Experiments with NetLogo for Distributed Channel Assignment in Dense WLAN Networks

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***Abstract*—Dense IEEE 802.11 networks require thorough radio network planning to minimize the detrimental effects of channel interference and achieve good performance. When multiple IEEE 802.11 networks operating under different administrations reside in the same geographical area (as defined by the bounds of radio signal transmission and reception), the problem of efficient radio resource allocation arises. Particularly, in highly populated urban environments, efficient radio resource allocation can be quite a challenge, due to the finite number of interference-free channels available in the IEEE 802.11 family of standards. To address this shortcoming, we have investigated how the radio resource management capabilities provided by the IEEE 802.11k standard can support a distributed approach in channel assignment. In this paper, we present our family of distributed channel assignment algorithms and elaborate on their pros and cons. Furthermore, we present our simulation environment for simulating distributed channel assignment scenarios based on the NetLogo agent modeling environment.**

***Index Terms*—Channel assignment; Channel Allocation; WLAN; Distributed Systems; Simulation; NetLogo**

I. INTRODUCTION

The establishment of wireless communication as a technological commodity in modern society and the insatiable demand for Internet connectivity paved the way to the tremendous popularity and commercial success of the IEEE 802.11 family of standards. With IEEE 802.11b/g as their flagship and—currently—most widespread standard, IEEE 802.11 Wireless LAN (WLAN) type systems feature an ever increasing footprint worldwide. Standardisation work is feverish and the IEEE 802.11 working groups regularly publishes improvements to existing standards and introduces additional features to address emerging use cases.

To address the complexity associated with the management of large scale deployments in the Future Internet (FI), the vision of self-managing systems has been proposed and adopted. Under the umbrella self-management concept, self-awareness, self-configuration, self-protection, self-healing and self-optimization stand as prominent properties. The fundamental premise of the self-management paradigm is to constrain the operational expenses resulting by the current network management practices in the Internet. To this end, three thematic areas have converged [1] under the umbrella of the Future Internet architecture: autonomic computing, cognitive networking and self-organization.

A. Radio interference

In a wireless infrastructure, network management and, in particular radio network planning, becomes a challenging task due to the volatile and unpredictable nature of the wireless medium and the mobility patterns of terminal devices. The increasing footprint and diversification of IEEE 802.11 type components in ICT equipment suggests that a large portion of FI devices will employ one or more types of wireless access technology. Due to the lack of coordination that characterizes applications in the mass consumer market segments, static and centralized solutions to wireless network planning prove inefficient in this chaotic environment [2].

Minimizing radio interference is essential to realizing good system performance in IEEE 802.11 networks due to the finite number of interference-free channels available [3]. However, the volatile nature of the wireless medium, combined with varying patterns of traffic demand, render any radio network planning exercise a challenging task. The latter is in iteself a sophisticated task, which, besides expert knowledge, may also require a solid understanding of the propagation profile at each radio site [4]. In cases where IEEE 802.11 networks operating under different administrations reside in the same geographical area (as defined by the bounds of radio signal transmission and reception, efficient radio resource allocation becomes quickly problematic. This is due to available technology solutions for radio resource management of IEEE 802.11 installations supporting only a centralized model of administration (e.g., Cisco Unified Wireless Access [5], HP MultiService Mobility Controller [6]). In addition, they lack support for peer-to-peer communication between IEEE 802.11 access points in regard to radio resource management procedures. Thereupon, as IEEE 802.11 systems continue to increase their footprint in the residential and enterprise market segments, efficient radio resource allocation in a dense urban environment rapidly devolves to a chaotic situation [2].

B. Channel assignment

Channel assignment in IEEE 802.11 systems may result in a conflict where more than one adjacent (in terms of radio coverage) wireless access points use the same channel, thus causing a substantial drop in performance. In addition, as Fig. 2 illustrates for the case of IEEE 802.11b/g/n, adjacent access points may use different channels but still experience a substantial spectrum overlap [3], thus still suffering from interference and degraded performance. Performance can also be degraded by the so-called hidden terminal problem whereby the transmission of interfering access points are beyond each other's reception range but within the reception range of client devices, thus limiting throughput (e.g., client device D_1 and access points AP_1 and AP_2 in Fig. 1).

