Higher Order Sliding Mode Control of Robot Manipulator

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***Abstract*—This paper presents the development of a nonlinear control strategy for a robot manipulator model, using a robust higher order sliding mode control structure. In the present work, a traditional sliding mode control is presented, the robustness of the controller in the context of stabilization and trajectory tracking, is analytically proved using Lyapunov approach. In order to reduce the chattering in sliding mode controller (SMC) we used the higher order sliding mode control algorithm (Super twisting and Twisting). The simulation results presented in this paper indicate that the suggested approach has considerable advantages compared to the classical sliding mode control.**

Keywords-robot manipulator; higher order sliding mode control; Twisting; Super twisting

I. INTRODUCTION

Variable structure systems with a sliding mode were discussed first in the Soviet literature [1], and have been widely developed in recent years. The sliding mode control (SMC) is a powerful method to control high-order nonlinear dynamic systems operating under uncertainty conditions [2][3]. A SMC law is designed such that the representative points' trajectories of the closed-loop system are attracted to the sliding surface and once on the sliding surface they slide towards the origin. As the sliding surface is hit, the system response is governed by the surface dynamic; consequently, the robustness to the uncertainty or disturbance is achieved. In spite of claimed robustness properties, high frequency oscillations of the state trajectories around the sliding manifold known as chattering phenomenon [2][4] are the major obstacles for the implementation of SMC in a wide range of applications.

Several methods of chattering reduction have been reported [5][6]. One approach [7] places a boundary layer around the switching surface such that the relay control is replaced by a saturation function. Another method higher order SMC [8][9][10], latter approach have been proposed for a Flexible Robot Arm in [11] [12]. The current papers result is based on this latter approach and its main idea can be described as follows:

Let $s(x,t)$ ($x \in \Re^n$ is the state variable, $t \in \Re^+$ the time variable) be the sliding variable and $r \in N$ the sliding order. The control forces to zero in finite time s and its $(r-1)$ first higher time derivatives by acting discontinuously on the rth time derivative of s. Keeping the main advantages of standard SMC, the chattering effect is eliminated and higher order precision is provided. In the case of "real" SMC [9], if τ is the sampling time, the error is $o(\tau)$ in the case of standard SMC [13] whereas it is $o(\tau^r)$ in the rth order SMC [14].

In the case of second order SMC ($r = 2$), many works have given solutions. Several second order sliding mode algorithms are proposed in [9][14][15] [16].

The present paper proposes a multi-input multi-output (MIMO) second order sliding mode strategy, for this purpose we have chosen as an application PUMA 560 robot manipulator with three degrees of freedom model obtained using Lagragian's equations [17]. The proposed controller based on sliding mode control approach.

The paper is arranged as follows: Section 2 introduces a general PUM 560 robot manipulator model. Section 3 presents traditional sliding mode controller design. Section 4 displays the design of the second order SMC (the Twisting and de Super Twisting algorithm). Section 5 presents the simulation results obtained with the full dynamic model. Finally, we present the comparative study to show the effectiveness and feasibility of the proposed control strategy.

II. DYNAMIC MODEL OF ROBOT MANIPULATOR

To control the manipulator arms, we chose the model of industrial PUMA 560 robot manipulator presented in Figure 1. We considered only the first three rotationals joints q_1, q_2 and q_3. The dynamic model [17] is given in simplified matrix form in (1).

$$M(q)\ddot{q} + V_m(q,\dot{q}).\dot{q} + G(q) + u_{m_0} = u \quad (1)$$

with:

$q \in \Re^n$: Vector of joint positions;

$\dot{q} \in \Re^n$: Vector of joint velocities;

$\ddot{q} \in \Re^n$: Vector of joint accelerations;

