A Control Method for Bipedal Trunk Spring Loaded Inverted Pendulum Model

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***Abstract*—In this paper, we present a control method for bipedal robotic walking with compliant legs and upright trunk. The proposed method is validated with dynamic simulation of a planar, reduced-order biped walking model, consisting of a rigid trunk and compliant legs. Existing literatures have found that simple mass-spring model can describe dynamic characteristics of bipedal locomotion, in terms of Ground Reaction Force (GRF) and the Center of Mass (CoM) profile. In order to explain trunk-upright walking mechanics, a control method named the Virtual Pivot Point (VPP) method based on the Virtual Pendulum (VP) concept has been previously introduced. In this method, the axial force of a compliant leg is redirected towards the VPP of the model, which is located above the CoM of the model, in order to provide restoring moment about trunk motion. The resulting behaviour of the model would resemble a virtual pendulum rotating about the VPP, thus upright trunk while walking is pursued. However, we have found that for some cases this method provides upsetting moment, instead of restoring moment, which degrades the performance of the control. Inspired from this analysis, we propose a new force-redirecting method as a controller for robotic biped walking. We consider a dynamic simulation of a simple, planar simple walking model to validate the performance of the proposed method under random initial condition and under the presence of force disturbance. The proposed method shows stable and robust walking performance compared to the VPP method.**

Keywords–Bipedal walking control; compliant leg; reduced-order walking model; force redirection rule.

I. Introduction

Bipedal robotic locomotion has been an extensive field of research for many years in robotics research community. Zero-moment-point (ZMP) approaches have been successful in demonstrating robust and versatile bipedal locomotion due to its mathematical tractability [1]–[4]. The joint trajectories are planned assuming level-height motion of the center of mass (CoM) of the robot with zero angular momentum and fully-actuated legs (i.e., at least one foot is always flat on the ground with ankle actuation), while in practice under-actuation is common in bipedal robot. In order to handle nonlinear, underactuated, hybrid dynamics of bipedal robot, a formal method such as hybrid zero dynamics (HZD)-based approaches [5]–[7], or model-based trajectory optimization and stabilization [8] have been studied deeply. However, these methods usually require precise dynamic models. They also require to compute periodic trajectories based on the model and transition among different trajectories for versatile locomotion is not a trivial problem [9]. On the other hand, some end up with an idea that the bipedal walking controller does not need to be complex and one can find proper target physical behaviours based on simple reduced-order models; then, joint trajectories will be determined by dynamic interaction of the robot, controller, and the environment [10]–[13].

Traditionally, the mechanics of walking and running have been modelled with different paradigm, i.e., inverted pendulum model for walking [14] and spring-mass model for running [15]. After many evidences that the inverted pendulum model cannot reproduce mechanical characteristics of walking ([10], [16], [17]), Geyer et al. [18] have suggested the spring-mass model as a unifying template for bipedal locomotion; both the fundamental characteristics of walking and running could be reproduced by the same model with different set of parameters. However, the point-mass conceptual models cannot explain the upright posture of walking with non-zero angular momentum. Therefore, robots based on the spring-mass model concept controlled trunk position be zero and this control was independent from the compliant leg behavior [9], [19], [20].

Regarding this issue, Maus et al. [21] and Gruben and Boehm [22] have investigated experimental results of different animals and propose that the ground reaction forces (GRF) are intersecting a single fixed point above the CoM. This point is called either the virtual pivot point (VPP) [21], or the divergent point (DP) [22]. According to these studies, the GRFs directing to this point generate uprighting torque to trunk, thus upright posture is maintained while walking or running. As the force direction could be calculated by geometrical property only, without global orientation of the trunk, the upright posture could be achieved with inherent mechanical property without complex control algorithms. The concept affected controller for biped hopping [23] but required a high-level stabilizing controller around a pre-computed periodic trajectory as in the formal method. Stable robotic walking with a simple controller based on the VPP method seems hard to achieve yet [24].