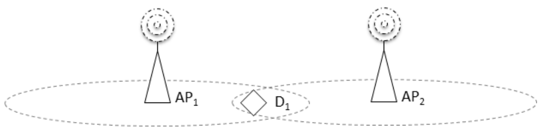

Fig. 1. The hidden terminal case in an IEEE 802.11 access point network.

The main set of IEEE 802.11 standards defines the basic capacities of the Medium Access Control (MAC) layer that enable wireless data exchange and mechanisms [3]. Radio resource management concerns (e.g., measurements of the radio channel) are addressed by the IEEE 802.11k standard [7]. The latter defines the exchange of radio resource measurement information to facilitate network management tasks. To this end, it provides measurements related to the physical and the (wireless) link layer of the protocol stack [7]. In regard to channel assignment, IEEE 802.11k defines channel load measurements as well as noise and time histograms.

C. Research motivation

By making detailed measurements of the radio environment avaialble, several key optimizations in terms of performance become feasible. Particularly deployments in harsh radio environments (e.g., aeroplanes, factories, municipalities, etc.) are greatly facilitated. Utilizing such measurements in a distributed manner whereby each access point operates autonomously in deciding which channel to assign presents a particularly promising proposal. The challenges entailed in distributed approaches include the engineering of convergence and stability properties to the distributed algorithm that is collectively effected by the access points. In prior work we studied the convergence properties between different distributed algorithms for channel assignment in dense IEEE 802.11k systems. In continuation of that work, we explore the parameter space of these algorithms using the NetLogo environment for multi-agent simulation.

We maintain focus on dense deployments of independently managed IEEE 802.11k systems in an urban environment where central coordination of radio resource management tasks is not a realistic assumption. Independent administrations may be insufficiently skilled, unable or even unwilling to coordinate their radio resource management strategies, assuming they have any. Hence, dynamic distributed resource management strategies without operator involvement are the most realistic approach to optimize the use of common radio resources in this chaotic environment. In developing our discussion further, we focus exclusively on IEEE 802.11 systems operating in infrastructure mode.

The rest of the paper is organized as follows: Section II presents the network model of our studies and outlines related work. Section III introduces the NetLogo agent modeling enrivonment and presents its salient features. Section IV presents our approach of modeling in NetLogo dense IEEE 802.11 networks operating under an autonomic setting. Section V presents the family of distributed channel assignment algorithms we have integrated in our NetLogo model. Section VI presents the experimentation settings and shares early results. Finally, Section VII concludes the paper and sets directions for future work.

II. System model

A. Related work

In [8], the authors take into account the traffic load at the MAC layer and propose a heuristic method to minimize the maximum effective channel utilization at the access point with the highest load. In [9] each access point monitors the number of active stations and tries to minimize the maximum effective channel utilization. We note that the set of active stations includes stations associated with a particular access point as well as stations using the same channel and whose transmission power levels is above the access point's receiver sensitivity threshold. The work in [10] proposes a new channel assignment mechanism for infrastructure-based IEEE 802.11 wireless networks in decentralized scenarios. The proposed mechanism operates at the access point level to select the operating channel automatically based on client station measurements exchanged through the IEEE 802.11k standard [3]. In [11], the authors evaluate the throughput performance of different channel assignment strategies in an experimental trial involving IEEE 802.11b systems while [12] proposes a set of algorithms to simultaneously solve the channel selection and user association problems in a fully distributed manner.