$u \in \Re^n$: Vector of forces and / or torques of motor;

$$u = [u_1, \ u_2, \ u_3]^T \quad (2)$$

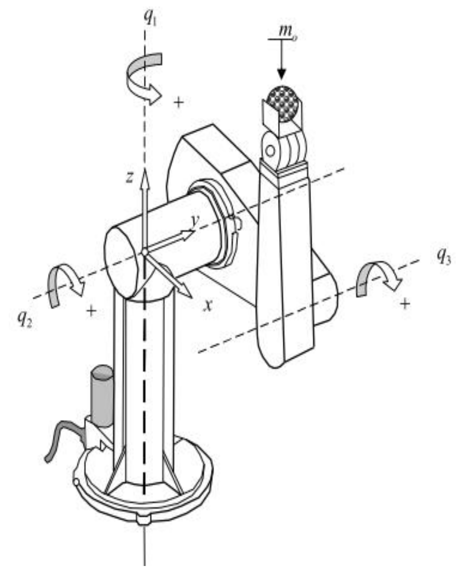

Figure 1. PUMA 560 robot manipulator

$u_{m_o}(t) \in \Re^3$: Vector of torque due to the load m_o.

$$u_{m_o} = m_0 J^T(q)\left[J(q)\ddot{q} + \dot{J}(q,\dot{q})\dot{q} + g\right] \tag{3}$$

where the Jacobian matrix is defined by:

$$J(q) = \begin{bmatrix} -s_1(l_2c_2 + l_3c_{23}) - d_2c_1 & -c_1(l_2s_2 + l_3s_{23}) & -c_1(l_3s_{23}) \\ c_1(l_2c_2 + l_3c_{23}) - d_2s_1 & -s_1(l_2s_2 + l_3s_{23}) & -s_1(l_3s_{23}) \\ 0 & -(l_2c_2 + l_3c_{23}) & -(l_3c_{23}) \end{bmatrix} \tag{4}$$

$\dot{J}(q,\dot{q})$ derived from the Jacobian matrix obtained from the differentiation with respect to time.

$M(q) \in \Re^{n\times n}$: Symmetric positive definite matrix of inertial accelerations;

$$M(q) = \begin{bmatrix} I_1 + I_2c_{23}^2 + I_3c_2^2 + I_4c_2c_{23} & I_5s_{23} + I_6s_2 & I_5s_{23} \\ I_5s_{23} + I_6s_2 & I_7 + I_4c_3 & I_8 + 0.5I_4c_3 \\ I_5s_{23} & I_8 + 0.5I_4c_3 & I_9 \end{bmatrix} \tag{5}$$

$G(q) \in \Re^n$: Vector of forces and / or couples due to gravitational forces;

$$G(q) = \begin{bmatrix} 0 \\ -(m_3l_2 + 0.5m_2l_2)gc_2 - 0.5m_3l_3gc_{23} \\ -0.5m_3l_3gc_{23} \end{bmatrix} \tag{7}$$

$V_m(q,\dot{q}) \in \Re^{n\times n}$: Matrix of forces and / or torques due to centrifugal and Coriolis accelerations;

$$V_m(q,\dot{q}).\dot{q} = \begin{bmatrix} -\left(2(I_3s_2c_2 + I_2s_{23}c_{23}) + I_4(c_2s_{23} + s_2c_{23})\right)\dot{q}_1\dot{q}_2 \\ -\left(2I_2s_{23}c_{23} + I_4c_2s_{23}\right)\dot{q}_1\dot{q}_3 \\ +\left(I_6c_2 + I_5c_{23}\right)\dot{q}_2^2 + (2I_5c_{23})\dot{q}_2\dot{q}_3 + (I_5c_{23})\dot{q}_3^2 \\ \left(I_3c_2s_2 + I_2c_{23}s_{23} + 0.5I_4\left(s_2c_{23} + c_2s_{23}\right)\right)\dot{q}_1^2 \\ -\left(I_4s_3\right)\dot{q}_2\dot{q}_3 - \left(0.5I_4s_3\right)\dot{q}_3^2 \\ \left(I_2s_{23}c_{23} + 0.5I_4c_2s_{23}\right)\dot{q}_1^2 + \left(0.5I_4s_3\right)\dot{q}_2^2 \end{bmatrix} \tag{6}$$

with the following notations:

$$\begin{cases} c_i = \cos(q_i), \quad c_{ij} = \cos(q_i + q_j), \\ s_i = \sin(q_i), \quad s_{ij} = \sin(q_i + q_j), \end{cases} \tag{8}$$

