Formal Performance Measures for Asymmetric Communication

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***Abstract* — Typically, any asymmetric network is characterized either by non-uniform link transmission bit rate or by uneven traffic intensity. Through monitoring and asymmetric network status control one can improve performance of the service-sector. In this paper, some exact analysis methods, oriented to achieve comparatively high utilization of the data packet transmission link and satisfy the service quality of the asymmetric loss and queueing systems, are proposed. The developed mathematical approach turns out to be extremely useful for the analysis of the network node with asymmetric transmission links. Some comments concerning the application of a few different strategies to the selection of an unoccupied data packet transmission link are discussed. The straight process analysis in the asymmetric loss and queueing systems is carried out using continuous-time Markov chains. Performance of the asymmetric system is measured using accurate expressions. Finally, in the case of Poisson arrivals and exponential transmission times, an exact analytical model is applied to the system.**

Keywords - queueing system; quality of service; asymmetric system; Markov process.

I. INTRODUCTION

Computation of the performance measures is simple enough if all data packet transmission links have the same parameters and packet transmission times are distributed exponentially. Though, in the data transmission network, heterogeneous transmission links appear often and often. Performance of the data transmission over an asymmetric system with multiple links depends on a particular link that transmits a data packet. Therefore, it is highly expedient and useful to compute accurately the performance measures of the said asymmetric systems. The more detailed description of the latter systems is presented in [1, 2]. In [3], it is shown that a well-known approach to investigating symmetric M/M/m and GI/G/m systems (Kendall's notation is used) can be applied also to the suchlike asymmetric systems, provided the service rates of data packet transmission links differ only slightly (by a ratio <10) and the system utilization is high enough. Some interesting methods, used to compute several characteristics of the asymmetric queueing systems, are described in [4] wherein computations reduce to solving equations associated with continuous-time Markov chains. In such a way, some accurate results are obtained for elementary asymmetric lossless M/M/m queueing systems. In [5], the asymmetric finite capacity queueing systems are analyzed with the use of analytical and simulation models based on Moore and Mealy automata. In [6], performance measures for an asymmetric node, with a priority flow and two data transmission links, are estimated applying analytical and simulation models. Advanced resources sharing methods, in asymmetric networks, are proposed in [7]. In [8], Lakshman et al. produce the network control mechanism facilitating and supporting TCP/IP data transmission over the asymmetric networks. Authors determine the throughput as a function of buffering, and state conditions under which the transmission link is fully utilized. In [9], Krithikaivasan et al., employing control and routing, outline rigorously how to improve performance in congested parts of the asymmetric network.

Not going into minor details, we here emphasize that asymmetric systems, as well as their performance analysis, are far from being investigated thoroughly. Further research is necessary.

The rest of the paper is organized as follows. Section II introduces an asymmetric loss system. Section III evaluates data packets arrival rate impact on the performance measures of the asymmetric loss system. Analysis of the data packet rate impact on the performance measures of the asymmetric queueing system is presented in Section IV. Some conclusive commentary is presented in Section V.

II. ANALYSIS OF THE ASYMMETRIC LOSS SYSTEM

An asymmetric loss system with different rates of the data packet transmission links is modelled. The functional diagram of the system is presented in Fig. 1.

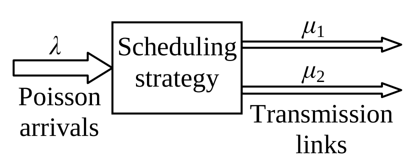

Figure 1. The functional architecture of an asymmetric loss system.

The best performance measures of the system, such as the link utilization and the data packet loss, are obtained by switching between transmission links.

Let us denote the data packet transmission rate over a link i (i=1, 2) by μ_i. We shall take an exact analytical model

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of the asymmetric loss system with Poisson data packet arrivals (intensity λ) and exponential data packet transmission time over each link (intensities μ_1 and μ_2, $\mu_1 > \mu_2$). Such an asymmetric system can be represented as the continuous-time Markov chain (the system itself being in a stable state). Two parameters (components of the vector *XY*) are attached to each state of the system, where *X* represents the state of the data packet transmission link 1, and *Y* represents that of the data packet transmission link 2. If *X* or *Y* equals zero the respective link is free (unoccupied), otherwise (*X* or *Y* equals 1), the respective link is busy. Let us consider a loss system, provided a few different strategies for selecting transmission link are applied (Fig. 2 – Fig. 4).

