Empirical Comparison of Fuzzy Cognitive Maps and Dynamic Rule-based Fuzzy Cognitive Maps

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***Abstract*— Among the soft computing techniques that can be used effectively to model decision tasks in autonomous robotics are Fuzzy Cognitive Maps. Dynamic Rule-based Fuzzy Cognitive Maps (DRBFCMs) are a Fuzzy Cognitive Map variant that allows modeling of dynamic causal maps, where influence weights are determined dynamically at simulation time using Fuzzy Inference Systems, in order to adapt to new conditions. We aim in this work to compare and contrast DRBFCM to a conventional Fuzzy Cognitive Map in application of cotton yield in precision farming. The cotton yield model shows the relationships between soil properties like pH, K, P, Mg, N, Ca, Na and cotton yield. DRBFCM was evaluated for 360 cases measured for three years (2001, 2003 and 2006) in a 5 ha experimental cotton field. The results revealed an accuracy of predictions of 85.55%, 87.22% and 73.33%, against 73.80%, 67.20% and 69.65% for the conventional FCM model, and against 75.55%, 68.86% and 71.32% for the FCM model with the Nonlinear Hebbian Learning algorithm, for the years 2001, 2003 and 2006 respectively. DRBFCM proved, in this case study, to predict more accurately the yield while being faithful to the real world model.**

Keywords*— fuzzy cognitive maps; fuzzy inference systems; dynamic rule-based fuzzy cognitive maps; cotton yield prediction

I. INTRODUCTION

A Fuzzy Cognitive Map (FCM) is a soft computing technique and semi-quantitative technique that can be used as an intuitive elicitation tool to transfer individuals' tacit knowledge into a causal network. FCMs can be used effectively as an approach to bridge the gap between the design of causal loops and their effective use in any decision making process. FCM's graph structure facilitates causal reasoning to study systems' dynamics in complex problems. FCMs also inherently support vagueness and ambiguities, and causal reasoning in FCMs allows handling of feedback loops. Moreover, FCM models produced by different individuals or groups can be combined to produce a larger and more reliable knowledge base, which can help solving knowledge inconsistencies by generating aggregated system complexities, using multiple participants or experts. FCM simulations can model the evolution of scenarios over time, and produce projections by evolving forward and letting concepts interact with one another.

FCMs have been applied successfully in many scientific fields, including autonomous robotics. For example the authors in [1] used FCM causal inference as a mechanism to derive required control values from the FCM's motion concepts and their interaction. The authors in [2] used a new FCM variant, named Event Driven-Fuzzy Cognitive Maps (ED-FCM), to model decision tasks in autonomous navigation. The authors in [3] [4] proposed Hybrid-Dynamic Fuzzy Cognitive Maps (HD-FCM) which incorporate different types of concepts and causal relations able to circumvent the main drawbacks of FCM modeling. In a recent work [5], the authors propose Dynamic Fuzzy Cognitive Maps (DFCM), with a multi-agent approach to develop an autonomous navigation system that has skills for learning, self-adaptation, behavior management and cooperative data sharing. A general review on FCMs' research during the last decade can be found in [6].

Many implementations of FCMs exist in the literature, but most of them focus on depicting causalities between system variables, rather than cause and effect relationships, and FCM inference allows drawing conclusions about what is caused and what is not caused, which is a major limitation in dynamic systems where reasoning is characterized by magnitudes of change and effects. Another shortcoming with FCMs is that the links' weights are forced into a static value in the range [-1, 1]. In dynamic systems, the weight should be a function of other factors' influences, which allows modeling of non-monotonic, nonlinear and dynamic relationships; in fact the effect of a variable X on Y should depend on the value of X. Moreover, variables in real models map to a universe of discourse within their minimum and maximum state limits; however, in FCM models, a threshold function normalizes all values between 0 and 1.

A Dynamic Rule-based Fuzzy Cognitive Map (DRBFCM) is a FCM extension that allows reasoning in terms of deterministic magnitudes of effects. A major difference between DRBFCM and a conventional FCM model is that the weights are not fixed prior to running simulations, but are rather adapted dynamically during every step of a simulation, by using Fuzzy Inference Systems (FISs) [7].