Another approach [13], [14] introduces a fully decentralised stochastic algorithm for graph colouring and applies it to the isomorphic channel assignment problem in IEEE 802.11 WLAN systems. Their algorithm does not require communication among WLAN access points and employs learning to ultimately converge to a conflict-free channel assignment. Given the widespread deployment of WLAN technology in urban areas and the lack of coordination among WLAN administrations with regard to channel assignment, we find this approach particularly interesting. Hence, we adopt it for our studies and extend it to accommodate the existence of communication among WLAN access points for purposes of radio resource management tasks. An obvious question is whether the performance of the learning algorithm can be further improved by allowing for the exchange of radio resource information among adjacent WLAN access points and what kind of information can support such an improvement.

B. Working assumptions

The IEEE 802.11 standards define a finite set of wireless channel resources and the respective access and usage controlling mechanisms [3]. For instance, IEEE 802.11b/g specifies the details of 14 distinct channels, with minor differences between world regions due to regulatory constraints. As Fig.

2 depicts, only channels 1, 6 and 11 are interference-free to each other and some amount of spectrum overlap occurs in the remaining subset of the IEEE 802.11 channels.

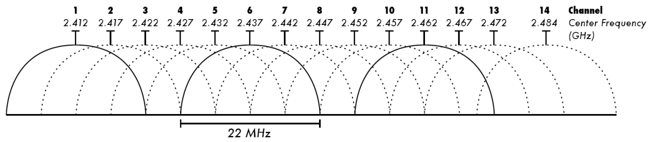

Fig. 2. IEEE 802.11b/g channel overlap (2.4 GHz band) [3].

The extent to which interference attributed to partially overlapping channels in IEEE 802.11 type systems is a determinant of system performance remains a matter of debate [15]–[17]. Analytical and simulation studies [15] suggest that the assignment of partially overlapping channels to neighbouring wireless access points can be justified, depending on the relative utilization of each channel. Increasing path loss decreases the level of interference, therefore, the actual physical distance among wireless stations is an important performance factor [17]. When multiple types of IEEE 802.11 systems coexist in the same geographical area, their topological distribution will largely affect overall performance, with proximal transceivers experiencing dramatic degradation in their channel throughput [16]. Nonetheless, it remains that a finite set of non-overlapping channels is available to wireless devices, a fact aligned to our design assumptions.

C. Channel assignment as graph colouring

Channel assignment in IEEE 802.11 type systems is equivalent to the graph colouring problem [18], a well-known NP-hard problem in graph theory [19]. For a simple graph $G = [V, E]$, colouring is about determining the number ψ of colours sufficient for assigning a color to each vertex $v \in V$ without any two adjacent vertices being assigned the same colour. Several algorithms based on contraction and sequential ordering have been developed to attack the graph colouring problem, including Largest First (LF), Recursive-Largest-First (RLF), Backtracking Sequential Coloring (BSC) and Degree of Saturation (DSATUR) [20].

The channel assignment problem is isomorphically mapped to the graph colouring problem as follows:

- The set of access points $U = \{u_1, u_2, \ldots, u_N\}$ is mapped to the set of vertices V in the graph.
- The pairs of access points $\{(u, v) : u, v \in U\}$ within transmission and reception range of each other is mapped to the set of edges E.
- The set of available channels $C = \{c_1, c_2, \ldots, c_D\}$ is mapped to the set of available colours.

We note that, depending on whether cross-channel interference is assumed to be substantial or not, the set of channels available for assignment by an access point may or may not include those with partially overlapping spectrum bands. The following section discusses prior work on these issues.

III. THE NETLOGO AGENT MODELING ENVIRONMENT

A. Principles

NetLogo is an integrated modeling environment for developing multi-agent simulations in the synonymous programming language. It facilitates rapid development of simulation models with a focus on emergent behavior in systems with a large number of agents (e.g., predator-prey relationships in an ecosystem). As such, it has been used in developing models in a variety of domains, such as economics, biology, physics, chemistry, psychology and system dynamics, to name a few.

NetLogo models consist of independently operating agents in a shared finite environment. The environment is termed world and is composed as a grid of square patches. Thus it provides a two dimensional model that define the space coordinates within which agents will exhibit their behavior and interact with each other. Within this space, distance is quantified, but not qualified.