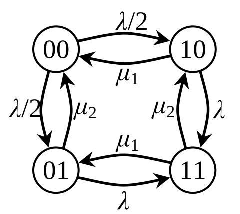

Figure 2. Markov process for the asymmetric loss system (the data packet transmission links are occupied, with the same intensities; *Case* 1).

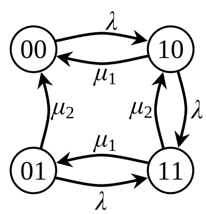

Figure 3. Markov process for the asymmetric loss system (the faster transmission link 1 is occupied first; *Case* 2).

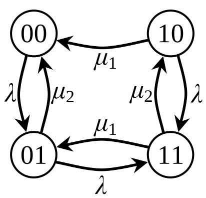

Figure 4. Markov process for the asymmetric loss system (the slower transmission link 2 is occupied first; *Case* 3).

The usage of the global balance concept for the Markov chains enables us to put down the following equations (for evaluation of the system state probabilities P_{XY}):

In *Case* 1 (Fig. 2),

$$\begin{cases} P_{00} + P_{10} + P_{01} + P_{11} = 1, \\ \lambda P_{00} - \mu_1 P_{10} - \mu_2 P_{01} = 0, \\ (\lambda + \mu_1) P_{10} - \dfrac{\lambda}{2} P_{00} - \mu_2 P_{11} = 0, \\ (\lambda + \mu_2) P_{01} - \dfrac{\lambda}{2} P_{00} - \mu_1 P_{11} = 0, \\ (\mu_1 + \mu_2) P_{11} - \lambda P_{10} - \lambda P_{01} = 0. \end{cases} \quad (1)$$

In *Case* 2 (Fig. 3),

$$\begin{cases} P_{00} + P_{10} + P_{01} + P_{11} = 1, \\ \lambda P_{00} - \mu_1 P_{10} - \mu_2 P_{01} = 0, \\ (\lambda + \mu_1) P_{10} - \lambda P_{00} - \mu_2 P_{11} = 0, \\ (\lambda + \mu_2) P_{01} - \mu_1 P_{11} = 0, \\ (\mu_1 + \mu_2) P_{11} - \lambda P_{10} - \lambda P_{01} = 0. \end{cases} \quad (2)$$

In *Case* 3 (Fig. 4),

$$\begin{cases} P_{00} + P_{10} + P_{01} + P_{11} = 1, \\ \lambda P_{00} - \mu_1 P_{10} - \mu_2 P_{01} = 0, \\ (\lambda + \mu_1) P_{10} - \mu_2 P_{11} = 0, \\ (\lambda + \mu_2) P_{01} - \lambda P_{00} - \mu_1 P_{11} = 0, \\ (\mu_1 + \mu_2) P_{11} - \lambda P_{10} - \lambda P_{01} = 0. \end{cases} \quad (3)$$

The asymmetric system state probabilities P_{XY} are obtained by solving the above linear systems. In particular, one can easily find some other system performance measures, such as:

- the data packet transmission link utilization

$$\rho_1 = P_{11} + P_{10}, \\ \rho_2 = P_{11} + P_{01}; \quad (4)$$

- the data packet loss probability

$$P_{loss} = P_{11}. \quad (5)$$

III. DATA PACKETS ARRIVAL RATE IMPACT ON THE PERFORMANCE MEASURES OF THE ASYMMETRIC LOSS SYSTEM

Performance measures of the asymmetric loss system, represented in the form of a function of the data packets arrival rate λ, are shown in Fig. 5 and Fig. 6.