In this paper, we propose a very simple form of control for bipedal walking with trunk, inspired by the Virtual Pendulum (VP) concept. We first argue that having GRF towards a single fixed point such as VPP or DP is not sufficient for maintaining upright posture, even in a simple model with massless legs; a

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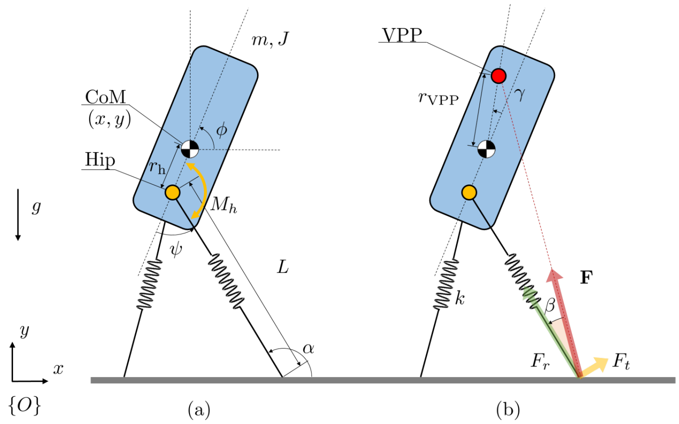

Figure 1. BTSLIP model (a). The VPP model suggested by [21] (b). Notations for parameters and variables are redefined in this paper.

simple analysis supports this argument. Based on this analysis, we propose a new GRF-redirecting control method for hip torques; the law no longer constrains the direction of GRF towards a single point but to the direction which is always providing a restoring moment to the body. A dynamic simulation result demonstrates the effectiveness of the proposed method. Here, we only consider a planar simple bipedal model with massless springy legs. The simulation results indicate that the proposed method is a promising method for achieving stable and robust bipedal walking.

The paper is structured as follows. In Section II, we first analyse the previous VPP method and propose a new method. Section III gives the main results of this paper, *i.e.*, planar bipedal walking simulation. Finally, we state concluding remarks in Section

II. Control for Upright Trunk

We seek for a control strategy for bipedal walking with upright trunk that is compatible with the compliant leg scheme, originated from spring-mass model. This type of model is called a bipedal trunk spring loaded inverted pendulum model (BTSLIP model [25]). An intriguing strategy named the VP concept and the VPP method [21] have inspired our method proposed in this section. We first review the VPP method and then proceed to a new method.

A. The VPP Model

The VPP model presented in [21] is illustrated in Figure 1. A planar bipedal walking model consists of a rigid trunk, of which center of mass (CoM) is located above hip, and two massless springy legs. This model is named the Bipedal Trunk Spring Loaded Inverted Pendulum (BTSLIP) model [25]. The springy legs are assumed to produce axial forces dependent on leg deflection from its rest length, and each leg is assumed to have hip torques that eventually generates tangential component of the GRF, perpendicular to its axial direction, between its point foot and the ground.

By introducing a variable β, a representative angle between the direction of the GRF and the leg axis, two components of the GRF (axial force F_r and tangential force F_t) are described as

$$\begin{cases} F_r = k(L_0 - L) \\ F_t = F_r \tan\beta, \end{cases} \tag{1}$$

where k is the stiffness of the leg and L_0 is the rest length of the spring. L is the length of the leg. In the VPP method, the hip torque is controlled to generate tangential force F_t in order to redirect the ground reaction force towards the VPP located above the CoM,

$$M_h = LF_t = LF_r \tan\beta,$$

where β is geometrically computed because the VPP is fixed on the trunk.

$$\tan\beta = \frac{r_h \sin\psi + r_{\mathrm{VPP}} \sin(\psi - \gamma)}{L + r_h \cos\psi + r_{\mathrm{VPP}} \cos(\psi - \gamma)}. \tag{2}$$

Here, r_{h} is the distance between CoM and hip, r_{VPP} is the distance between CoM and VPP, and γ is angle between torso longitudinal axis and a line connecting CoM and VPP. L and ψ are current leg length and angle, respectively. As originally proposed in [21], we regard $\gamma = 0$ throughout this paper. The insight behind the method is that the resultant GRF will *always* provide restoring moment about body tilting and the model can walk with upright trunk with this inherent mechanical property. Although a direct feedback of orientation of the trunk is not input to the hip torque controller, the trunk can maintain upright position with small oscillation as if a pendulum has a stable equilibrium point at its downright position. However, we have found that having a *single intersection point* for GRFs is not sufficient to always provide restoring moment to the body.