In NetLogo the evolution of time is modeled through a so-called ticks counter, which is typically incremented through integer arithmetic, though floating point increments are also possible. Thus modeling of time supports both discrete time and continuous time models, albeit the latter would require additional development effort to complete.

NetLogo also supports extensions, with several extensions being already contributed and leveraged by the research community (e.g., associative arrays, network graphs, an interface to the R projecct for statistical computing, etc.).

B. Agents

An agent in NetLogo may comprise state information (i.e., state variables and their values). The set of agents that comprise the same set of state variables form a so-called breed. In manifesting its behaviour, an agent may access the values of its own state variables, as well as the values of any global variables defined in the environment. NetLogo offers powerful filtering and selection mechanisms that operate on agent sets (i.e., breeds). These include generic primitives that allow an agent to select the set of agents (possibly out of a breed) with particular values in their state variables.

IV. AUTONOMIC IEEE 802.11 NETWORKS IN NETLOGO

A. Modeling wireless communication

Aligned to the autonomic networking premise, IEEE 802.11 access points map to autonomous agents that manifest their behavior driven by their individual policies. To model system properties resulting from the wireless nature of IEEE 802.11 networks, we employed directed links between agents. Given that the propagation profile of the radio signal varies in space and time, we understand that topological proximity will be determined individually for each pair of access points. For any given pair of access points (u, v) in the IEEE 802.11 network, reception by u of a radio transmission by v will depend on the the propagation profile $R_{(u,v)}$ of the radio signal paths followed between v and u, the power P_v employed at the transmitter at v, and the sensitivity S_u of

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the receiver employed at u. Hence, for any given pair of access points (u, v), the capacity to exchange information (i.e., to receive a transmission) is determined directionally. Consequently, for any access point, being able to receiver another access point's transmission does not necessarily imply the reverse proposition. The typical disk graph model results by relaxing the assumptions of this propagation model to the same transmission power and receiver sensitivity for all access points.

In the NetLogo environment, for any given pair of access points (u, v), we model the capacity to communicate as a link $e = (u, v)$ between the respective agents. NetLogo provides directed and undirected links between agents, thus supporting both approaches in modeling radio propagation. To control the complexity of the simulation model at this stage of our work, we opted for the disk graph model and used undirectional links between agents. This is supported elegantly by NetLogo through its builtin *in-radius* filtering primitive that selects all agents whose center coordinates are within a given distance of the center coordinates of the invoking agent in the world model. Fig. 3 illustrates a screenshot of the NetLogo model we developed for the distributed channel assignment algorithms, including simulation parameters and monitors of state information.

B. Modeling autonomous behavior

In our NetLogo model we leverage each agent's state variables (as defined in Table I) to drive the autonomous behavior of each access point. The set of available channels and the channel in use are common resources management variables included in the Simple Network Management Protocol (SNMP) Management Information Base (MIB) definitions for an IEEE 802.11 access point. The set of neighboring access points can be built and maintained by an access point that is periodically surveying its known channels for foreign Beacon management frames, a capacity that is supported by the IEEE 802.11 standard. The remaining variables in Table I are not standardized and would have to be included in the state information of an autonomous access point.

Within the NetLogo environment's control flow, the bevahiour of each access point is invoked in its respective state scope. When addressing a group of access point with a particular simulation command (e.g., to scan the radio channel and identify neighboring access points), NetLogo randomly iterates over each access point in the group. This simplifies model development by omitting timing concerns from its design and aligns to the real world situation where independently operating access points do not comply to a global timing sequence related to state changes. For instance, an IEEE 802.11 access point is able to iterate over its known channels and observe foreign Beacon management frames, but the timing sequence of these observations (e.g., the order in which known channels are observed, the amount of time spent in observing each channel, etc.) may differ from those of other IEEE 802.11 access points, even if those are geographically collocated.

TABLE I
STATE VARIABLES OF EACH ACCESS POINT.