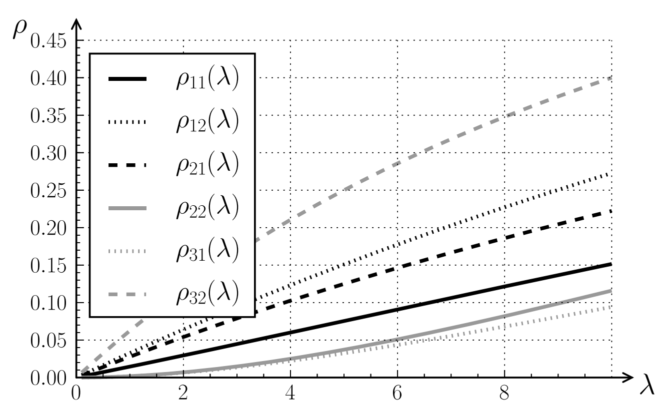

Figure 5. The data packet transmission link utilizations as a function of λ (*Cases* 1,2,3; μ_1=35 and μ_2=15).

In Fig. 5, the dependence of the data packet transmission link utilizations ρ_{i1} and ρ_{i2} on the strategy used to select an unoccupied data transmission link i $(i = 1,2,3)$ facilitates selection of the data transmission link (*Cases* $1,2,3$).

The data packet loss probability P_{loss} attains its maximal value in *Case* 3 and minimal value in *Case* 2 (the faster transmission link is occupied first; Fig. 6).

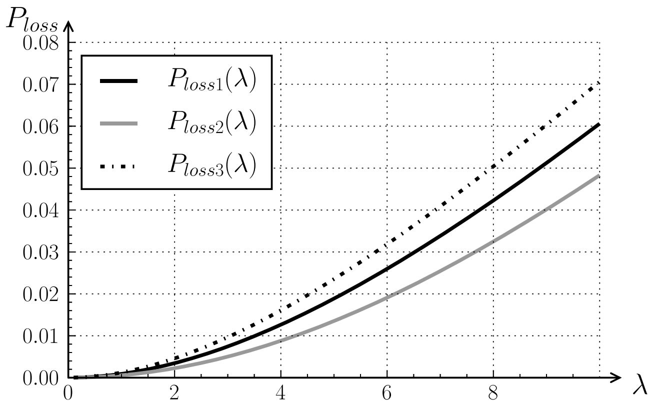

Figure 6. The data packet loss probabilities as a function of the data packet arrival rate λ, assuming different data packet transmission link scheduling strategies are applied (*Cases* 1,2,3; μ_1=35, μ_2=15).

We here observe that the analytical model of the queueing system is accurate only in the case of Poisson arrivals and exponential data packet transmission times (in the links).

IV. Data Packets Arrival Rate Impact on the Performance Measures Of The Asymmetric Queueing System

In this section, an asymmetric queueing system with two data packet transmission links is analysed. To estimate the queueing system performance measures, an exact analytical model has been developed. The queueing system itself is characterized by different data packet transmission rates μ_1, μ_2 and a finite buffer of size K (the lower part; Fig. 7).

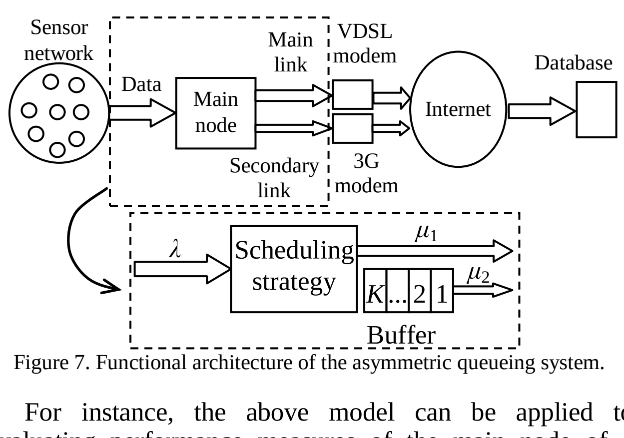

Figure 7. Functional architecture of the asymmetric queueing system.

For instance, the above model can be applied to evaluating performance measures of the main node of a sensor network. As the basis for calculations, the situation shown in the upper part of Fig. 7 is chosen. The data flow, from the network of sensors, is directed to the main node which sends the data over the Internet to the remote database. The main node has two data transmission links: the primary link that is connected to the Internet over VDSL modem, and the secondary link that is connected to the Internet over 3G modem. The mean data transmission rates over the primary and the secondary links are equal to C_1=4Mb/s=500000B/s and C_2=2Mb/s=250000B/s, respectively. The secondary link is used if and only if the primary link is busy. The mean length of the data packet equals L=1000B. Therefore, the buffer of B=16KB can store up to K=B/L=16 data packets. The data packet transmission intensities over the first and the second links equal μ_1=C_1/L=500 packets/s and μ_2=C_2/L=250 packets/s, respectively.