In this work, we will validate DRBFCM numerically, and compare the accuracy of its predictions to a conventional FCM model in application of cotton yield in precision farming, by building on the work of Papageorgiou et al. [8].

The cotton yield model shows the relationships between soil properties, like Nitrogen (N), Phosphorus (P), Potassium (K), Sodium (Na), Clay (Cl), Sand (S), Calcium (Ca), Magnesium (Mg), pH, Electrical Conductivity (EC), Organic Matter (OM), and Cotton Yield (Y). DRBFCM is evaluated for 360 cases measured for three years (2001, 2003 and 2006) in a 5 ha experimental cotton yield.

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The paper is organized as follows. Section II presents a general background about FCM models, FCM models with Fuzzy Weights, and DRBFCM models. In Section III, we describe the cotton yield DRBFCM model, and we discuss the results of predicting cotton yield and compare them to the conventional FCM model. We highlight some conclusions and directions for future work in Section IV.

II. MATERIALS AND METHODS

In this section, we introduce the FCM soft computing technique, we also give background about the DRBFCM extension that leverages Fuzzy Logic inference to compute the system variables' states, using quantified perturbations produced by FISs.

A. Fuzzy Cognitive Maps

At the structural level, a FCM is a directed graph where nodes represent system concepts or variables, and links represent perceived causal relationships (caused or not) between concepts. According to Kosko [9], all the values are fuzzy; each edge between two concepts C_i and C_j is associated with a weight w_{ij}, which varies from -1 to 1. There are three different types of possible causalities between two concepts C_i and C_j:

- A positive weight reflects an excitation relation.
- A negative weight designates an inhibition relation.
- A weight of zero indicates that C_i and C_j do not exert any influence on each other's.

These causal links (also called FCM connections) and their respective weights can be encoded into an NxN (N being the number of concepts) matrix, which is referred to as the Connection or Weight Matrix E.

FCM's graph structure facilitates causal reasoning; it is a decision support system where calculations can be made to perform an assessment of the consequences of a specific system state. FCM forward inference is very close to Artificial Neural Network (ANN) mechanisms, Kosko calculates each subsequent value of the causal state using previous state and weight matrix multiplication [9]. The concepts take values, which are also called *activation levels,* between 0 and 1, where zero means the concept being deactivated or not important, and 1 being activated or important. The State Vector of activations evolves in time according to the influences between concepts. By feeding the FCM with an initial stimulus $S^{(t)}$ (state vector at time (t)), it can model the evolution of a scenario over time by evolving forward and letting concepts interact with one another.

The next state of the system $S^{(t+1)}$ is produced by multiplying previous state vector $S^{(t)}$ by the graph's weight adjacency matrix E. In auto-associative ANNs, neurons are considered to have a memory with a self-feedback link weight of 1.

The next state value of each concept C_i is hence elaborated, during simulation, by retrieving its value at the previous iteration, and adding it to the propagated weighted values of all concepts C_j that have a direct influence on the concept according to (1).

$$C_i^{(k+1)} = F\left(C_i^{(k)} + \sum C_j^{(k)} * w_{ji}\right) \quad (1)$$

where $C_i^{(k+1)}$ is the activation value of concept C_i at iteration k+1, $C_i^{(k)}$ is the value of node C_i at iteration k, w_{ji} is the weight of the cause-effect link between C_j and C_i, and F is a threshold function like sigmoid used to normalize the values within the range [0, 1] as shown by (2) [10].

$$S(x) = \frac{1}{(1 + e^{(-x)})} \quad (2)$$

In order to generate projections based on a given simulated scenario, a series of vector-matrix multiplications is performed, until a fixed point attractor; that is the vector-matrix multiplication yields an equilibrium state, where the same vector is repeated over a number of iterations [9].

B. FCM Approach with Fuzzy Weights

A variant of FCMs was proposed by Papageorgiou et al. [8]; it consists in generating weights using fuzzy linguistic terms, extracted from rules collected from domain experts. The approach consists in pooling domain knowledge from experts in the form of Fuzzy Rules [11] from experts for each interconnection of the FCM model. Thus, a number of linguistic weights are obtained for each interconnection by considering the consequent of the rule only, ignoring hence the antecedent part [8].

