve the constrained optimization problem of minCCIS [2][23], which can also be recast as a constrained convex minimization/optimization problem [23]. In essence, PGM endeavors to find locally optimal solutions to a continuous relaxation of the convex optimization problem [1]. While PGM can be useful for convex optimization problems with simple constraints, such as minCCIS with LTI dynamics, other methods may be needed given more complex constraints. For example, various works are examining minCCIS amidst uncertainties (minCCIS-u), such as time delays [24] (e.g., does the ability to effectuate a CM persist beyond the immediate time period, and is it available when desired). The aforementioned paradigm is delineated in Fig. 1 below.

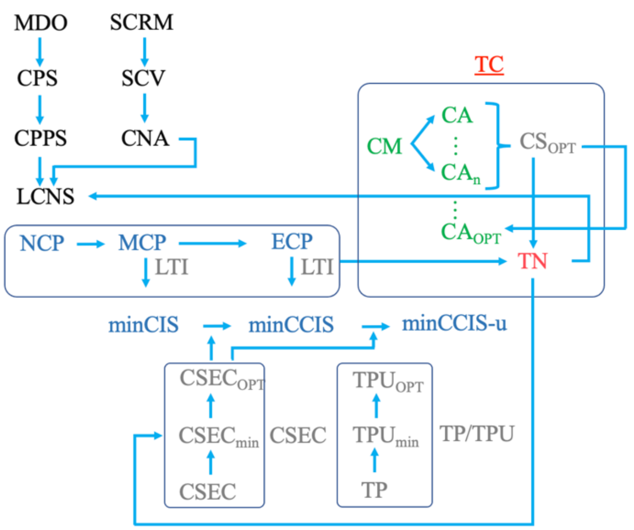

Figure 1. Targeted Control (TC), via Targeted Nodes (TN), in the Described Environs of minCCIS-u and TPU_{OPT}

Given the myriad of uncertainties for the minCCIS-u problem, the continuous relaxations involve successive convex optimization problems, wherein varied approaches might be utilized to optimally resolve each problem. After all, as previously observed in [25], even when the input set is specifically formulated to segue to a convex optimization problem, the resultant may still turn out to be nonconvex, thereby necessitating a transformation to the desired form of a convex optimization problem, via certain relaxation techniques; however, the transformation itself may spawn other nonconvex optimization problems. In fact, when the objective and constraint functions are nonconvex, these problems turn out to be NP-Hard Mixed Integer Non-Linear Programming (MINLP) nonconvex optimization problems that need to be optimally solved.

The referenced ERCR, which was equipped with a bespoke numerical stability paradigm, was utilized to handle these nonconvex optimization problems and reduce the complexity class from NP-Hard to polynomial time; to further unpack this handling, by way of background information, pertinent approach vectors are typically classified into two methods: (1) exact (i.e., complete), and (2) relaxed (i.e., incomplete). Prototypical exact verifiers are predicated upon Mixed Integer Programming (MIP) (specifically, MINLP, for the experimentation discussed herein), Branch-and-Bound (BnB), or Satisfiability Modulo Theories (SMT) (which, by definition, are not beset by false positives or false negatives). The challenge of utilizing exact verifiers is that they must contend with resolving NP-hard optimization problems, which in turn, obviates their scalability. Prototypical relaxed verifiers are predicated upon Mixed Integer Convex Programming (MICP) or Mixed Integer Linear Programming (MILP). MILP/MICP can be more quickly resolved and are more scalable, but the effectiveness (i.e., increased false negative rates) degrades quickly [42], thereby potentially obviating the ability to verify robustness. Hence, addressing robustness, such as via robust convex relaxations (i.e., effectuating the tightest possible relaxation [42]) becomes central for the experimentation/simulation. The utilized pathways to a convex paradigm are set within a Discrete, Continuous/Discontinuous (y-axis) and Non-Linear, Linear (x-axis) quad chart shown in Fig. 2 below.