Consider the Poisson data packet arrival flow, with intensity λ, and the data packet transmission time (over each link) distributed exponentially. The data packet transmission link scheduling strategy is such that the link with data packet transmission rate μ_1 is occupied first. The data packet from the buffer (finite capacity) is transmitted only over the second transmission link with transmission rate μ_2. A birth-and-death Markov model of the suchlike queueing system is shown in Fig. 8.

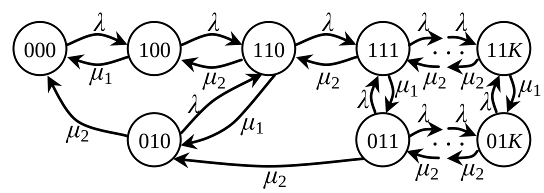

Figure 8. The continuous-time Markov chain for the asymmetric queueing system.

Each steady state of the system is described using three parameters (components of the vector XYZ), where X represents the state of the first link (0 – unoccupied, 1 –busy) Y represents that of the second link and Z represents the number of data packets in the buffer (in our case, from 0 to K).

For finding the system state probabilities, the following system of algebraic equations is used:

$$\begin{cases} \lambda P_{000} - \mu_1 P_{100} - \mu_2 P_{010} = 0; \\ (\lambda + \mu_1) P_{100} - \lambda P_{000} - \mu_2 P_{110} = 0; \\ (\lambda + \mu_2) P_{010} - \mu_1 P_{110} - \mu_2 P_{011} = 0; \\ (\lambda + \mu_1 + \mu_2) P_{110} - \lambda P_{100} - \lambda P_{010} - \mu_2 P_{111} = 0; \\ (\lambda + \mu_2) P_{011} - \mu_1 P_{111} - \mu_2 P_{012} = 0; \\ (\lambda + \mu_1 + \mu_2) P_{111} - \lambda P_{011} - \lambda P_{110} - \mu_2 P_{112} = 0; \\ (\lambda + \mu_2) P_{012} - \mu_1 P_{112} - \mu_2 P_{013} = 0; \\ (\lambda + \mu_1 + \mu_2) P_{112} - \lambda P_{111} - \lambda P_{012} - \mu_2 P_{113} = 0; \\ ------------------ \\ (\lambda + \mu_2) P_{01K} - \mu_1 P_{11K} = 0; \\ (\mu_1 + \mu_2) P_{01K} - \lambda P_{11(K-1)} - \lambda P_{01K} = 0. \end{cases} \quad (6)$$

The obtained state probabilities P_{XYZ} can be applied to finding performance measures of the above asymmetric system, such as:

- data packet loss probability

$$P_{loss} = P_{11K}\,; \tag{7}$$

- data packet transmission link utilizations

$$\rho_1 = P_{100} + \sum_{i=0}^{K} P_{11i}\,; \tag{8}$$

$$\rho_2 = \sum_{i=0}^{K} P_{11i} + \sum_{i=0}^{K} P_{01i}. \tag{9}$$

Let us denote the data packet arrival (to the first and the second links) intensities by λ_1 and λ_2, respectively. Then the link utilization can be alternatively computed this way:

$$\rho_1 = \lambda_1 / \mu_1\,;\ \rho_2 = \lambda_2 / \mu_2\,; \tag{10}$$

here

$$\lambda_1 = \lambda(P_{000} + \sum_{i=0}^{K} P_{01i})\,; \tag{11}$$

$$\lambda_2 = \lambda(P_{100} + \sum_{i=0}^{K-1} P_{11i})\,. \tag{12}$$

The average number of the data packets in the buffer equals

$$\overline{N_q} = \sum_{i=1}^{K} i \cdot P_{11i} + \sum_{i=1}^{K} i \cdot P_{01i}\,. \tag{13}$$

The mean waiting time value (for the data packet) in the queue is obtained in accordance with Little's theorem, i.e.