1) *Property 1:* The matrix $M(q)$ is symmetric, positive definite and bounded, and its inverse is existing and also bounded. The matrix verifies the following equality $\dot{M}(q) - 2V_m(q,\dot{q})$ for none a zero vector X :

$$X^T\left[\dot{M}(q) - 2V_m(q,\dot{q})\right]X = 0 \tag{9}$$

III. Sliding Mode Controller Design

This section focuses on the design of a sliding mode control for the stabilization of nonlinear systems; we will apply it on a highly nonlinear system which is the PUMA 560 robot manipulator. This control must meet the specifications defining the objectives, including stability, speed, accuracy and robustness. The simulations are performed in the case of trajectory tracking, and we passed the drop test load as the test of robustness. The PUMA 560 robot model without taking into account the effect of the load is as follows:

$$M\ddot{q} + V_m\dot{q} + G = u \tag{10}$$

This model describes the dynamics of a robot manipulator with three degrees of freedom, which requires the synthesis of three controls, and as each joint is considered as a subsystem whose relative degree is $r_i = 2$, which means that each surface s_i is of order $r_i - 1$. And the error e is defined by: $e = q - q_d$, with $e_i = q_i - q_{id}$

Therefore, the sliding surface is chosen, $s = [s_1, s_2, s_3]$, such as:

$$s_i = \dot{e}_i + \lambda_i e_i \tag{11}$$

we can write, $s = \dot{q} - (\dot{q}_d - \lambda e)$

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where,

$$s = \dot{q} - \dot{q}_r \tag{12}$$

with, $\dot{q}_r = \left(\dot{q}_d - \lambda e\right)$ is considered a reference for the joint velocity.

From (10) and (12), we can set:

$$\begin{aligned} M \cdot \dot{s} &= u - V_m \cdot \dot{q} - G - M \cdot \ddot{q}_r \\ M \cdot \dot{s} &= u - V_m \cdot \left(s + \dot{q}_r\right) - G - M \cdot \ddot{q}_r \\ M \cdot \dot{s} &= u - M \cdot \ddot{q}_r - V_m \cdot s - V_m \cdot \dot{q}_r - G \end{aligned} \tag{13}$$

A. Proposition 1

We define the Lyapunov function as follows:

$$V = \frac{1}{2} s^T M s \tag{14}$$

From property (1), the matrix M is positive definite, and we also $V > 0$ for $s \neq 0$. So, according to the study of Lyapunov stability, the system is stable when, $\dot{V} < 0$ with $\dot{V}$ is the time derivative of V such that:

$$\begin{cases} \dot{V} = s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s \\ \quad = s^T \left[u - M\ddot{q}_r - V_m s - V_m \dot{q}_r - G\right] + \frac{1}{2} s^T \dot{M} s \end{cases} \tag{15}$$

and from (9), we have $s^T \left[\dot{M} - 2V_m\right] s = 0$.

This implies that:

$$\dot{V} = s^T \left[u - M\ddot{q}_r - V_m \dot{q}_r - G\right] \tag{16}$$

B. Proposition 2

The control signal u, be given by

$$u = u_{eq} - K^T sign\left(s\right) \tag{17}$$

and

$$u_{eq} = M\ddot{q}_r + V_m \dot{q}_r + G \tag{18}$$

with the gains of switching $K = \left[K_1, \quad K_2, \quad K_2\right]^T$, $K_i > 0$.

C. Proof 1

Thus, with this choice of the control u from (17), we obtain:

$$\dot{V} = -s^T K^T sign\left(s\right) = -\left|s\right| K^T < 0 \tag{19}$$

Therefore, with the control (17) the system is stable in closed loop (the equilibrium point $e_i = 0$, with $i = 1,3$, is asymptotically stable).