In Figure 2, we present all possible postures of the schematic model (trunk with a single compliant leg). The trunk may be upright or tilted in clockwise/counter-clockwise (CW/CCW) direction. At the same time, the foot of its stance leg (leg in touch with the ground) may be located right below or posterior/anterior the hip. Three possible postures of the trunk and three possible locations of foot relative to hip give eleven different postures to analyse. Mostly, as intended, the GRF pointing the VPP provides restoring moment, or at worst, zero moment. This allows the trunk be settle down in some region without specifying a desired posture. However, in some cases, the GRF pointing the VPP provides upsetting moment, which would cause the trunk to fall. The postures correspond to this case are depicted with red and shaded cells in the figure. Although this is an analysis to a static posture, we interpret that this property limits the performance of the model as will be shown in Section III.

In order to provide mathematical criterion, we introduce two variables $\tilde{\phi}$ and $\tilde{\beta}$, pitch orientation from upright posture and the angle between GRF and a virtual line connecting foot and CoM, respectively, as shown in Figure 3 (a). Note that the direction of the GRFs with respect to the CoM direction ($\tilde{\beta}$) is considered rather than β, because eventually this will determine the direction of the moment applied. Then, it can be shown that the cases which would destabilize the trunk, shown in Figure 2, can be represented by the following condition,

$$\tilde{\phi} \cdot \tilde{\beta} > 0. \tag{3}$$

For example, if we examine a posture in the left column and bottom row of the Figure 2, the trunk is tilted in clockwise direction and foot is posterior to the hip in both postures. However, if the controller tends to generate GRF towards the fixed VPP above the hip, in one model the resultant GRF would provide moment in counter-clockwise direction, which is a righting moment, whereas the moment in the other model

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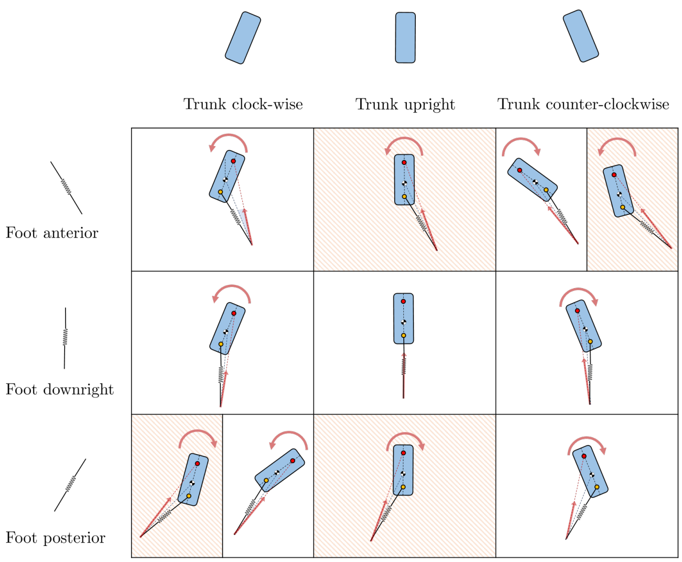

Figure 2. Representative possible postures of the VPP model are illustrated. Among these, in four postures(red, shaded), GRF provides the trunk with moments in destabilizing direction.