$$\overline{W} = \frac{\overline{N_q}}{\lambda_2} = \frac{\overline{N_q}}{\mu_2 \rho_2}. \tag{14}$$

The probability that a new data packet will enter the queue is given by

$$P_{wait} = P(W > 0) = \sum_{i=0}^{K} P_{11i}. \tag{15}$$

The average number of data packets in the asymmetric queueing system

$$\overline{N_s} = \overline{N_q} + \rho_1 + \rho_2\,. \tag{16}$$

The average time, spent by the data packet in the asymmetric queueing system, equals

$$\overline{T_s} = \frac{\lambda_1}{\mu_1(\lambda_1 + \lambda_2)} + \frac{\lambda_2}{\mu_2(\lambda_1 + \lambda_2)} + \overline{W}. \tag{17}$$

Performance measures of the queueing system, expressed in the form of a function of the queueing system parameters λ and K, are shown in Fig. 9 - Fig. 15.

The data packet loss probability increases considerably when the data packet transmission link utilization achieves 0.5 (λ>400) (Fig. 9).

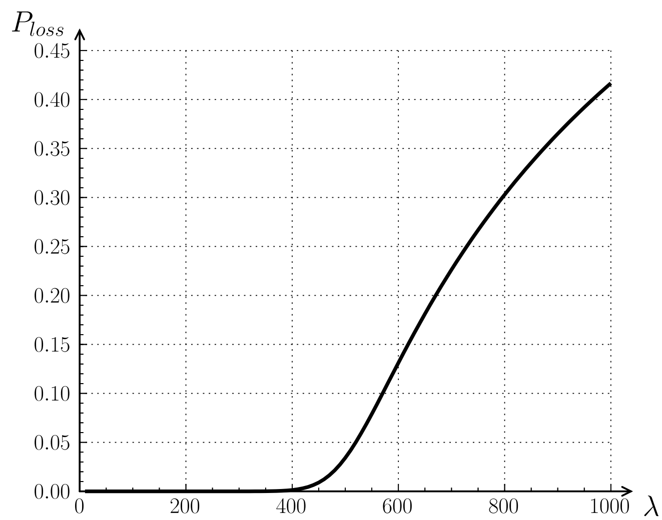

Figure 9. The data packet loss probability P_{loss} as a function of λ, given μ_1=500, μ_2=250, K=16.

The data packet loss in the system occurs if and only if the buffer is full. The data packet loss probability can be lessened in several ways: by increasing the buffer capacity, by increasing the data transmission rate over the links or by decreasing the data packet arrival rate. In the given example, the decrease of the data packet arrival rate is achieved by limiting the number of data collection sensors. For instance, if one of the data sensors produces 10 data packets per second, then the main node can serve 40 sensors with minimal risk of data packet loss.

The link utilization level turns out to be another important concern. The right estimation of the link utilization level is used to guarantee that the packet loss will not occur. Also, the estimation results can be used to evaluate economic aspects of the link usage. For instance, the cost of the data transmission over the secondary link over the 3G Internet connection can be higher. Thus, the given model can be explored to estimate how intensively the links will be used, calculate the usage price or make a decision concerning data transmission rates operable in the links.

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In Fig. 10, dependence of the link utilization on the data packet arrival intensity λ, is demonstrated. It can be seen that the links are used according to the selected scheduling strategy: first of all, the primary link (with greater data transmission rate) is occupied, the secondary link is used if and only if the first one is busy.

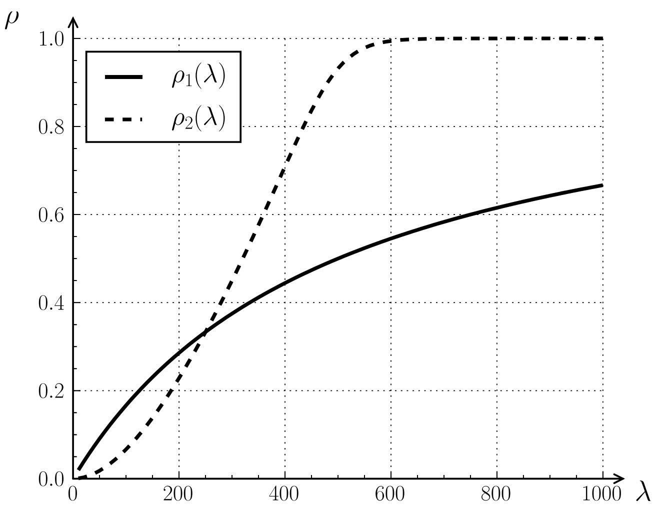

Figure 10. The data packet transmission link utilization ρ_1 and ρ_2 as a function of λ, given μ_1=500, μ_2=250, K=16.