IV. 2-Sliding Mode Controller Design

The problem of 2-sliding mode control is to constrain the trajectories of the system to evolve on the sliding manifold in a finite time [15]:

$$s_2 = \left\{x \in \mathrm{O} : s(t,x) = \dot{s}(t,x) = 0\right\} \tag{20}$$

Let us write again the dynamic model of PUMA 560 robot manipulator as follows:

$$M\ddot{q} + V_m . \dot{q} + G = u \tag{21}$$

For each joint, the constraint is chosen linear function:

$$s_i = \dot{e}_i + \lambda_i \, e_i, \ \ \lambda_i > 0, \ \ i = 1:3 \tag{22}$$

The equivalent control is given by:

$$u_{eq} = M\ddot{q}_r + V_m \dot{q}_r + G \tag{23}$$

For each subsystem, the effective control u is composed of two terms: the equivalent control u_{eq} and the discontinuous control.

To calculate the latter, we use the algorithms of 2-sliding mode control. When we take local coordinates $\left[y_1 \quad y_2\right]^T = \left[s \quad \dot{s}\right]^T$, the problem of 2-sliding mode for each subsystem (joint) is reduced to the stabilization in finite time of the auxiliary system of order tow below:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_1 = \chi(t,x) + \varsigma(t,x)\nu \end{cases} \tag{24}$$

where ν represents the derivative of the control by respect to time $\nu = \dot{u}$; in this case, the control u is considered a state variable.

$$\begin{cases} \chi(x,t) = \frac{\partial}{\partial t}\dot{s}(t,x,u) + \frac{\partial}{\partial x}\dot{s}(t,x,u)\left\{f(x) + g(x)u\right\} \\ \varsigma(t,x) = \frac{\partial}{\partial u}\dot{s}(t,x,u) \end{cases} \tag{25}$$

$0 < K_m \leq \left|\frac{\partial \ddot{s}}{\partial u}\right| \leq K_M$, with the necessary condition of existence of the equivalent control in sliding mode $\frac{\partial \ddot{s}}{\partial u} \neq 0$ and $\left|\frac{\partial}{\partial t}\dot{s}(t,x,u) + \frac{\partial}{\partial x}\dot{s}(t,x,u)\left\{f(x) + g(x)u\right\}\right| \leq \psi$.

Two 2-sliding algorithms (the twisting and Super Twisting) are applied to demonstrate their ability to stabilize the system, reduce chattering and improve accuracy in a problem tracking.

A. Twisting Algorithm Controller Design

The control law for a system of relative degree $r=1$ is as follows in (26) [15]:

$$v=\dot{u}=\begin{cases}-u & if \quad |u|>u_M \\ -a_m sign(y_1) & if \quad y_1 y_2 \le 0 \quad |u|\le u_M \\ -a_M sign(y_1) & if \quad y_1 y_2 > 0 \quad |u|\le u_M\end{cases} \quad (26)$$

with sufficient conditions for finite time convergence are:

$$a_M > 4\frac{K_m}{\varepsilon_0}, a_m > \frac{\Psi}{K_m}, K_m a_M - \Psi > K_M a_m + \Psi \quad (27)$$

The effective control is as follows: $u=u_{eq}+\int \dot{u}dt$, we note $u_{TW}=\int \dot{u}dt$.

In practice for an real sliding, instead of y_2, determination of the sign of $y_1 y_2$ is made by the first difference of y_1 as:

$$\Delta s=\begin{cases}0 & for \quad K=0 \\ (y_1(K\tau)-y_1((K-1)\tau)) & for \quad K\ge 1\end{cases} \quad (28)$$

where τ is the sampling period.