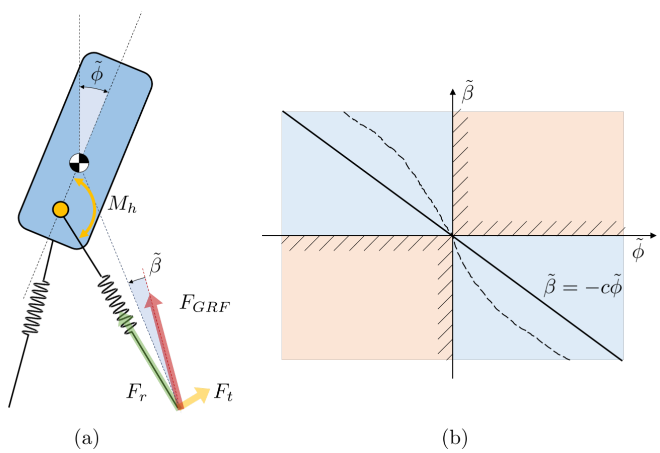

Figure 3. In order for analysis, a new variable $\tilde{\beta}$ and $\tilde{\phi}$ are introduced (a). In the domain of variables, the first and the third quadrant are not allowed (b). The controlled relation between $\tilde{\beta}$ and $\tilde{\phi}$ must lie within the second and the fourth quadrant.

in the shaded region will be in clock-wise direction, which results in a upsetting moment.

From this simple analysis, we end up with several remarks. First, a single fixed intersection point such as that in the VPP method is not sufficient for upright trunk. Actually, experimental findings indeed show that a single intersection point is not necessary for a VP system and cannot always be found [21]. Second, the *direction* of the GRF with respect to the CoM is what important factor to consider, rather than the *position* of the point where the GRF is redirected to, in order to approach the fascinating VP concept.

B. Proposed Control Strategy

Although the fixed VPP seems not sufficient, the general argument of the VP concept is still persuasive for us, in that, the direction of force generated at foot is important in upright trunk walking. Therefore, we keep the format of the controller (1) such that the hip torque redirect the force generated by the springy leg. It is also compatible with springy leg concept which seems important in dynamic walking. The difference is how to define the angle β.

Based on the above analysis, a proper GRF redirecting controller should aim $\tilde{\phi} \cdot \tilde{\beta} < 0$, which is represented by the region without shade in Figure 3 (a). We would like to argue that if the generated GRF by a suitable redirecting controller satisfies this condition, any form of control would work. The simplest form we can propose for a redirecting controller is the linear relationship as shown in Figure 3 (b).

$$\tilde{\beta} = -c\tilde{\phi}, \tag{4}$$

where c is a positive gain which should be properly designed.

Noting that the speed and the direction of motion of the planar robot model should also be considered in determining the direction of the GRFs, we end up with the following control form for force direction control.

$$\tilde{\beta} = -c\tilde{\phi} - d\dot{\tilde{\phi}}, \tag{5}$$

where d is another positive gain which should be properly designed. Corresponding hip torques will be computed to redirect axial leg force to desired direction with respect to CoM. The angle between the actual leg and the virtual line from foot to CoM can be computed from geometric information as well; denoting this angle as $\angle$, $\tan\beta = \tan(\tilde{\beta} + \angle)$ can be easily computed.

C. Feasible Direction of Foot-ground Interaction Force

As the direction of the GRF is a target control parameter, considering the constraints on the GRF becomes trivial in this framework. Typical constraints on the ground reaction force are friction cone and that the force is unilateral, i.e., it can only push the ground. These constraints put limit on determining $\tilde{\beta}$. In other words, the hip torques are uprighting the trunk as much as possible within feasible region.

III. SIMULATION RESULT

A. Model Description

In order to verify the proposed control method, we use dynamic simulation. The model configuration is illustrated in Figure 1 (a). The model parameters are listed in Table I.

The equations of motion of the model can be described as follows.

$$\begin{cases} m\ddot{x} = \sum_i F_x \\ m\ddot{y} = \sum_i F_y - mg \\ J\ddot{\phi} = \sum_i (\mathbf{r}_\mathrm{f} - \mathbf{r}_\mathrm{CoM}) \times \mathbf{F} \end{cases} \tag{6}$$

where $\mathbf{r}_\mathrm{CoM} = [x, y]^T$ is the CoM position, $\mathbf{r}_\mathrm{f}$ is the foot-in-contact position, ϕ is pitch angle in sagittal plane simulation.

