Funnel Control for a Class of High-Order Nonlinear Systems

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***Abstract*—This paper addresses the problem of funnel output tracking control for a class of unknown high-order nonlinear systems with state feedbacks, which requires to achieve output tracking with prescribed accuracy when both the system nonlinearities and the powers of the system are unknown. Therefore, a robust funnel control algorithm, i.e., a continuous, static, universal, state-feedback controller is explicitly constructed, which ensures that the state errors evolve within the predesigned performance space. The advantages of the proposed funnel output tracking controller when compared with the current approaches lie in the fact that no *a priori* knowledge of system nonlinearities, including generally required bounding functions, is needed. Furthermore, all the powers in each high-order subsystem are not required to be known as well. A simulation example is provided to demonstrate the effectiveness of the proposed algorithm.**

Keywords–nonlinear systems; output tracking; funnel control; unstabilizable linearization.

I. Introduction

Owing to its practical significance and theoretical challenge, the control problem of high-order uncertain nonlinear systems has attracted considerable research effort. Significant progress in different directions, including adaptive regulation, output tracking control with state feedbacks, and finite-time stabilization [1]-[4], has been achieved by adding a power integrator technique and a homogeneous domination method. However, in all aforementioned developments, a priori knowledge of the system nonlinearities and the powers in each subsystem is needed.

Another important issue associated with the control design of unknown high-order nonlinear systems is the prescribed transient behaviour of the closed loop system. Recently, the work [5] introduced the concept of funnel control, which not only deals with unknown system nonlinearities, but also achieves the output tracking with prescribed performance. In particular, via the backstepping procedure, the funnel control methodology has been employed for various classes of nonlinear systems, such as Brunovsky, strict-feedback and pure-feedback systems. Working independently, an alternative approach, called Prescribed Performance Control, was proposed to achieve the same control objective [6]. Unfortunately, both schemes mentioned in [5]-[6] cannot be directly applied to high-order nonlinear systems even if the powers are precisely known, due to the singularity around the origin.

Motivated by the above discussions, this paper focuses on the output tracking problem with prescribed performance via state feedbacks for high-order nonlinear systems with unknown powers and functions. By combining the funnel control technique with barrier Lyapunov functions, the difficulty involved with the singularity problem can be avoided and a continuous, static, universal, state-feedback controller is explicitly constructed, which ensures the predesigned performance. In the proposed universal approach, the barrier Lyapunov functions are employed to enforce the unknown system nonlinearities to be bounded, making constructions of the adaptive laws or function approximators not necessary. Furthermore, the precise knowledge of all the powers in each subsystem is not needed to be known *a priori*. Thus, compared with the current state-of-the-art of the output tracking control, the proposed scheme relaxes significantly the common assumptions in the related works and represents a structurally simple and computationally inexpensive strategy. Finally, simulation results illustrate the effectiveness of the proposed theoretical findings.

The paper is organized as follows: In Section II, the problem addressed is stated. In Section III, the main result of this paper is presented without rigorous stability analysis. Further, in Section IV, a simulation example is provided to demonstrate the effectiveness of the proposed scheme. Conclusions are drawn in Section V.

II. Problem Formulation

Notations: R denotes the set of real numbers. $R_{\geq 0}$ denotes the set of nonnegative real numbers. $R_{>0}$ denotes the set of positive real numbers. R^n denotes the real n-dimensional space. $\mathcal{W}^{1,\infty}(R_{\geq 0}, R_{>0})$ denotes the set of differential functions $\rho : R_{\geq 0} \to R_{>0}$ with ρ and $\dot{\rho}$ being essentially bounded on $R_{\geq 0}$.