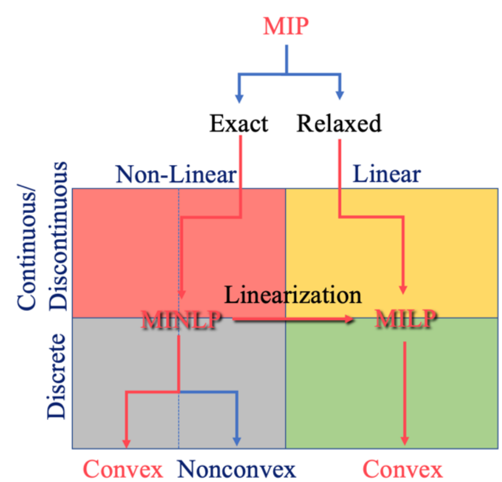

Figure 2. Computational Pathways for Attaining a Convex Paradigm

Let us then take the case of a prototypical Command and Control (C2) architecture (even an advanced one [26]), such as within the energy ecosystem, which typically involves Control Center (CC)-related node data and remote, distributed hyper-locale (specific to the area conditions) node data that need to be effectively fused so as to create actionable quality data [27]. Under exigency circumstances, control may devolve to Back-up CCs. If the exigency is limited, the devolution may only involve one Back-up CC. However, if the exigency is large-scale and widespread, the needs may be varied, and, consequently, multiple Back-up CCs may be involved.

Throughout it all, the interests of the original CC likely remain overarching (if not paramount), thereby necessitating non-zero-sum game theory success (i.e., ideally, all winners and no losers among the involved original CC, regional/area CCs, and Back-up CCs) [27]. However, this is often not the case due to the practicality of limited capacities and capabilities during large-scale and widespread exigencies. For example, some involved areas may have not blackstart (i.e., the ability to restart and recover from a blackout without external reliance) or quickstart (i.e., the ability to come back on-line quickly) capabilities. As has been observed from various Just-In-Time (JIT) case studies, issues with even a single component within the supply chain can have a cascading effect and impact a myriad of organizations [40]; this paradigm can, potentially, lead to a decrease in a country's overall total industrial output. Thus, if the criticality of a particular component is known, and the involved manufacturing resides in an area with no blackstart or quickstart capabilities, then the original CC may prioritize that area; alternatively, the CC may prioritize other areas, as the circumstances and/or involved decision engineering posits dictate. In any case, the follow-on research of [28] in 2020 and 2021 have shown that the involved objective and constraint functions, which include TPU and minCCIS-u-related considerations, are likely to be nonconvex.

As the involved circumstances change with time, the involved MINLP problems will vary. For example, the CSEC associated with minCCIS-u might be considerably higher when the normal CCs are at play than when the Backup-up CCs are at play. Regardless, prototypical approaches to solving these nonconvex MINLP problems involve transforming them into convex surrogates (e.g., via reformulations, convex approximations, or a series of convex relaxations) [25]. It turns out that the particular instantiation of the ERCR utilized, with the bespoke numerical stability paradigm, is well suited for this requisite series of convex relaxations. The utilized ERCR, which was based on [25], could not only resolve the minCCIS-u problem, but it could also leverage the same ERCR mechanisms for tuning its own hyperparameters; the utilized ERCR architectural stack achieved this with three key design/architectural elements: (1) effectuating an ERCR paradigm, via a bespoke Modified Squeezed "You Only Look Once" (YOLO) v3 (a PyTorch implementation, as contrasted to, for example, v4, which is a Darknet implementation) [Deep Convolutional Generative Adversarial Network (DCGAN)] Implementation (MSY3I), (2) utilizing Particle Swarm Optimization (PSO) to tune the MSY3I so as to reduce the associated computational costs, and (3) operationalizing the PSO via an Adaptive Inertial Weighting Mechanism (AIWM) (to mitigate against potential stagnation at local optima) facilitated by a modified GNU Octave platform (m-GNU-O). The particulars of this ERCR architecture are delineated in [25]; the utilized architectural stack and components are presented, the experimental setup of a stable RCR, composed of two MSY3I implementations that are augmented with a third DCGAN is delineated, and a sampling of the numerical issues found in various ML libraries/toolkits is discussed.