TABLE I. MODEL PARAMETERS FOR THE SIMULATION

Parameter	Meaning	Value [unit]
m	total mass	80 [kg]
J	moment of inertia	4.58 [kg·m^2]
g	gravitational acceleration	9.81 [m/s^2]
k	leg stiffness	20,000 [N/m]
r_h	distance between CoM and hip	0.1 [m]

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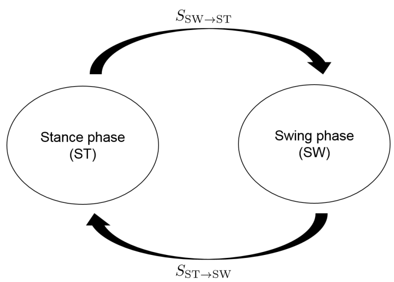


Figure 4. State transition of each leg.

$\mathbf{F} = [F_x, F_y]^T$ is the GRF at the foot in stance and can be described as

$$\begin{cases} F_x = F_r \cos\alpha + F_t \sin\alpha \\ F_y = F_r \sin\alpha - F_t \cos\alpha \end{cases} \tag{7}$$

where α is the angle between leg and the ground. $i\in$ [left, right] denotes leg in stance.

The model can have double support, single support, and possibly flight phase as shown in Figure 4. Transition between the phase and will not require additional computation nor changing the form of dynamical equations, because we do not consider impact dynamics when a swing leg touches down the ground when the leg is assumed massless. We only need to choose which leg provides forces to the body, *i.e.*, choose i properly. State transition of dynamical system is thus determined by the state transition of each leg, of which condition can be described as follows, where $\mathbf{x} = [x, y, \phi, \dot{x}, \dot{y}, \dot{\phi}]^T$ denote the state vector and $\mathbb{C}$ denote the admissible configuration space.

$$S_{\mathrm{SW}\rightarrow\mathrm{ST}} = \{\mathbf{x} \in \mathbb{C} | y_h - L_0 \sin\alpha_0 = 0\}$$
$$S_{\mathrm{ST}\rightarrow\mathrm{SW}} = \{\mathbf{x} \in \mathbb{C} | F_r = 0\}$$

In other words, swing to stance transition happens when the swing leg touches down the ground with the predefined angle of attack α_0, and the stance leg takes off the ground when ground reaction force becomes zero. Here, y_h is the vertical position of the hip.

B. Sagittal Plane Walking Simulation

The dynamics of the bipedal walking model with trunk and compliant legs is simulated for 20 seconds. Two methods, the VPP approach and the proposed control approach, are compared with the planar walking model in sagittal plane, of which control parameters are listed in Table II. In Figure 5 (a) and Figure 6 (a), the trajectories of the center of mass forward (x) and vertial (y) motion and pitch (ϕ) of the model are presented. Both methods have periodic solutions, and the simulation results are started from initial conditions a bit deviated from the periodic solution. With this initial condition, the proposed method quickly stabilize the model to its steady-state motion, whereas the model with the VPP method does not. The method cannot reject this errors. In the proposed method, pitch motion of the model is directly fed back to the controller and therefore the pitch stabilization is directly achieved, whereas in the VPP model, the pitch oscillation is maintained to some extent.

Figure 5 (b) and Figure 6 (b) present the models with the same initial conditions but with external force disturbance of $F_{\mathrm{dist}} = [45, 0]^T$[N] that is applied at the right foot of the model during 0.2 seconds. This would affect both translational motion and rotational motion. Both models could reject small disturbances in the sense of not falling down, and this value is chosen to show the difference between the two. It is clear that the proposed method is more robust in the sense of not falling down than the VPP method. The pitch motion is immediately stabilized and at the same time, the translational motion is indirectly stabilized by the self-stabilizing property of mass-spring walking system [18]. However, in the VPP method, destabilized pitch motion induced gait instability.