Consider the following class of single-input-single-output (SISO) nonlinear systems:

$$\begin{aligned} \dot{x}_i &= d_i(t,x,u)x_{i+1}^{p_i} + \phi_i(t,x,u), \quad i = 1,...,n-1, \\ \dot{x}_n &= d_n(t,x,u)u^{p_n} + \phi_n(t,x,u), \\ y &= x_1, \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1,...,x_i]^T \in R^i$, $i = 1,...,n$; $x = \bar{x}_n = [x_1,...,x_n]^T \in R^n$ are the system states with initial condition $x^0 = [x_1^0,....,x_n^0]^T$, $u \in R$ is the control input, $y \in R$ is the output; p_i, $i = 1,...,n$ are the powers of the system; The system nonlinearities $d_i, \phi_i : R_{\geq 0} \times R^n \times R \to R$, $i = 1,...,n$ are locally Lipschitz in x and u, and piecewise continuous in t.

For simplicity of presentation, denote $x_{n+1} = u$. The following assumptions are made.

Assumption 1: The powers p_i, $i = 1,...,n$ are positive odd integers, which may be *unknown*.

Assumption 2: There exist unknown continuous and strictly positive functions $c_i : R^i \to R$ and $\bar{c}_i : R^{i+1} \to R$, $i = 1, ..., n$ such that

$$0 < c_i(\bar{x}_i) \leq d_i(t, x, u) \leq \bar{c}_i(\bar{x}_{i+1}), \; i = 1, ..., n. \quad (2)$$

Assumption 3: There exist unknown continuous non-negative functions $\bar{\phi}_{ij} : R^i \to R$, $i = 1, ..., n$, $j = 0, ..., p_i - 1$ such that

$$|\phi_i(t, x, u)| \leq \sum_{j=0}^{p_i-1} |x_{i+1}|^j \bar{\phi}_{ij}(\bar{x}_i), \quad i = 1, ..., n. \quad (3)$$

Assumption 4: The desired trajectory y_r is bounded, continuous and available, and $\dot{y}_r$ is bounded but its bound may not be available.

Remark 1: Assumptions 1-3 are sufficient conditions for global controllability of the system (1), which are extensively used in the literature [3]-[4]. It should be stressed that the developed controller in the sequel does not require the analytical expressions of system nonlinearities $d_i(t, x, u)$, $\phi_i(t, x, u)$ and their bounding functions $c_i(\bar{x}_i)$, $\bar{c}_i(\bar{x}_{i+1})$, $\bar{\phi}_{il}(\bar{x}_i)$, in contrast to some results in [3]-[4].

The *control objective* is to design a state-feedback controller

$$u = \alpha(t, x, y_r) \quad (4)$$

such that

- all signals in the closed loop system are globally bounded;
- the tracking error $e = y - y_r$ evolves within a prescribed performance funnel

$$\mathcal{F}_\rho := \Big\{(t, e) \in R_{\geq 0} \times R \Big| |e| < \rho_1\Big\}, \quad (5)$$

  which is determined by a performance function $\rho_1 \in \mathcal{W}^{1,\infty}(R_{\geq 0}, R_{>0})$ incorporating the desired performance specifications.

III. Funnel Controller Design

In this section, we will construct a funnel controller for system (1) via barrier Lyapunov functions [7]. The design procedures of the proposed funnel controller are given as follows.

Step 1 : Preselect the first performance function $\rho_1 \in \mathcal{W}^{1,\infty}(R_{\geq 0}, R_{>0})$ that satisfies $\rho_1(0) > |x_1(0) - y_r(0)|$ and guarantees the desired performance specifications regarding the steady state error and the speed of convergence. Let $z_1 := e = x_1 - y_r$ and $\xi_1 := \frac{z_1}{\rho_1}$, then, the first virtual law is designed as

$$\alpha_1 = \frac{-k_1\xi_1}{1 - \xi_1^2}, \quad (6)$$

where k_1 is a positive constant.