In order to build the FCM weight matrix, the linguistic weights, obtained from all experts, are combined using Fuzzy Logic operators [12]. For every link connecting two concepts, the linguistic terms are aggregated by typically using the fuzzy *Union* operator [11]. The membership function of the Union of two Fuzzy Sets *A* and *B*, defined over the set *X*, with membership functions μ_A and μ_B respectively is defined by a *T-conorm* mapping. One of the most common used mappings is the *maximum* as shown by (3):

$$\mu_{A \cup B}(x) = \max\left[\mu_A(x), \mu_B(x)\right] \quad (3)$$

Then, a defuzzification method is employed to calculate a single numerical weight value of the link. Several methods have been used in practice for defuzzification, the most popular method is the "centroid" method [11], which calculates the center of gravity of the aggregated fuzzy set as shown by (4):

$$CoG = \int \mu_A(x)\, x\, dx \div \int \mu_A(x)\, dx \quad (4)$$

Thus, a numerical weight (w_{ij}) is calculated for the link between every pair of concepts C_i and C_j, prior to starting simulations. To demonstrate how the linguistic terms are aggregated, let us consider the relation between K (Potassium) and Y (cotton yield) using expert knowledge:

1st Expert

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"IF K IS med THEN Y IS med. Infer: influence IS med"
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2nd Expert

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"IF K IS med THEN Y IS high. Infer: influence IS high"
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3rd Expert:

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"IF K IS high THEN Y IS very high. Infer: influence IS very high"
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The linguistic influence terms ('med', 'high' and 'very high') are summed and an overall weight is produced, transforming hence the influence into the numerical constant weight $W_{K\text{-}Y}$ = 0.65. The process of aggregation and defuzzification for this example is shown by Figure 1.

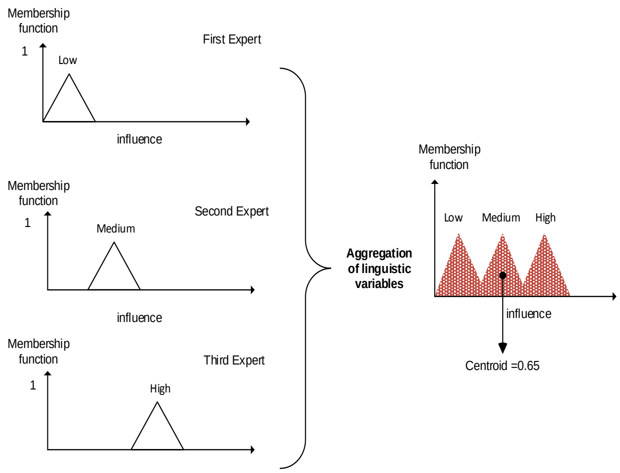

Figure 1. Aggregation and defuzzification of the three linguistic variables.

In [8], FCMs were also enriched with the unsupervised Nonlinear Hebbian Learning (NHL) algorithm. The technique was used to overcome inadequate knowledge of experts or non-acceptable FCM simulation results. The weight adaptation procedure is based on the Hebbian Learning rule proposed in [13]. The nonlinear Hebbian-type rule for ANNs learning has been adapted and modified for FCM models as proposed by the authors in [14].

C. Dynamic Rule-based Fuzzy Cognitive Maps

In this work, we build on the work of Mourhir et al. that proposes DRBFCM as an alternative to System Dynamics, using FCMs and Rule-based Systems [7]. DRBFCM models have three main properties. First, the fuzzy set theory adds true fuzziness to DRBFCM, and resolves ambiguities and subjectivity usually faced in complex real world problems [12]. Another main property is that the weights represent deterministic real values and not fuzzy binaries. The most fundamental property of a DRBFCM model is the ability to depict dynamic causalities between concepts, the influence induced on a given concept is not static, but depends on the initial state of influencing nodes. DRBFCM can adapt the weights dynamically by describing causal relationships using FISs.

In DRBFCM, concepts represent causes or effects that collectively characterize a system state at a given time. Each concept, analyzed by experts in the model, is divided into a number of intervals to determine linguistically descriptions corresponding to threshold intervals, or possible states it can exist in using membership functions [11].