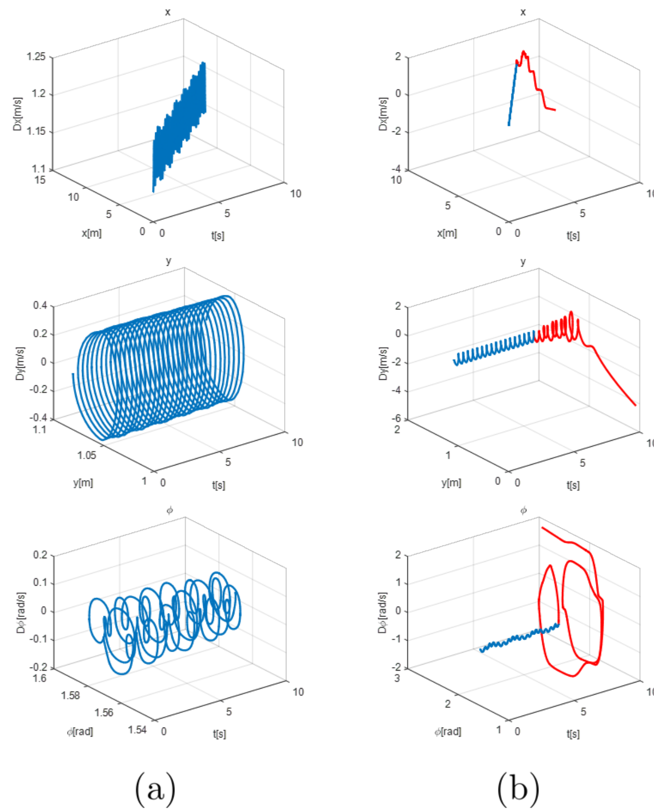


Figure 5. Simulation results with planar bipedal walking model with the VPP method on its sagittal plane. The response of the system without (a) and with (b) disturbance. The trajectories after the moment that force disturbance is applied are plotted with red.

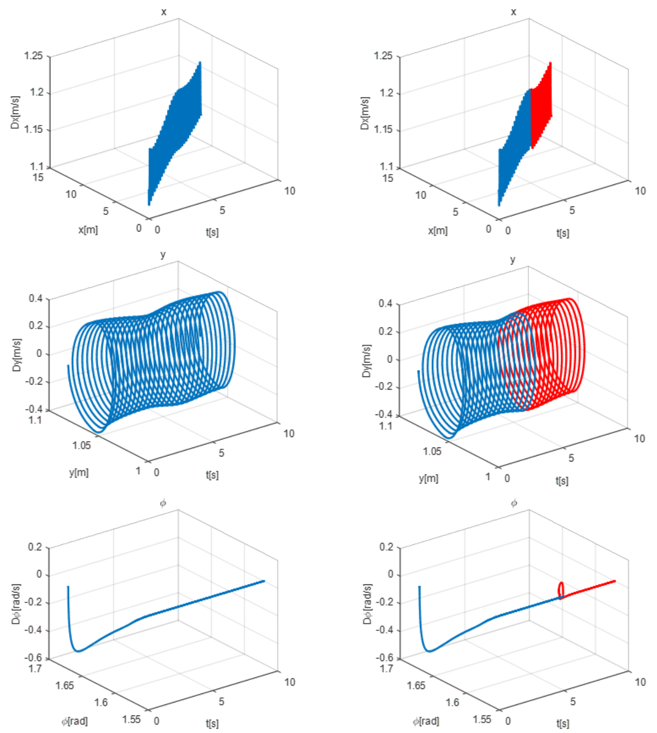
IV. FUTURE WORK

We investigate the simple reduced-order models and develop the control method in order to build and control a real bipedal robotic platform. Regarding this, we present interesting future works of our great interest.