Variable	Description
Channels	The number M of channels available for assignment to this access point.
Channel	The channel c currently assigned to this access point.
Distribution	The distribution $P = \{p_1, p_2, \ldots, p_D\}$ defining the probability of each channel to be assigned to this access point by a random selection process.
Neighbor	The set access points $N = \{u_{w_1}, w_{a_2}, \ldots, w_{a_{n(u)}}\}$ that are neighbors to this access point u (according to the disk graph model).
Conflicts	The set of access points $W = \{w_{b_1}, w_{b_2}, \ldots, w_{b_{c(u)}}\}$ that are assigned a conflicting channel to the channel assigned to this access point u.
Penalty	The percentage $b \in [0, 1]$ by which the assignment probability of an assigned channel is discounted when existence of a channel conflicts is determined.
Stability	A boolean variable q set to true when this access point is locked on to the currently assigned channel and false otherwise. For convenience, an access point locked onto its assigned channel is termed a stable access point.

Some of the channel assignment algorithms addressed here assume the capacity to exchange state information between IEEE 802.11 access points in a peer-to-peer manner, subject to sufficient radio coverage of course. The format of the Beacon Request and Radio Measurement Response management frames defined in the IEEE 802.11 specification provides for the definition of extension fields, thus supporting this assumption [3]. In this regard, the IEEE 802.11k standard provides a more efficient and versatile level of support by explicitly supporting peer-to-peer communications through the Neighbor Request and Neighbor Response management frames [7]. Hence, the assumptions underpinning the NetLogo environment regarding peer-to-peer communication between access points are properly met by the IEEE 802.11 standard and its extensions.

V. DISTRIBUTED CHANNEL ASSIGNMENT ALGORITHMS

Herein, we detail each distributed channel assignment algorithm under study, in order of increasing complexity.

Random Walk—The Random Walk (RW) algorithm provides the crudest variant of distributed channel assignment. In each iteration, the algorithm checks if any of the neighboring access points is assinged a channel that interferes with the channel assigned to its hosting access point. If such a so-called conflicting access point is found, the algorithm probabilistically selects and assigns a channel anew; otherwise, it takes no action.

Random Walk with Stickiness—The Random Walk with Stickiness (RWS) algorithm differs from the RW algorithm in that once an access point finds its currently assigned channel to be conflict-free, it locks on to that channel. This is realized by