III. THEORETICAL APPROACH

The theoretical approach centers upon the issue of the accuracy of the controllability (as the actual controllability is also probabilistic). As discussed, an ERCR framework, as shown in Fig. 3, is utilized, and enhancing the tightness of the ERCR bounds is an ongoing challenge. The PSO and AIWM tuning of the involved MSY3Is is central, as is minimizing the convex relaxation barrier, the inherent gap between the actual and lower bound of robustness provided by verifiers (i.e., verification algorithms for verifying the involved DCGANs, or MSY3Is in this case).

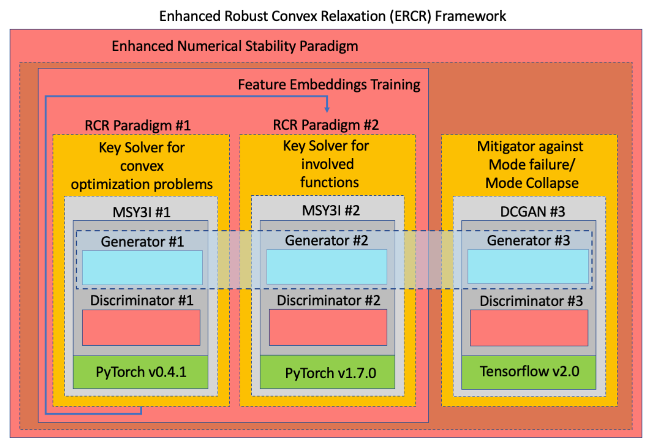

Figure 3. Enhanced Robust Convex Relaxation (ERCR) Architectural Stack Utilized

As contextualizing background information, Machine Learning (ML) is a subfield of Artificial Intelligence (AI). In turn, Deep Learning (DL) is a subfield of ML, and DL Neural Networks (NN) are a mainstay of DL algorithms ("deep" refers to the number of layers of the involved NN). A NN with just a few layers may produce a model that is not quite acceptable for the task at hand. Conversely, a NN that is fully connected, with many layers, dramatically increases the computational complexity and cost. Consequently, the goal is to arrive at a DNN architecture with sufficiently reduced connectivity, and therefore, reduced computational complexity and cost, that is still fit for purpose and, ideally, sufficiently robust. A commonly used DNN, with such reduced connectivity, is a Convolutional Neural Network (CNN). By way of example, Zhu et al. have asserted that CNNs are promising for condition monitoring [49]. Huang et al. have noted that specific implementations of CNN, such as the Multi-Scale Cascade CNN (MC-CNN), can robustly classify faults [50]. Others have noted that CNNs are the architectural elements of choice for Generative Adversarial Networks (GANs). Radford et al. have noted that a Deep Convolutional GAN (DCGAN) can produce robust results that were not present in the training set [51].

The utilized ERCR framework, which is underpinned by DCGANs (or MYS3Is, in this case) was utilized to tackle the Complex Network Analysis (CNA) challenge of optimal controllability of certain Large Complex Networked System (LCNS), particularly a Cyber-Physical Power System (CPPS) within the overarching rubric of Cyber-Physical Systems (CPS). An underlying challenge was to identify control/driver Target Nodes (TN) that are amenable to “Targeted Control” (TC). Other underlying challenges included discerning the topological structure of the LCNS-CPPS, as the structure diffusiveness (a.k.a., permeability) can facilitate or prevent the diffusion of control signals (CS) and/or augmented control signals (ACS), which in turn informs the computations, which leverage a set of control/driver TN_{n-1} to influence yet another set of control/driver TN_n so as to steer the LCNS-CPPS to a target state. Further granularity regarding the topological structure (e.g., directed/ undirected links; weighted/unweighted links) is necessary to address the issues of exact controllability, actual controllability, and accuracy of controllability. The involved sub-challenges include, among others, the Network Controllability Problem (NCP), Minimum Controllability Problem (MCP), Efficient Controllability Problems (ECP), Control Signal Energy Cost (CSEC) problem, minimum Constrained Input Selection (minCIS) problem, minimum Cost Constrained Input Selection (minCCIS) problem, minCCIS amidst uncertainties (minCCIS-u) (e.g., time delays), and adequately addressing the associated Temporal Problems (TPs) with uncertainty (TPUs). Essentially, the aforementioned problems can be recast as constrained convex minimization/optimization problems.