A general Fuzzy variable called "Variation" is used consistently to represent the influence between concepts. The variation variable has the fuzzy sets like positively or negatively 'low', 'medium', 'high' or 'very high'. DRBFCM model structure is shown by Figure 2.

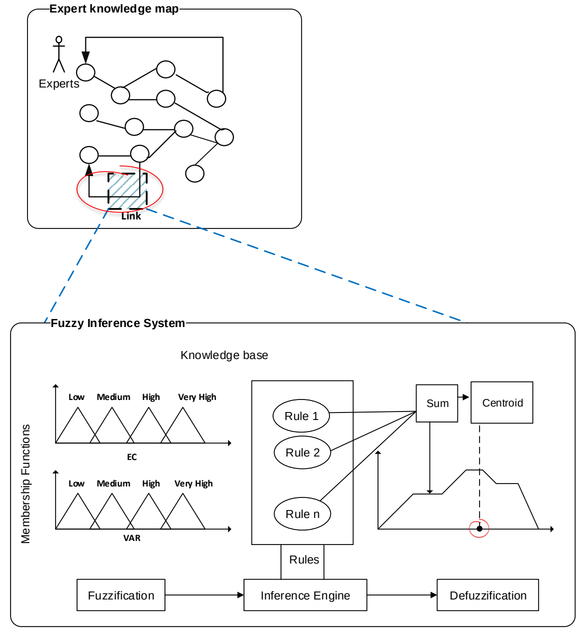

Figure 2. DRBFCM model structure.

The concepts' set of linguistic terms is used to describe the causal relationships and links between input concepts and outputs using fuzzy "if-then" rules. A link between two concepts C_i and C_j, depicted by a connection in a DRBFCM model is represented by a FIS [11]. Each FIS is described using the Fuzzy Control Language (FCL) [15]. In FCL, a FIS inference system is usually composed of one or more Function Blocks (FB). In DRBFCM models, each FB has one input variable, and an output variable as well as a Rule Block (RB). The rule block is composed of a set of rules, as well as the *aggregation*, *activation* and *accumulation* methods [11]. In DRBFCM, the rules have a single antecedent related to a concept state a or concept's variation, and output the variation strength which represents a perturbation in the target concept:

$$\texttt{IF } C_i \texttt{ is A THEN Variation is } V_{ij} \texttt{ ON } C_j$$

$$\texttt{IF Variation is } V_i \texttt{ IN } C_i \texttt{ THEN Variation is } V_{ij} \texttt{ ON } C_j$$

Since FCL supports only rules that map input concept states to output concepts states, the authors modified the FCL grammar to cope with rules describing concept variations by adding two clauses to the condition part and the conclusion part: (i) an in_clause: IN^ ID in the subcondition, that is used to specify the causal variation, where "IN" is a keyword and ID is the cause variable, and (ii) an on_clause: ON^ ID in the subconclusion, that is used to specify the effect variation, where "ON" is a keyword and ID is the effect variable.

Inference is carried according to an algorithm for combining effects on a given concept and dealing with feedback. In a given scenario, the concepts are activated with their real deterministic values.

To run a simulation, the DRBFCM model is fed with the initial stimulus for a given scenario, and while the system does not converge or does not reach a minimum number of iterations, the inference algorithm is executed to update the activation value of each concept.

To update the state value of a given concept, all the incoming connections on that concept are retrieved, and then the variation induced by every incoming connection is evaluated. Since every connection between two concepts is a FIS, fuzzy inference is used to compute the output variation. DRBFCM models make use of the Mamdani's fuzzy inference process [16]. The implication method used is the "Min", the aggregation method is "Max", and the defuzzification method used to quantify the variation is Centroid [11]. Once the different variations are obtained, the state of the concept is updated by using the sate vector and weight matrix multiplication. Concepts are considered to have memory with a self-feedback link weight equal to 1, so the activation value of the concept is updated by recalling its old value, and adding it to the summation of weighted input activations.

Since DRBFCM models operate on real deterministic values, concepts are considered to have a maximum and a minimum state value. So when these are exceeded, the concept value is set to the maximum or the minimum value.