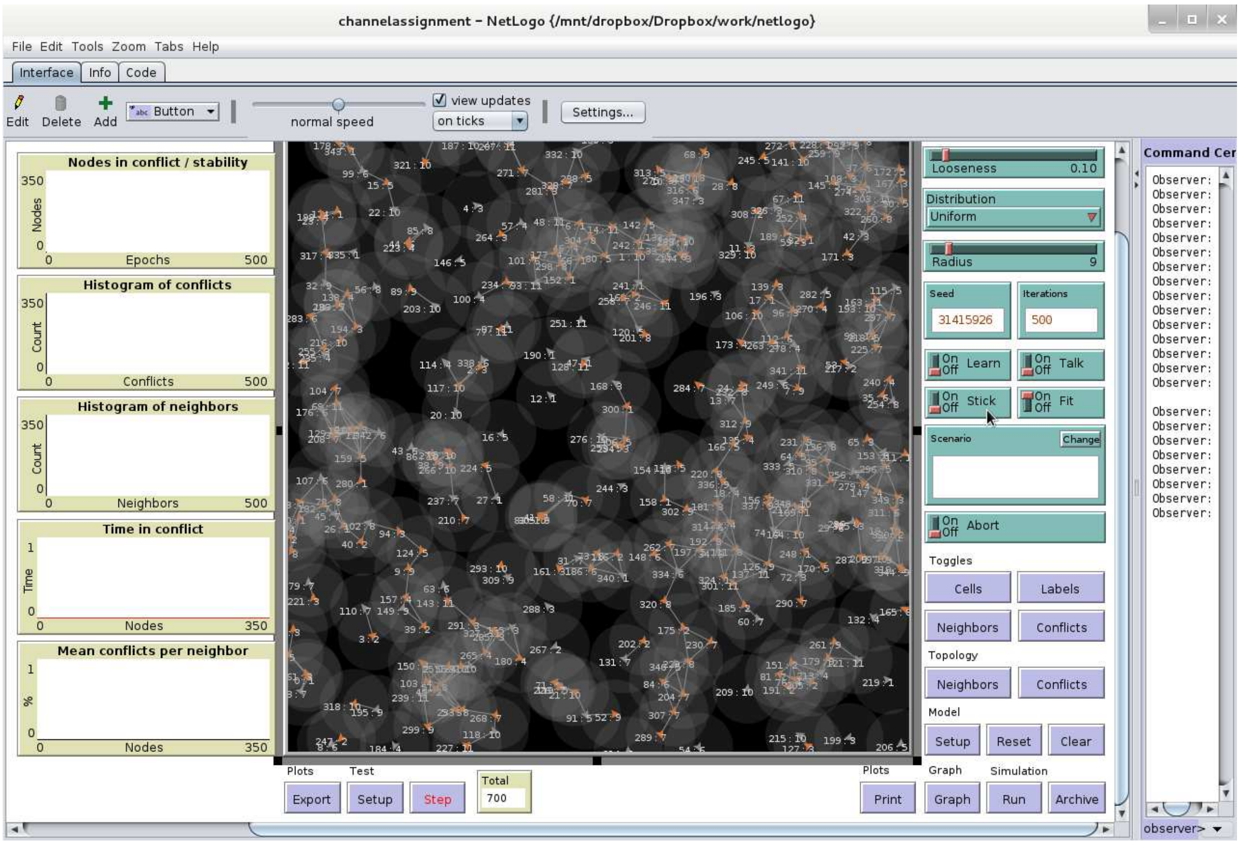

Fig. 3. Settings of the NetLogo modeling environment.

setting the assignment probability for that particular channel to one and for all other channels to zero.

Commumication Free Learning—The Commumication Free Learning (CFL) algorithm [13], [14] provides the basic learning-capable algorithm that uses past observations regarding each channel in adapting its probability of being assinged in the future. More specifically, whenever the channel currently assigned to the hosting access point is found in conflict to the channel assigned by a neighboring access point, its assignment probability is discounted by the penalty factor (see Table I).

Commumication Free Learning with Stickiness—The Commumication Free Learning with Stickiness (CFLS) algorithm [13], [14] differs from the CFL algorithm in that once an access point finds its currently assigned channel to be conflict-free, it locks on to that channel.

Learning From Communication—The Learning From Communication (LFC) algorithm [21] leverages communication with neighboring access points in adapting the probability of each channel being assigned by the access point hosting the LFC algorithm. Each access point communicates its conflict status (i.e., indicating thus whether it will probabilistically attempt to change its assigned channel in the next iteration) to its neighboring access points. By knowing which of its neighboring access points will preserve their currently assigned channel, an access point with a conflicting channel assignment is able to render its future channel assignments less prone to conflict, by zeroing out the assignment probability for all the channels its so-called stable neighboring access points are currently occupying.

Learning From Communication with Stickiness—The Learning From Communication with Stickiness (LFCS) algorithm differs from the LFC algorithm in that once an access point finds its currently assigned channel to be conflict-free, it locks on to that channel.

VI. Experiments

We establish a square world environment with an area of 160×160 units and a patch size of 4 units. Within it, we instantiate different populations of agents representing autonomous access points distributed randomly in 2D space according to the uniform distribution. Each access point runs one of the aforementioned channel assignment algorithms independently of other access points. The performance of a particular algorithm is assessed under different radius settings for the disk graph model which, in combination with the coordinates of access points, determines the connectivity of the resulting network graph (e.g., Fig. 4).