In addition to analyzing its performance as pertains to the convex optimization problems, the involved MSY3Is of the ERCR framework must be examined for robustness. As noted in [25], this often relates to the performance of the layer-wise optimal convex relaxations implemented within the involved DCGAN (also MSY3Is in this case); in essence, a certain convex relaxation is posited for the purpose of ascertaining an upper bound for a worst-case instability scenario. This is of critical import, as prototypical DCGANs exhibit non-graceful degradation in performance even at imperceptible perturbation levels, which results in numerical instability; this is also why the bespoke numerical stability paradigm discussed in [25] is invaluable. For this paper, the numerical stability paradigm employed by the ERCR framework is, potentially, of scientific gain and shows promise in contending with certain round-off errors, thereby better facilitating the transformation of certain uncontrollable cases into controllable cases; moreover, Ohtsuka et al. has noted there is equivalence between the convex relaxation and sparsity constrained controllability problems, wherein the controllability Gramian is used as a metric for the ease of control [52]. In essence, the discernment of the controllability Gramian is directly related to the involved convex relaxation. In particular, the minimum/optimal TN selection (i.e., sparsity constraint) is, in essence, a selection problem, wherein TNs are selected for their efficacy of control while minimizing CSEC. This sparse optimization problem, as applied to a LCNS-CCP controllability maximization problem, has equivalency to its convex relaxation. As a consequence, the ERCR — conjoined with its bespoke numerical stability paradigm — by its very design (i.e., more robust convex relaxations) might, potentially, warrant further examination for its efficacy in treating sparse optimization controllability maximization problems.

IV. EXPERIMENTATION

A. Heuristical Pre-Processing

For the 2020 follow-on work from [28], three regions were examined: A, B, and C. It was found that B had no blackstart and quickstart capabilities. Yet, B contained manufacturing sites producing components that would impact the supply chain affecting A, B, and C. In many ways, B’s criticality surpassed that of A and C, and from a SCV Criticality (SCVC) perspective — for the specific manufacturing analysis at hand — B was, potentially, the most vulnerable. For this case, the aggregate network of A, B, and C, hereinafter $LCNS_{ABC}$, did not have to be treated in its entirety. The heuristical determination was that an examination of the sub-network of B ($LCNS_B$), would suffice. Hence, it was not necessary to compute the CSEC for $LCNS_{ABC}$ ($CSEC_{ABC}$); computing the CSEC for $LCNS_B$ ($CSEC_B$), would suffice. Also, by simply treating $LCNS_B$, the considered time frame could be further constrained (as contrasted by treating the entirety of $LCNS_{ABC}$); hence, the involved TPU component could be reduced and simplified (TPU_B), and accordingly, the involved CSEC could also be reduced and simplified ($CSEC_B$). Moreover, Chen et al. had found that CSEC could be reduced significantly when the addition of input CS could be accomplished while minimizing the path lengths from control/driver nodes to non-control/driver nodes, via optimal placements of the involved nodes [29]; the longest path of the set of involved paths is known as the Longest Control Chain (LCC). As $LCNS_B$ was considered in isolation, as contrasted to considering $LCNS_{ABC}$, it was found that the LCC_B for $LCNS_B \ll LCC_{ABC}$ for $LCNS_{ABC}$; correspondingly, $CSEC_B \ll CSEC_{ABC}$.