III. Numerical Comparison of FCM and DRBFCM Models in a Cotton Yield Application

In this section, we compare and contrast DRBFCM to a conventional cotton yield FCM model, developed by Papageorgiou et al. [8] in a precision farming application.

A. DRBFCM Cotton Yield Model

We developed the cotton yield model, following the DRBFCM approach described in the previous section. However, in our work, although we use crisp real values to determine the influence, we stimulated the FCM model using normalized values in order to generate results that can be contrasted and compared consistently to the approach proposed by the authors in [8]. The initial values of concepts are transformed into the range [0,1] using a linear transformation based on the universe of discourse of fuzzy variables as shown by (5).

$$C_{jnormalized} = \frac{C_j - C_{jmin}}{C_{jmax} - C_{jmin}} \qquad (5)$$

Moreover, since there are no feedback loops in the cotton yield model, we simplified the inference algorithm by generating the weight matrix once for every record. Hence, simulations are carried according to the algorithm of Figure 3. The knowledge and data were obtained from the work of Papageorgiou et al. [8] to predict cotton yield data. Three experts contributed to the development of the cotton yield map, one experienced cotton farmer, and two experienced soil scientists, one from Technological Educational Institute of Larissa, Greece, and the other from the Laboratory of Regional Soil Analysis and Agricultural Applications of Larissa, Greece [8].

The experts stated that there are elven soil parameters that can be used to determine cotton yield (t/ha) in precision farming, which are: Soil shallow Electrical Conductivity (mS/m), Magnesium (ppm), Calcium (ppm), Sodium (ppm), Potassium (ppm), Phosphorus (ppm), Nitrogen (ppm), Organic Matter (%), pH, Sand (%) and Clay (%).

The experts described the soil parameters and their threshold values using membership functions as depicted in Table I. The experts were also requested to define the degree of influence exerted by one concept on another, using "if-then" rules, representing the causal relationships between soil parameters and the cotton yield.

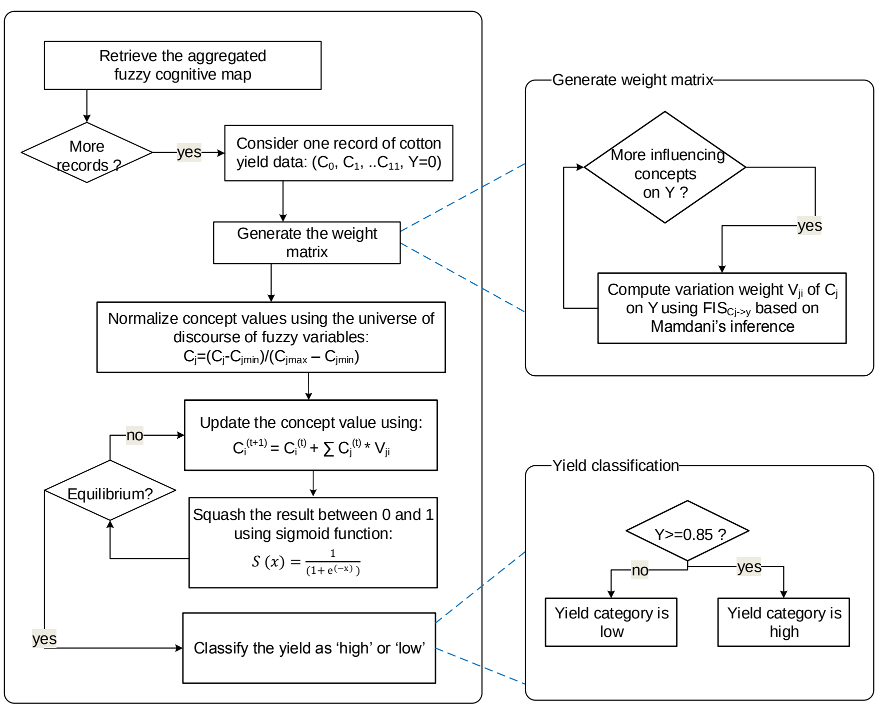

Figure 3. Cotton yield inference algorithm.

The list of fuzzy rules aggregated from the three experts is shown in Table II, where "VAR" is the variation fuzzy variable.