We study the performance of the CFLS algorithm by varying the radio coverage experienced by each access point and the number of access points in the environment. The CFLS algorithm is a fitting subject for this as it was also studied in [13], [14]. We consider from 50 to 400 access points in increments of 50 and a radius of 10 to 60 for the disk graph model. We measure the number of iterations it takes to converge the entire network to a conflict-free channel assignment (Fig. 4).

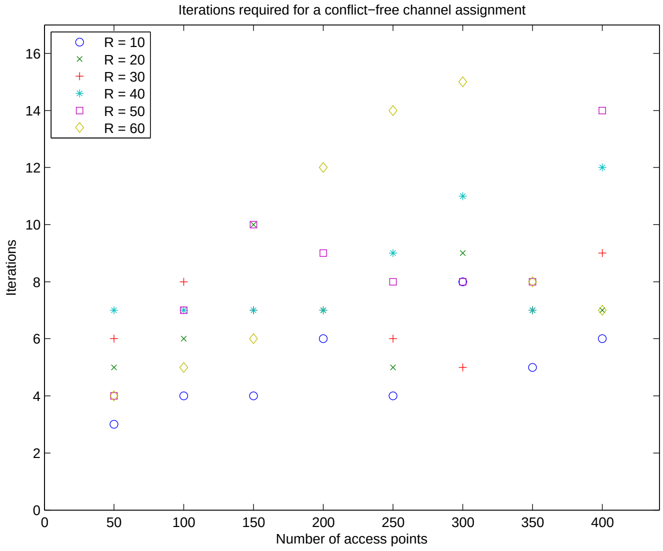

Fig. 4. Performance of the CFLS algorithm.

Aligned to our prior work in this area [21] where we leveraged the Matlab environment for simulation tasks, we find that the convergence delay of the CFLS algorithm scales nicely with an increase in the IEEE 802.11 network density.

VII. Conclusions

With the deployment of IEEE 802.11 type systems increasing in density, particularly in urban areas, attention is drawn to optimization aspects. And though the IEEE 802.11k standard has been developed specifically to support radio resource management applications, an understanding of radio resource management approaches and algorithms suitable for these settings is not fully established. To address this shortcoming, we have developed a simulation model for distributed channel assignment problems in an autonomic networking arrangement. We intend to continue studying dense IEEE 802.11 networks under a more dynamic arrangement where a mix of channel assignment algorithms may manifest.

References

[1] V. Gazis, A. Kousaridas, C. Polychronopoulos, T. Raptis, and N. Alonistioti, "Self-Management Capacities in Future Internet Wireless Systems," in *The First International Conference on Adaptive and Self-adaptive Systems and Applications*, 2009.

[2] A. Akella, G. Judd, S. Seshan, and P. Steenkiste, "Self-management in chaotic wireless deployments," in *MobiCom '05: Proceedings of the 11th annual international conference on Mobile computing and networking*. New York, NY, USA: ACM, 2005, pp. 185–199.

[3] IEEE Standards Association. (2007) IEEE Standard for Information Technology Telecommunications and Information Exchange Between Systems Local and Metropolitan Area Networks Specific Requirements Part 11: Wireless Lan Medium Access Control (MAC) and Physical Layer (PHY) Specifications. [Online]. Available: http://standards.ieee.org/getieee802/download/802.11-2007.pdf

[4] A. Mishra, S. Banerjee, and W. Arbaugh, "Weighted coloring based channel assignment for wlans," *SIGMOBILE Mob. Comput. Commun. Rev.*, vol. 9, no. 3, pp. 19–31, 2005.

[5] Cisco, "Radio resource management under unified wireless networks," Online, Tech. Rep., 2010. [Online]. Available: http://www.cisco.com/en/US/tech/tk722/tk809/technologies_tech_note09186a008072c759.shtml

[6] HP, "Radio resource management and spectrum analysis best practices," Online, Tech. Rep., 2012 2012. [Online]. Available: http://h20195.www2.hp.com/V2/GetPDF.aspx%2F4AA4-4997ENW.pdf

[7] IEEE Standards Association. (2008, June) IEEE Standard for Information Technology Telecommunications and Information Exchange Between Systems Local and Metropolitan Area Networks Specific Requirements Part 11: Wireless Lan Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment 1: Radio Resource Measurement of Wireless LANs.