B. Algorithmic Pre-Processing

To further minimize $CSEC_B$ and attain $CSEC_{OPT}$, algorithmic processing was used to ascertain the potentially greatest impact $LCNS_{Bn}$ (a sub-region of $LCNS_B$). In this way, LCC_{Bn} for $LCNS_{Bn} \ll LCC_B$ for $LCNS_B$, $CSEC_{Bn} \ll CSEC_B$, and correspondingly, TN_{Bn} for $CSEC_{Bn} \ll TN_B$ for $CSEC_B$. With the same mechanism utilized for [30], selective updating of an optimal Adaptive Impact Vector (AIV_{OPT}) was undertaken for helping derive the potentially greatest impact $LCNS_{Bn}$. In essence, AIV_{Bn} can be derived, via minimizing a recast TN_{Bn} criterion subject to a similarity

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constraint; the AIV can also be validated, and more finely-tuned, via a decomposition-based evolutionary algorithm coupled with the AIV. The associated constrained paradigm can be transformed into a convex optimization problem, via various Semi-Definite Programming (SDP) algorithms, which were implemented on a m-GNU-O as delineated in [30]. Then, a Quadratically Constrained Quadratic Programming (QCQP) Step-Down Algorithm (QCQP-SDA) can compute the [QCQP special class] resultant convex optimization problem in polynomial time; historically, this had been tested in Ilog Cplex Optimizer (a commercial software package for optimization); subsequent testing migrated to AD Model Builder (ADMB) (an open source software package for non-linear statistical modeling) as well as Interior Point OPTimizer (IPOPT) (a software package for large-scale nonlinear optimization) [31], and experimentation has also been conducted with Advanpix (a multi-precision computing toolbox for Matlab). The significance of deriving CSEC$_{Bn}$, and subsequently, TN$_{Bn}$, is to have a sufficiently small TN, such that a particular approach proposed by Klickstein et al., the controllability Gramian of lattice graphs [33], could be practically used for further testing and winnowing to a TN$_{Bno}$ of LCNS$_{Bno}$ (a sub-area of LCNS$_{Bn}$), as graph-related computations can be computationally less prohibitive as contrasted to algebraic computations and is well suited to the task at hand [45]. While certain methods, such as greedy approximation algorithms, which have been proposed by Summers et al. [32] and others, as well as low-rank approximation algorithms, which have been proposed by Benner et al. and others, are of mathematical interest, as noted by Klickstein et al., they do not necessarily provide the requisite discernment into the connections among the optimal distribution of input CS (i.e., CS$_{OPT}$) and the topological properties of the involved LCNS [33]; this discernment is necessary, as it is an important aspect of the assessment process [34]. Ultimately, it provides validation that, by way of example, LCNS$_{Bno}$ has been "properly selected" [6], that CSEC$_{Bno}$ is reasonable, and that TN$_{Bno}$ makes practical sense.

C. *Hybridized Processing*

For the involved experimentation, the full node set of LCNS$_{ABC}$ had been heuristically reduced to LCNS$_{Bno}$, its corresponding CSEC$_{Bno}$, and its corresponding TN$_{Bno}$. Li et al. had previously proposed PGM to iteratively search for the energy optimal placement of CS [2] (i.e., for an optimized CSEC or CSEC$_{OPT}$). Ding et al. proposed a Revisited Projected Gradient Method Extension (R-PGME) for even better performance [4]. Numerous other works have also contributed to deriving CSEC$_{OPT}$. However, generally speaking, the notion of complete control is typically considered, wherein the CS steer the full node set towards the predefined goal state vector. Klickstein et al. have noted that a smaller TN set, such as TN$_{Bno}$, might be all that is needed [4][33] to effectuate the cascading effect of LCNS$_{Bno}$, LCNS$_{Bn}$, LCNS$_{B}$, and LCNS$_{ABC}$ converging to the desired "final state in the prespecified time within a predefined precision" [7], thereby providing a physically controllable case. A TN$_{Bnp}$ accelerant might also serve to assist TN$_{Bno}$ (i.e., TN$_{Bno}$-TN$_{Bnp}$ Amalgam) in effectuating this paradigm, which is depicted in Fig. 4 below. Ideally, the TN$_{Bno}$-TN$_{Bnp}$ Amalgam still remains optimally small (i.e., TN$_{OPT}$). In this case, the involved Gramian matrix is well-behaved (i.e., the condition number or sensitivity of the least squares polynomial approximation and the CSEC are not unrealistically large), which is the desired state [7]. This is contrasted to the case of when the Gramian matrix is ill-conditioned (i.e., the condition number and CSEC are unrealistically large), wherein, the LCNS is unable to reach the "final state in the prespecified time within a predefined precision" [7]. Hence, a suitable approach to addressing the Gramian matrix is critical; after all, some approaches, as noted by Lindmark et al., can only be "computed in closed form … when the time of the transfer tends to infinity" [8] (i.e., actual control will likely never devolve).