TABLE I. CONCEPTS' MEMBERSHIP FUNCTION PARAMETERS.

Concept	Membership functions
C1: Shallow EC (EC)	VL := TRAPE 0 0 7.5 15; L := TRIAN 10 18 25; M := TRIAN 25 28 35; H := TRIAN 30 38 45; VH := TRAPE 40 45 100 100;
C2: Magnesium (Mg)	VL := TRAPE 0 0 60 120; L := TRIAN 60 140 240; M := TRIAN 160 290 360; H := TRIAN 300 500 1400; VH := TRAPE 700 950 1400 1400;
C3: Calcium (Ca)	VL := TRAPE 0 0 455 1000; L := TRIAN 545 1273 2000; M := TRIAN 1363 2455 3000; H := TRIAN 2637 3909 5000; VH :=TRAPE 4000 4380 5000 5000;
C4: Sodium (Na)	VL := TRAPE 0 0 26 59; L := TRIAN 32 70 123; M := TRIAN 80 140 200; H := TRIAN 156 250 600; VH := TRAPE 350 450 600 600;
C5: Potassium (K)	VL := TRAPE 0 0 24 65; L := TRIAN 30 81 135; M := TRIAN 88 152 230; H := TRIAN 190 275 600; VH := TRAPE 300 470 600 600;
C6:Phosphorous (P)	VL := TRAPE 0 0 5 10; L := TRIAN 5 12.5 20; M := TRIAN 12.5 22 31.5; H := TRIAN 25 38 50; VH := TRAPE 40 45 50 60;
C7:Nitrogen (N)	VL := TRAPE 0 0 3 8; L := TRIAN 5 8 17.5; M := TRIAN 12 20 27.5; H := TRIAN 22 32 45; VH := TRAPE 35 40 45 45;
C8:Organic Matter (OM)	L := TRAPE 0 0 0.6 1.1; M := TRIAN 0.5 1.5 2.5; H := TRAPE 1.8 2.1 3 3;
C9: pH	VL := TRAPE 0 0 4 5; SL := TRIAN 5 6 7; L := TRIAN 4 5 6; M := TRIAN 6 7 8; SH := TRIAN 7 8 9; H := TRIAN 8 9 10; VH := TRAPE 9 10 11 11;
C10: Sand (S)	L := TRAPE 0 0 15 30; M := TRIAN 20 45 70; H := TRIAN 60 75 90; VH := TRAPE 80 90 100 100;
C11: Clay (Cl)	L := TRAPE 0 0 12.5 20; M := TRIAN 10 22.5 35; H := TRAPE 30 37.7 60 60;

TABLE II. COTTON YIELD FUZZY RULES.