First of all, we consider investigating the role of foot and ankle is crucial in developing more comprehensive walking

TABLE II. CONTROL PARAMETERS FOR THE SIMULATION

Parameter	Meaning	Value [unit]
r_{VPP}	distance between VPP and CoM	0.1 [m]
α_0	fixed leg touchdown angle in sagittal plane	107 [deg]
c	position-proportional gain	5 [·]
d	velocity-proportional gain	1 [s]



(a) (b)

Figure 6. Simulation result with planar bipedal walking model with the proposed method on its sagittal plane. The response of the system without (a) and with (b) disturbance. The trajectories after the moment that force disturbance is applied are plotted with red.

model. In robotics, in order for the task of standing still, the foot and ankle seem crucial. On the other hand, in biomechanics, the role of foot and ankle have been studied in variety of perspective, including mechanical advantage of ankle push-off [26]. In [27], the authors have analyzed the role of ankle and foot in static posture, based on a similar trunk-upright control scheme. A similar study regarding dynamic behaviour would be interesting.

Second, as the model is based on a reduced-order model with massless legs, implementing the principles gained from the model onto more realistic articulated model is another important task. There are many available techniques developed in existing literatures, for example, readers are referred to [28]. We aim to apply the model to rigid-body articulated robotic model and develop a detailed control algorithm.

Third, we seek for developing a full 3-d robot in the near future, and therefore extending the model to 3-d would be important. we believe that the same control strategy would be valid in frontal plane (roll motion) balance, as the structure of the proposed strategy consisting of compliant legs and force redirecting hip torque is not restrained from sagittal plane walking. Once planar model in frontal plane is investigated, combining the sagittal and frontal plane control method in a single 3-D model would be possible.

V. CONCLUSION

In this paper, we proposed a novel method to stabilize trunk orientation of bipedal walking. The method is inspired from the VP concept and the VPP method. We focus on the fact that if the direction of the ground reaction force is computed based on rotational motion of the trunk and the magnitude of force which is computed based on axial compliance of the leg, it is possible to realize bipedal walking with dynamic characteristic of human walking. As for our future work, we aim to employ the method in controlling articulated robot simulation. Extending the simple model in 3-D is also of our great interest.

REFERENCES

[1] M. Vukobratović and B. Borovac, "Zero-moment pointthirty five years of its life," *International Journal of Humanoid Robotics*, vol. 1, no. 01, pp. 157–173, 2004.

[2] Y. S. et al., "The intelligent asimo: system overview and integration," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 3, 2002, pp. 2478–2483.

[3] Y. Choi, D. Kim, Y. Oh, and B. J. You, "Posture/walking control for humanoid robot based on kinematic resolution of com jacobian with embedded motion," *IEEE Transactions on Robotics*, vol. 23, no. 6, pp. 1285–1293, 2007.

[4] K. Kaneko, K. Harada, F. Kanehiro, G. Miyamori, and K. Akachi, "Humanoid robot hrp-3," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2008, pp. 2471–2478.

[5] E. R. Westervelt, J. W. Grizzle, and D. E. Koditschek, "Hybrid zero dynamics of planar biped walkers," *IEEE transactions on automatic control*, vol. 48, no. 1, pp. 42–56, 2003.

[6] E. R. Westervelt, J. W. Grizzle, C. Chevallereau, J. H. Choi, and B. Morris, *Feedback control of dynamic bipedal robot locomotion*. CRC press, 2007, vol. 28.

[7] K. Sreenath, H.-W. Park, I. Poulakakis, and J. W. Grizzle, "Embedding active force control within the compliant hybrid zero dynamics to achieve stable, fast running on mabel," *The International Journal of Robotics Research*, vol. 32, no. 3, pp. 324–345, 2013.

[8] I. R. Manchester, U. Mettin, F. Iida, and R. Tedrake, "Stable dynamic walking over uneven terrain," *The International Journal of Robotics Research*, vol. 30, no. 3, pp. 265–279, 2011.

[9] S. e. a. Rezazadeh, "Spring-mass walking with atrias in 3d: Robust gait control spanning zero to 4.3 kph on a heavily underactuated bipedal robot," in *ASME 2015 Dynamic Systems and Control Conference*. American Society of Mechanical Engineers, 2015, p. V001T04A003.