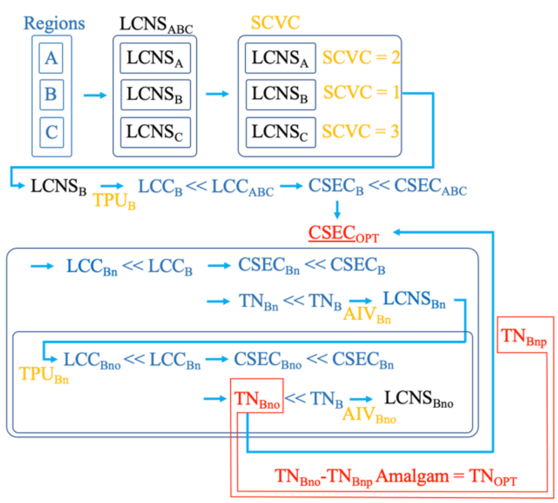

Figure 4. TN$_{Bno}$, CSEC$_{OPT}$, and the Cascading Effect for Convergence to the Desired Final State, while the Amalgam of TN$_{Bno}$ and TN$_{Bnp}$ (TN$_{OPT}$) Still Remains Small

Arriving at a well-behaved controllability Gramian matrix when using CSEC$_{OPT}$ illuminates the value of TN$_{Bnp}$, as an augmentation and accelerant to TN$_{Bno}$, to enhance the likelihood of actual controllability. This TN$_{Bno}$-TN$_{Bnp}$ Amalgam may have an even higher likelihood of actual controllability (i.e., robust controllability) and accurate controllability, particularly in the case of dense and homogeneous networks [35] (as contrasted to sparse and heterogeneous networks) with clustered sub-networks [48]. Moreover, temporal networks seem to be more controllable than their static counterparts, such as when considering link temporality for network controllability; Zhang et al. have noted that link temporality, such as by weight variation, can be equated to "attaching a virtual driver node to that link"

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[43]. Hence, the TN computational approach can also be used for specific optimal Target Links (TL_{OPT}). The TN_{Bno}-TN_{Bnp} Amalgam (a.k.a., TN_{OPT}) can be revised to include TL_{OPT} for a more accurate amalgam descriptor: TN_{Bno}-TN_{Bnp}-TL_{OPT} or TN_{OPT}- TL_{OPT} Amalgam. The TN_{Bno}-TN_{Bnp}-TL_{OPT} Amalgam need not necessarily effectuate an overarching controlling or cascading effect on $LCNS_B$ and/or $LCNS_{ABC}$; if TN_{Bno} can impact a peer TN (e.g., TN_{Bnn}, TN_{Bnm}, TN_{Bnl}, etc.) or other (e.g., TN_{Bn}, TN_{Bm}, TN_{Bl}, etc.) (i.e., one set of control/driver nodes influencing yet another set of control/driver nodes) so as to steer $LCNS_{Bn}$ and/or other pertinent peer LCNS and/or higher-order LCNS to a target state, then the desired state might be achieved.