B. Super Twisting Algorithm Controller Design

In this section, we will apply the super Twisting algorithm [15] to stabilize the robot manipulator. The effective control u for this algorithm consists of two terms: u_{eq} and u_{ST}, $u(t)=u_{eq}(t)+u_{ST}(t)$ with $u_{ST}(t)=\int \dot{u}_1(t)dt+u_2(t)$.

and

$$\dot{u}_1(t)=\begin{cases}-u & if \quad |u|>u_M \\ -Wsign(y_1) & if \quad |u|\le u_M\end{cases} \quad (29)$$

$$u_2(t)=\begin{cases}-\lambda\varepsilon_0^{\rho} sign(y_1) & if \quad y_1>\varepsilon_0 \\ -\lambda|y_1|^{\rho} sign(y_1) & if \quad y_1\le\varepsilon_0\end{cases} \quad (30)$$

In this case, sufficient conditions for convergence are:

$$W>\frac{\psi}{K_m}, \lambda^2 \ge 4\psi\frac{K_M}{K_m^2}\left(\frac{W+\psi}{W-\psi}\right), 0<\rho\le 0.5 \quad (31)$$

V. SIMULATION RESULTS

In this section, simulations are presented to illustrate the performance and robustness of proposed control law when applied to PUMA 560 robot manipulator. The parameters values used for the dynamic model are as follows [17],

$$M(q)\ddot{q}+V_m(q,\dot{q}).\dot{q}+G(q)+u_{m_O}=u \quad (32)$$

- Mass of various links

$m_2=17.40\,kg$, $m_3=5.04\,kg$, $m_4=0.82\,kg$, $m_5=0.35\,kg$, $m_6=0.09\,kg$, $m_t=m_4+m_5+m_6=1.26\,kg$.

- Geometrical parameters

$d_2=149.09\,mm$, $l_2=431.8\,mm$, $l_3=433.07\,mm$.

with $g=[0 \quad 0 \quad 9.8]^T$

To excite the any dynamics of robot there is a cycloidal trajectory test (33), where the different joints move respectively of the position $\{-50^\circ,-135^\circ,135^\circ\}$ to the position $\{45^\circ,-85^\circ,30^\circ\}$, in a time of movement equal to 1.5 sec.

$$q_{di}(t)=\begin{cases}q_{di}(0)+\frac{D_i}{2\pi}\left[2\pi\frac{t}{t_f}-\sin\left(2\pi\frac{t}{t_f}\right)\right] & for \quad 0\le t\le t_f \\ q_{di}(t_f) & for \quad t_f<t\end{cases} \quad (33)$$

with $D_i=q_{di}(t_f)-q_{di}(0)$: Displacement, and t_f the final instant of the movement.

The parameters of the simulations are,

- For the sliding surfaces $\lambda_j=5$, $j=1,2,3$
- For a SMC control, the control gains selected are $K^T=[50,50,50]$
- For the Twisting algorithm are: $a_m^T=[0.1, 0.1, 0.1]$, $a_M^T=[550, 550, 550]$, $u_M^T=[20, 20, 20]$.
- For the Super Twisting algorithm are: $\varepsilon_0=0.1$, $\rho=0.5$, $u_M^T=[20, 20, 20]$, $\lambda^T=[250, 250, 250]$, $W^T=[0.025, 0.025, 0.025]$.

with the sampling period $T=0.001$sec, and the simulation time $t=2$sec. To test the robustness of the control laws proposed, we chose the falling load. In this, the robot is a tracking trajectory with a load $(m_0\,kg)$ and if the maximum speed is pending, the load drops $(m_0=0 \text{ à } t=0.8)$.

VI. COMPARATIVE STUDY

The results of using conventional sliding mode controller are shown in Figure 2 and the results obtained when the 2-sliding controller has been used are shown in Figure 3 for Twisting algorithm, Figure 4 for Super Twisting algorithm.