[10] R. J. Full and D. E. Koditschek, "Templates and anchors: neuromechanical hypotheses of legged locomotion on land," *Journal of Experimental Biology*, vol. 202, no. 23, pp. 3325–3332, 1999.

[11] D. G. Hobbelen and M. Wisse, "Controlling the walking speed in limit cycle walking," *The International Journal of Robotics Research*, vol. 27, no. 9, pp. 989–1005, 2008.

[12] D. J. Braun and M. Goldfarb, "A control approach for actuated dynamic walking in biped robots," *IEEE Transactions on Robotics*, vol. 25, no. 6, pp. 1292–1303, 2009.

[13] T. Geng and J. Q. Gan, "Planar biped walking with an equilibrium point controller and state machines," *IEEE/ASME Transactions on Mechatronics*, vol. 15, no. 2, pp. 253–260, 2010.

[14] R. Alexander, "Mechanics of bipedal locomotion," *Perspectives in experimental biology*, vol. 1, pp. 493–504, 1976.

[15] R. Blickhan, "The spring-mass model for running and hopping," *Journal of biomechanics*, vol. 22, no. 11, pp. 1217–1227, 1989.

[16] C. R. Lee and C. T. Farley, "Determinants of the center of mass trajectory in human walking and running." *Journal of experimental biology*, vol. 201, no. 21, pp. 2935–2944, 1998.

[17] M. G. Pandy, "Simple and complex models for studying muscle function in walking," *Philosophical Transactions of the Royal Society B: Biological Sciences*, vol. 358, no. 1437, pp. 1501–1509, 2003.

[18] H. Geyer, A. Seyfarth, and R. Blickhan, "Compliant leg behaviour explains basic dynamics of walking and running," *Proceedings of the Royal Society of London B: Biological Sciences*, vol. 273, no. 1603, pp. 2861–2867, 2006.

[19] M. H. Raibert, *Legged robots that balance*. MIT press, 1986.

 ISBN: 978-1-61208-555-5

[20] C. e. a. Hubicki, "Atrias: Design and validation of a tether-free 3d-capable spring-mass bipedal robot," *The International Journal of Robotics Research*, vol. 35, no. 12, pp. 1497–1521, 2016.

[21] H.-M. Maus, S. Lipfert, M. Gross, J. Rummel, and A. Seyfarth, "Upright human gait did not provide a major mechanical challenge for our ancestors," *Nature Communications*, vol. 1, p. 70, 2010.

[22] K. G. Gruben and W. L. Boehm, "Force direction pattern stabilizes sagittal plane mechanics of human walking," *Human movement science*, vol. 31, no. 3, pp. 649–659, 2012.

[23] M. A. e. a. Sharbafi, "Controllers for robust hopping with upright trunk based on the virtual pendulum concept," in *IEEE/RSJ International Conference on Intelligent Robots and Systems(IROS)*. IEEE, 2012, pp. 2222–2227.

[24] A. T. Peekema, "Template-based control of the bipedal robot atrias," Master's thesis, OSU, 2015.

[25] M. A. Sharbafi, C. Maufroy, M. N. Ahmadabadi, M. J. Yazdanpanah, and A. Seyfarth, "Robust hopping based on virtual pendulum posture control," *Bioinspiration & biomimetics*, vol. 8, no. 3, p. 036002, 2013.

[26] A. D. Kuo, "Choosing your steps carefully," *IEEE Robotics & Automation Magazine*, vol. 14, no. 2, pp. 18–29, 2007.

[27] K. G. Gruben and W. L. Boehm, "Ankle torque control that shifts the center of pressure from heel to toe contributes non-zero sagittal plane angular momentum during human walking," *Journal of biomechanics*, vol. 47, no. 6, pp. 1389–1394, 2014.

[28] G. Garofalo, C. Ott, and A. Albu-Schffer, "Walking control of fully actuated robots based on the bipedal slip model," in *2012 IEEE International Conference on Robotics and Automation*, May 2012, pp. 1456–1463.