As noted by Roy et al., central to this task seems to be the principal submatrices of the controllability Gramian [37]. In particular, these Gramian submatrices well inform various metrics and optimal inputs. The LCNS diffusiveness (a.k.a., permeability) for CS and/or augmented CS (collectively, CS_{opt}) can be calculated in a variety of ways [38]; in turn, the permeability can be emblematic of the readiness of the LCNS to be controllable. For the specific cases studied, when the LCNS is uncontrollable, the inverse Gramian does not exist and CSEC approaches infinity [39]; conversely, when the LCNS is controllable, the inverse Gramian does exist. On the basis that a corresponding vanishing moment recovery matrix is a suitable approximation to the inverse Gramian and "guarantees *n* vanishing moments of the irregular framelets" [40], the ERCR framework endeavors to capitalize upon its efficacy for handling wavelet tight frames with *n* vanishing moments; as the number of vanishing moments increases, the polynomial degree of the wavelet increases and the involved underlying graph becomes smoother. The potential advantage of this is that, theoretically, wavelet tight frames can be derived from any multiresolution analysis [47].

Architecturally, to facilitate the requisite discernment into the LCNS diffusiveness, autodiff libraries (e.g., a C++ library that facilitate automatic differentiation of mathematical functions) are utilized by the ERCR framework to enable large-scale tuning of the myriad of parameters utilized, and the specialized workflow is comprised of the following: (1) iterative convolutions with ever smaller filters (wherein the filter depth is smaller than the input layer depth, such that kernel size is less than the channel size), (2) pointwise nonlinearities (which are relationships that are already equivariant to permutations of the input/output indices), and (3) constrained subsampling operations, such that, collectively, the resultant paradigm nicely bears semblance/emulates the wavelets [41]. Overall, the enhanced numerical stability paradigm utilized by the ERCR framework, which is based upon [25], shows promise in contending with select round-off errors, thereby facilitating the transformation of certain uncontrollable cases into controllable cases. For those paradigms, wherein the BTW cascading effect is a potentiality, this facilitation may constitute a deciding factor.

For the experiments described herein, two different ERCR paradigms with different versions of components at the MSY3I level (i.e., ERCR Component #1: MSY3I-1 and ERCR Component #2: MSY3I-2) were augmented with a TensorFlow-based DCGAN implementation. MSY3I-1 was utilized for solving the controllability-related convex optimization problems. As such, it required a high degree of numerical stability; accordingly, PyTorch v0.4.1 was utilized. MSY3I-2 was utilized for solving ERCR-related optimization problems. PyTorch v1.7.0 was utilized, which allowed MSY3I #2 to focus on its intrinsic stability training, so as to mitigate against numerical instability issues from PyTorch v1.7.0 (as contrasted to v0.4.1). A "forward stable" TensorFlow-based DCGAN implementation (i.e., ERCR Component #3: DCGAN) was utilized via an additional generator (hence, a mixture of generators) to assist in mitigating mode failure (a.k.a., mode collapse), which can occur when two competing neural networks that are being trained concurrently fail to converge or have an unusual convergence [25]. In addition, to validate the results for reasonableness, Advanpix was utilized for its multi-precision computation of the eigen-decomposition of the controllability Gramian (W_p), the invertible matrix (U_p), and the matrix M_p, where $M_p = U_p^{-1} W_p U_p^{-1}$ is the p x p symmetric, real, semi-positive definite matrix and has the same set of eigenvectors as W_p (W_p also has the same set of eigenvectors as U_p) [44]. As the Gramian is approximately proportional to the covariance matrix, sample covariance matrix computations were performed for Quality Assurance/Quality Control (QA/QC).

V. Further Heuristical Processing for LCNS Partioning and Potential TN_{BNO} Expansion

The process involved in the derivation of TN_{Bno}, $CSEC_{OPT}$, etc. is invaluable for it gives insight into the notion of network partitioning (i.e., LCNS partitioning) and the potential significance of various involved clusters. Of note, Pasqualetti et al. had proposed a decoupled control strategy that was scalable and amenable to a distributed implementation; central to the strategy was LCNS partitioning into strongly connected components [46]. Restated, interconnection matrices needed to be computed for the various involved clusters. Also of note, works in the area of control theory typically focus on simultaneous control of the clusters. Yet, certain prescient works in the area of SCV have noted the potential potency related to sequential control of the clusters. For example, Zhu et al. have noted that "the sequential attack is demonstrated to be statistically stronger than the simultaneous attack" [45]; along this vein, sequential control is likely to have more efficacy than simultaneous control of the clusters. A final point to note, Liu et al. have noted that "dense and homogeneous networks can be controlled using a few driver nodes" [35], and this sets the stage for the clustered sub-network and TL virtual driver experimentation for a decoupled and sequenced control strategy on a dense and homogeneous temporal LCNS described herein.