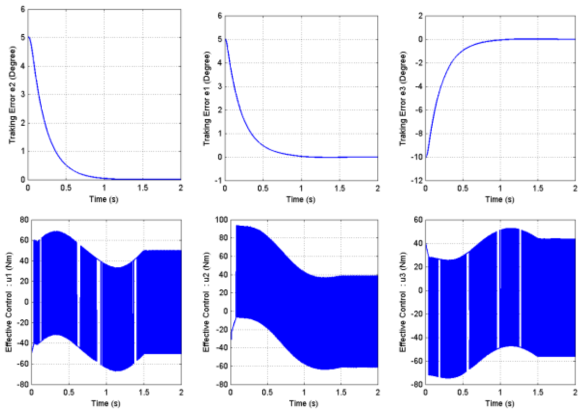

Figure 2. Simulation results of first-order SMC.

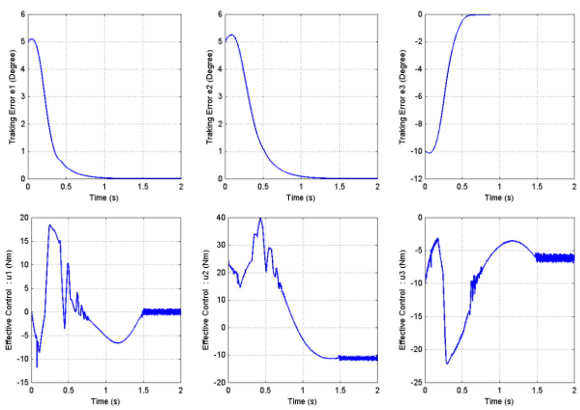

Figure 3. Simulation results of second order SMC: Twisting algorithm.

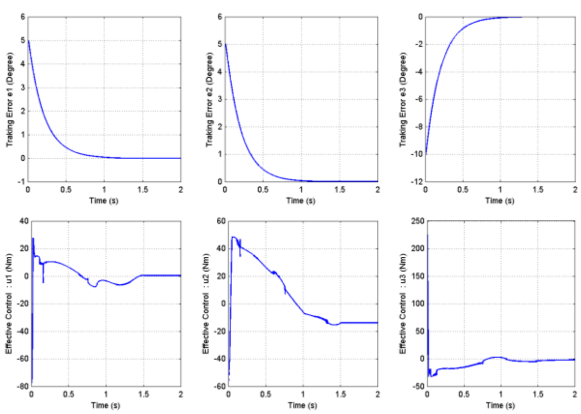

Figure 4. Simulation results of second order SMC: Super Twisting algorithm.

From the simulations results, we can find that the control result of conventional sliding mode controller produces a serious chattering phenomenon, Figure 2.

On the other hand, the chattering phenomenon of the controlled system is suppressed in the case of Super Twisting algorithm controller (Figure 4). Moreover, the proposed controller is a robust controller since the load drops hasn't influence on the control performances.

To compare the performance of 2-sliding controller with SMC, we define tow cost functions J_1 and J_2, such as:

$$J_1 = \frac{1}{2}.\sum_{k=1}^{P}(u^T.u) \tag{34}$$

$$J_2 = \frac{1}{2}.\sum_{k=1}^{P}(e^T.e) \tag{35}$$

The simulation results for each performance index are given in Table I:

TABLE I. COMPARATIVE STUDY

Controller	*Cost function* J_1	*Cost function* J_2
SMC	$6.6686\ 10^6$	2.6692
Twisting algorithm	$3.9922\ 10^5$	28.0488
Super Twisting algorithm	$2.769\ 10^5$	2.8437

Comparing the simulation results, it can be said that the proposed control strategy, second order SMC (Super Twisting algorithm), gave better performance than using the conventional sliding mode controller.

VII. CONCLUSION AND FUTURE WORK

In this paper, a second order SMC is proposed for a PUMA 560 robot manipulator system and simulation results are presented. Firstly, the classical sliding mode control of PUMA 560 robot manipulator system is developed. Secondly, the second order SMC control is used to smooth the discontinuous control term in order to alleviate the chattering phenomenon. The simulation results presented in this paper indicate that the suggested approach has considerable advantages compared to the classical sliding mode control. As future works, we would like making an experimental study of the control approach on a real robot manipulator.

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