[8] K. Leung and B.-J. Kim, "Frequency assignment for ieee 802.11 wireless networks," in *Vehicular Technology Conference, 2003. VTC 2003-Fall. 2003 IEEE 58th*, vol. 3, 6-9 2003, pp. 1422 – 1426 Vol.3.

[9] M. Yu, H. Luo, and K. Leung, "A dynamic radio resource management technique for multiple aps in wlans," *Wireless Communications, IEEE Transactions on*, vol. 5, no. 7, pp. 1910 –1919, july 2006.

[10] M. da Silva and J. Ferreira de Rezende, "A dynamic channel allocation mechanism for ieee 802.11 networks," Sept. 2006, pp. 225–230.

[11] J. Riihijarvi, M. Petrova, P. Mahonen, and J. Barbosa, "Performance evaluation of automatic channel assignment mechanism for ieee 802.11 based on graph colouring," Sept. 2006, pp. 1–5.

[12] B. Kauffmann, F. Baccelli, A. Chaintreau, V. Mhatre, K. Papagiannaki, and C. Diot, "Measurement-based self organization of interfering 802.11 wireless access networks," in *INFOCOM 2007. 26th IEEE International Conference on Computer Communications. IEEE*, 6-12 2007, pp. 1451 –1459.

[13] D. Leith and P. Clifford, "A self-managed distributed channel selection algorithm for wlans," April 2006, pp. 1–9.

[14] D. Malone, P. Clifford, D. Reid, and D. Leith, "Experimental implementation of optimal wlan channel selection without communication," in *New Frontiers in Dynamic Spectrum Access Networks, 2007. DySPAN 2007. 2nd IEEE International Symposium on*, 17-20 2007, pp. 316 –319.

[15] E. G. Villegas, E. Lopez-Aguilera, R. Vidal, and J. Paradells, "Effect of adjacent-channel interference in ieee 802.11 wlans," in *Cognitive Radio Oriented Wireless Networks and Communications, 2007. CrownCom 2007. 2nd International Conference on*, 1-3 2007, pp. 118 –125.

[16] J. Nachtigall, A. Zubow, and J.-P. Redlich, "The impact of adjacent channel interference in multi-radio systems using ieee 802.11," in *Wireless Communications and Mobile Computing Conference, 2008. IWCMC '08. International*, 6-8 2008, pp. 874 –881.

[17] W. L. Tan, K. Bialkowski, and M. Portmann, "Evaluating adjacent channel interference in ieee 802.11 networks," in *Vehicular Technology Conference (VTC 2010-Spring), 2010 IEEE 71st*, 16-19 2010, pp. 1 –5.

[18] J. Riihijarvi, M. Petrova, and P. Mahonen, "Frequency allocation for wlans using graph colouring techniques," Jan. 2005, pp. 216–222.

[19] R. Diestel, *Graph Theory*, 3rd ed., ser. Graduate Texts in Mathematics. Springer-Verlag, 2005, vol. 173. [Online]. Available: http://www.math.uni-hamburg.de/home/diestel/books/graph.theory/

[20] W. Klotz, "Graph coloring algorithms," Technical University of Clausthal, Tech. Rep., 2002. [Online]. Available: http://www.math.tuclausthal.de/Arbeitsgruppen/Diskrete/Optimierung/publications/2002/gca.pdf

[21] V. Gazis, K. Sasloglou, N. Frangiadakis, P. Kikiras, A. Merentitis, K. Mathioudakis, and G. Mazarakis, "Cooperative communication in channel assignment strategies for IEEE 802.11k WLAN systems," in *IEEE 24th International Symposium on Personal, Indoor and Mobile Radio Communications: Mobile and Wireless Networks (PIMRC'13 - Mobile and Wireless Networks)*, London, United Kingdom, Sep. 2013.