Step $i(i = 2, \cdots, n)$: Preselect the i-th performance function $\rho_i \in \mathcal{W}^{1,\infty}(R_{\geq 0}, R_{>0})$ that satisfies $\rho_i(0) > |x_i(0) - \alpha_{i-1}(0)|$. Define $z_i := x_i - \alpha_{i-1}$ and $\xi_i := \frac{z_i}{\rho_i}$, then, the i-th virtual and actual control laws are designed as

$$\alpha_i = \frac{-k_i\xi_i}{1 - \xi_i^2}, \quad (7)$$

$$u = \alpha_n, \quad (8)$$

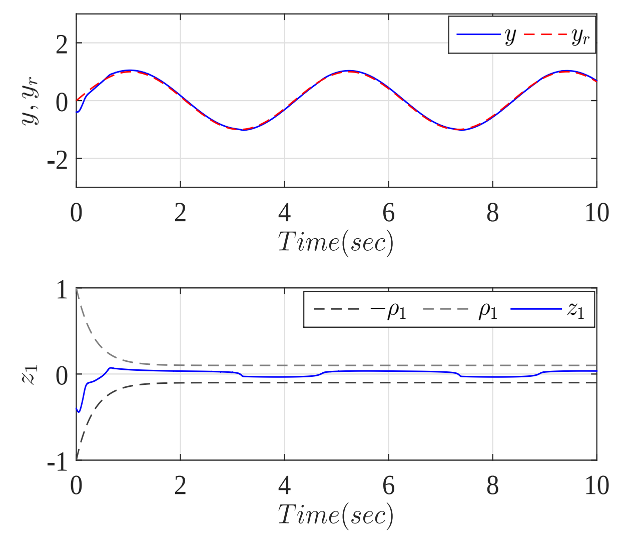

Figure 1. Output tracking performance.

where k_i is a positive constant.

Remark 2: The features of the proposed scheme lie in the fact that the exact knowledge of system nonlinearities, including generally required bounding functions, is not needed to be a priori, and all the powers in each high-order subsystem are allowed to be any unknown positive odd rational numbers. Moreover, compared with adaptive robust control approaches, no adaptive techniques are utilized in the developed controller.

Remark 3: In the proposed control design, the prescribed transient behaviour is imposed by appropriately selecting the performance function ρ_1, other controller parameters ρ_i, $i = 2, ..., n$, and k_i, $i = 1, ..., n$, are chosen flexibly according to the conditions $\rho_i(0) > |x_i(0) - \alpha_{i-1}(0)|$, $i = 2, ..., n$.

IV. A Simulation Example

To illustrate the correctness and effectiveness of the theoretical findings, we consider the following second order nonlinear system:

$$\begin{aligned} \dot{x}_1 &= (4 - \sin(x_1))x_2^3 + \sin(x_1)x_2 + x_1 e^{x_1\cos(x_2)}, \\ \dot{x}_2 &= (3 + \sin(t))u^3 + \cos(x_1)e^{x_2\sin(x_1)}, \\ y &= x_1, \end{aligned} \quad (9)$$

where the initial condition is $[x_1(0), x_2(0)]^T = [-0.4, 0.5]^T$. The control purpose is to force the output y to track the desired trajectory $y_r = \sin 1.5t$ with steady state error no more than 0.1 and minimum speed of convergence as obtained by the exponential e^{-3t}.

By selecting appropriately the design parameters and applying the proposed controller, the simulation result on the output tracking performance is presented in Figure 1, in which it can be observed that the prescribed performance of the tracking error is achieved.

V. Conclusion

This paper has studied the funnel output tracking problem for unknown high-order nonlinear systems. By combining the funnel control technique with barrier Lyapunov functions, we have exploited a constructive approach for designing the global universal controller, which achieves the predesigned performance of the state errors. Contrary to the current state-of-the-art of the output tracking control, the proposed funnel control does not incorporate any prior knowledge of system nonlinearities and the powers in each subsystem. Moreover, instead of utilizing adaptive laws or function approximators, the unknown system nonlinearities are guaranteed to be bounded via the barrier Lyapunov functions. Simulations performed on an illustrative example verify and clarify the theoretical findings. As a future work, we will apply the proposed method to an underactuated unstable two degree of freedom mechanical system [1].

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