The selected link usage (scheduled) strategy also affects other performance measures.

The average number of data packets in the queue (Fig. 11) rapidly increases when the primary link is busy and the loading of the secondary link goes up.

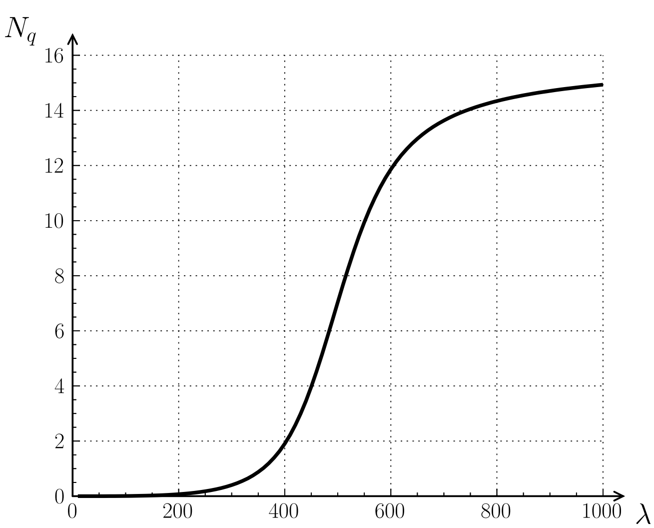

Figure 11. The mean value of data packets in the buffer N_q as a function of λ, given μ_1=500, μ_2=250, K=16.

The probability that an arriving data packet will enter the queue (Fig. 12) also increases when the intensity λ of data packet arrival is increased. Although, it is clear that it should be in this way, but the proposed model gives the exact values, which have an interesting nonlinear fashion.

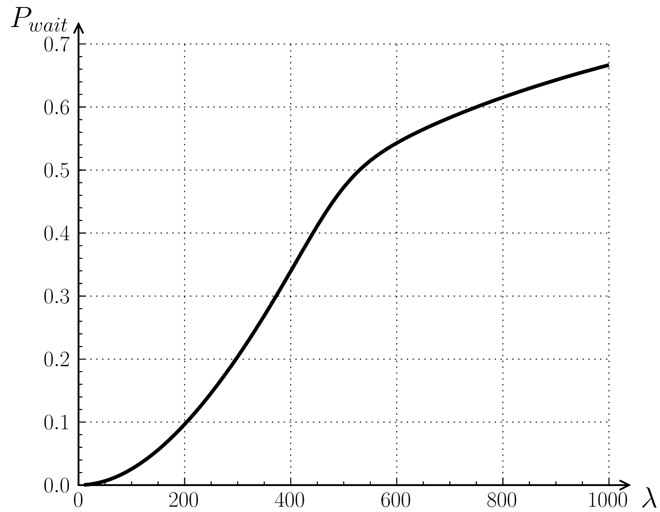

Figure 12. The probability that an arriving data packet will enter the queue $P(W>0)$ as a function of λ, given μ_1=500, μ_2=250, K=16.

The mean values of the time, spent by a data packet in the queue (W) and in the system (T_s), are very important parameters (Fig. 13). Those values facilitate evaluation of the data packet transmission delay or the processing rate.