Concept	If-then rules
C1: Shallow EC (EC)	IF EC IS VL THEN VAR IS PVL ON Y; IF EC IS M THEN VAR IS PL ON Y; IF EC IS H THEN VAR IS PM ON Y; IF EC IS H THEN VAR IS PH ON Y; IF EC IS VH THEN VAR IS PH ON Y; IF EC IS L THEN VAR IS PVL ON Y;
C2: Magnesium (Mg)	IF Mg IS VL THEN VAR IS NL ON Y; IF Mg IS L THEN VAR IS NL ON Y; IF Mg IS M THEN VAR IS NM ON Y; IF Mg IS H THEN VAR IS NM ON Y; IF Mg IS VH THEN VAR IS NM ON Y;
C3: Calcium (Ca)	IF Ca IS VL THEN VAR IS PM ON Y; IF Ca IS L THEN VAR IS PL ON Y; IF Ca IS M THEN VAR IS PM ON Y; IF Ca IS H THEN VAR IS PM ON Y; IF Ca IS VH THEN VAR IS PM ON Y; IF Ca IS VH THEN VAR IS PL ON Y;
C4: Sodium (Na)	IF Na IS VL THEN VAR IS NVH ON Y; IF Na IS VL THEN VAR IS NH ON Y; IF Na IS L THEN VAR IS NM ON Y; IF Na IS M THEN VAR IS NL ON Y; IF Na IS H THEN VAR IS NH ON Y; IF Na IS VH THEN VAR IS NH ON Y; IF Na IS VH THEN VAR IS NVH ON Y;
C5: Potassium (K)	IF K IS VL THEN VAR IS PVL ON Y; IF K IS L THEN VAR IS PVL ON Y; IF K IS M THEN VAR IS PM ON Y; IF K IS H THEN VAR IS PM ON Y; IF K IS VH THEN VAR IS PM ON Y; IF K IS VH THEN VAR IS PH ON Y;
C6:Phosphorous (P)	IF P IS VL THEN VAR IS PM ON Y; IF P IS L THEN VAR IS PM ON Y; IF P IS M THEN VAR IS PM ON Y; IF P IS H THEN VAR IS PM ON Y; IF P IS VH THEN VAR IS PM ON Y;
C7:Nitrogen (N)	IF N IS VL THEN VAR IS PVL ON Y; IF N IS L THEN VAR IS PL ON Y; IF N IS M THEN VAR IS PL ON Y; IF N IS H THEN VAR IS PL ON Y; IF N IS VH THEN VAR IS PM ON Y; IF N IS VH THEN VAR IS PL ON Y;
C8:Organic matter (OM)	IF OM IS L THEN VAR IS PL ON Y; IF OM IS M THEN VAR IS PL ON Y; IF OM IS H THEN VAR IS PM ON Y;
C9: pH	IF Ph IS VL THEN VAR IS PVL ON Y; IF Ph IS L THEN VAR IS PVL ON Y; IF Ph IS SL THEN VAR IS PVL ON Y; IF Ph IS M THEN VAR IS PVL ON Y; IF Ph IS SH THEN VAR IS PVL ON Y; IF Ph IS H THEN VAR IS PVL ON Y; IF Ph IS H THEN VAR IS PL ON Y; IF Ph IS VH THEN VAR IS PM ON Y;
C10: Sand (S)	IF S IS L THEN VAR IS NM ON Y; IF S IS M THEN VAR IS NM ON Y; IF S IS H THEN VAR IS NM ON Y; IF S IS VH THEN VAR IS NH ON Y;
C11: Clay (Cl)	IF Cl IS L THEN VAR IS PM ON Y; IF Cl IS M THEN VAR IS PM ON Y; IF Cl IS H THEN VAR IS PM ON Y;

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The degree of variation induced by one concept on another one is elaborated through the fuzzy variable "Var" of Figure 4, with the following fuzzy sets ("VVH: very very high", "VH: very high", "H: high", "M: medium", "L: low", "VL: very low", "VVL: very very low"). The influence can be positive or negative.

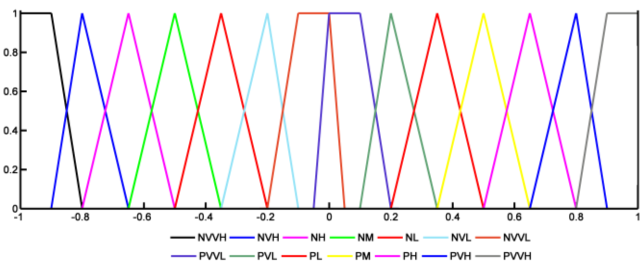

Figure 4.Variation fuzzy variable.

B. *Results and Discussion*

The used data consists of 360 entries measured for the years 2001, 2003 and 2006, as collected in a 5 ha field at Myrina, Karditsa prefecture, Central Greece. The FCM model has been developed based on raster data GIS approach, i.e., the data are stored in a two-dimensional matrix that represents the spatial distribution of every factor in the field. Each cell of the matrix corresponds to an area of 10 x 10 m, which is the spatial resolution of the yield data model. Measurements were collected in soil depth 0–30 cm.

FCM simulations were run on cotton yield data till the model reached convergence. In order to discriminate between the different yield categories, we used the threshold of 0.85 as in [8]. If the estimated yield value is less than 0.85, which means that the yield production is less than the 85% of desired cotton production, then yield is categorized as ''low''. If the estimated yield value is higher than 0.85, then yield is considered as ''high''.