Particular attention was paid to those clusters, whose associated CSEC were abnormally low. These might constitute areas of SCV, which might warrant further examination. When honing in on these areas, it might be prudent to review the relative criticality of vulnerability of the prospective control nodes via the Analytical Hierarchy Process (AHP). In particular, Sharma, et al. noted that the following factors might be non-trivial: (1) type of supply chain relationship (e.g., transactional, collaborative), (2) transparency with regards to supply chain-related information (e.g., ambiguity, uncertainty), (3) degree of control over alerting systems [6]. Amaeshi et al. had noted "boundaryless responsibility" and the potential liability associated with the actions of the suppliers' suppliers, and Liao et al. [41] had noted that "firms are building stronger relationships with their supply chain suppliers in order to gain flexibility, efficiency," etc.; the combination of these notions may have enticed larger organizations to migrate from transactional to more collaborative relationships. In some cases, collaborative relationships have led to more ambiguity, and Luthra, et al. have noted that "data vagueness and inaccuracy" … "may affect the results of AHP" [35]. Limited Vulnerability Design (LVD) efforts may also be affected by a skewed AHP.

Accordingly, in the treatment of abnormally low values related to CSEC, the principal submatrices of the Gramian and their inverses were treated. This informed the involved TC metrics and CS_{OPT}, which in turn informed the derivation of CA_{OPT} and the upstream CM. Hence, the overall notional sequence of involved transformations (not necessarily in this computational order), among others, is shown in Fig. 5 below.

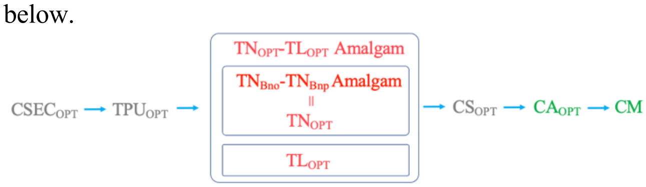

Figure. 5. Notional Sequence of Involved Transformations (Not Necessarily in this Computational Order)

Overall, this paradigm contributes towards informing both the actual as well as the accuracy of controllability.

VI. CONCLUSION

Optimal controllability of certain LCNS involves solving a succession of convex optimization problems. Since further nonconvex problems may be spawned amidst the solving of these convex optimization problems, an ERCR framework is leveraged. The utilized ERCR's bespoke numerical stability paradigm was useful in the facilitation of certain uncontrollable cases into controllable cases, and it was also able to facilitate discerning the involved LCNS's permeability so as to yield the apropos accelerant amalgam for use in the determination of $CSEC_{OPT}$, TPU_{OPT}, TN_{OPT}, TL_{OPT}, among others. Accordingly, in the treatment of abnormally low values related to CSEC, the principal submatrices of the Gramian and their inverses were treated. This helped to inform the involved TC metrics and CS_{OPT}, which in turn informed the derivation of CA_{OPT} and the upstream CM. The involved sequence of transformations contributed to enhancing the actual and accuracy of controllability (i.e., optimal controllability) of the LCNS involved in the preliminary experimentation described in this paper. Also of interest, it turns out that the involved interconnection matrices can well inform a potential TN_{Bno} expansion [46], as the clusters discerned from the LCNS portioning need to be assessed by their efficacy, as sequencing changes; conversely, the involved TN_{Bno} might also be winnowed as each cluster is assessed. Future work will involve more quantitative experimentation in this area.

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