It can be seen from Table III that the predictions made by DRBFCM are actually better than the ones obtained by Papageorgiou et al. [8], even when the NHL algorithm is used. The results of DRBFCM inference revealed an accuracy of predictions of 85.55%, 87.22% and 73.33%, against 73.80%, 67.20% and 69.65% for the conventional FCM model, and against 75.55%, 68.86% and 71.32% for the FCM model with the NHL algorithm, during the years of 2001, 2003 and 2006, respectively.

TABLE III. PREDICTION ACCURACY RESULTS

Year	**2001**	**2003**	**2006**
Conventional FCM accuracy	73.80%	67.20%	69.65%
FCM with NHL algorithm	75.55%	68.86%	71.32%
DRBFCM accuracy	85.55%	87.22%	73.33%

The DRBFCM modeling approach can hence enhance the results obtained by FCM modeling, while taking into consideration the ambiguities intrinsic to this type of applications. From the results of this case study, it can be also concluded that the simulations have proven DRBFCM to be more faithful with regards to the real world model structure. Indeed, in the traditional FCM, parts of the gathered rules are omitted. The antecedent of a rule would just be ignored, and the only thing that is used to draw a conclusion about the influence is the consequence: is the influence "high", "medium" or "low"? Only those are used to generate weights. This results in building a complex dynamic model with nonlinear relationships, and not fully exploiting it in simulating the real world system. The DRBFCM model, on the other hand, is very authentic to the knowledge that has been aggregated. The consequences of the rules are also used, provided that the antecedent is fulfilled: which concept affects the yield, and to what extent?

The differences in predictions are attributed to the generated weights. A recapitulation of weights for the traditional FCM model and DRBFCM are shown in Table IV, where we can clearly see that the influence of Potassium (K) is much lower in DRBFCM ($W_{K\text{-}Y}$ =0.22 ± 1.88E-04) compared to the traditional FCM ($W_{K\text{-}Y}$ =0.6).

TABLE IV. FCM AND DRBFCM WEIGHTS

	Cotton yield (Y)	
Concept	**FCM**	**DRBFCM** [a]
EC	0.25	0.22 ± 2.04E-04
Mg	-0.4	-0.48 ± 4.27E-02
Ca	0.5	0.48 ± 3.15E-02
Na	-0.7	-0.7 ± 2.00E-15
K	0.6	0.22 ± 1.88E-04
P	0.5	0.49 ± 5.24E-02
N	0.4	0.35 ± 1.51E-02
OM	0.4	0.35 ± 8.23E-03
pH	0.1	0.26 ± 2.37E-02
S	-0.6	-0.5 ± 7.07E-16
Cl	0.5	0.5 ± 5.47E-16

[a] Mean values shown with standard deviation obtained using the 360 cotton yield cases.

The low influence produced by DRBFCM seems to make sense as Potassium produces a 'high' variation when it is 'very high', nevertheless by looking at the 360 cases of cotton yield data, Potassium was classified as either 'medium' or 'low' all the time. Hence, DRBFCM seems to generate weights that are coherent with the model structure and collected knowledge, since it produces predictions that can be interpreted by tracing the rules that contributed to the results.

IV. CONCLUSION AND FUTURE WORK

DRBFCM is a rule-based FCM, where relationships between concepts are expressed in the form of fuzzy "if-then" rules that dynamically determine the influence of one concept on another one, while running a simulation for a specific scenario. In this work, we evaluated DRBFCM numerically using cotton yield knowledge and data. We used 360 entries of measured data, collected during three years in an experimental cotton field in Central Greece. The results revealed an accuracy of predictions of 85.55%, 87.22% and 73.33% for the years 2001, 2003 and 2006, respectively.

In comparison to the conventional FCM approach with Fuzzy Weights, DRBFCM proved, in the cotton yield case study, to predict more accurately the yield while being faithful to the real world model. The knowledge collected from the experts is fed into a standard inference system to map adequately the input space into the output space. Hence, the generated projections are more coherent with the model's collected knowledge, and the use of rules improves interpretation of the produced results.

In this study, we used the inference parameters and membership functions as defined in [8]. As a future work, we would like to perform an uncertainty and a sensitivity analysis to gain insights into the factors that would have an impact on the produced predictions, and to attribute the uncertainty in the output to the uncertainties in the input.

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