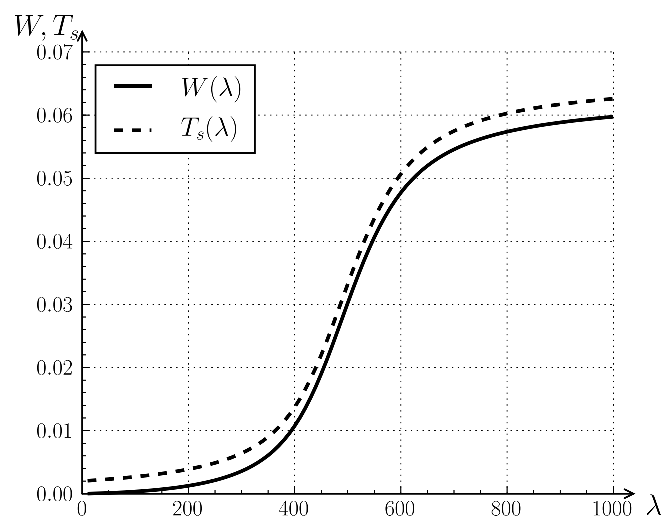

Figure 13. The mean values of the time, spent by a data packet in the queue W [seconds] and in the system T_s [seconds], as a function of λ, given μ_1=500, μ_2=250, K=16.

It is recommended to transmit data packets, which are sensitive to delay, via the link 1 (in the presented asymmetric system).

The size of the buffer also influences the system performance parameters. The influence degree can be estimated using the proposed model.

The data packet loss probabilities P_{loss}, expressed in terms of λ and K, are presented in Fig. 14. As it can be seen, the greater the buffer K, the lesser the probabilities P_{loss}. On the other hand, the difference is negligible, as the packet transmission link utilization approaches 1 (λ>750).

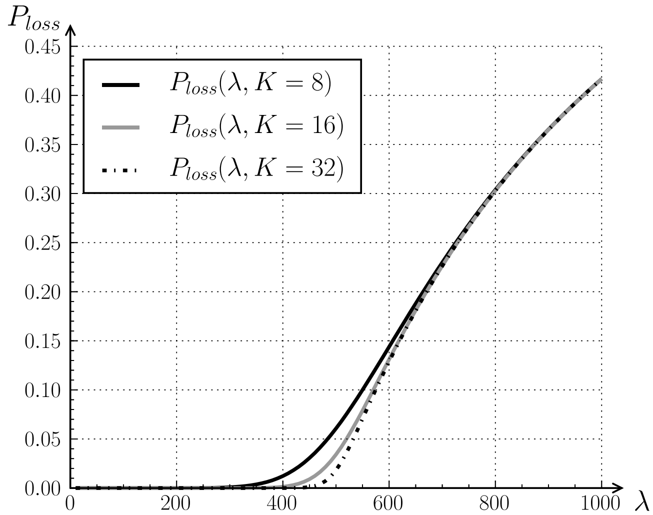

Figure 14. The data packet loss probability P_{loss} as a function of λ and K, given μ_1=500, μ_2=250, K=8,16,32.

The mean values of time spent by a data packet in the system T_s, expressed in terms of λ and K, are presented in Fig. 15.

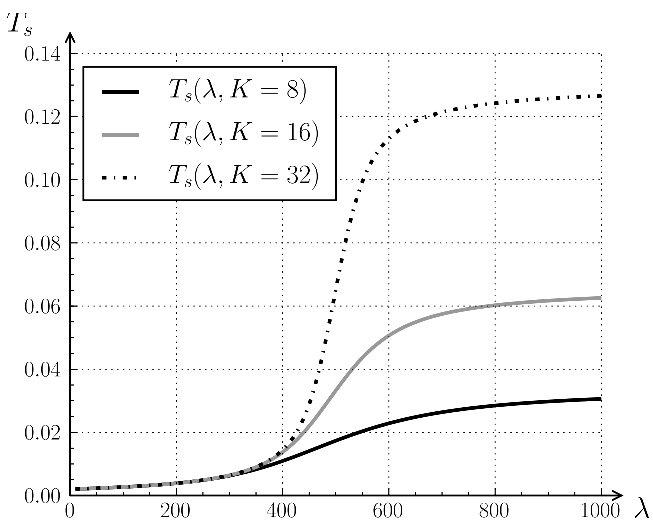

Figure 15. The mean values of time spent by a data packet in the system T_s [seconds] as a function of λ and K, given μ_1=500, μ_2=250, K=8,16,32.

The values of W appear to be greater for greater values of K. It can also be seen that the values of T_s grow apart, as the data packet transmission link utilization approaches 1 (λ>750).

V. Conclusion And Future Work

In the paper, the queueing performance measures, such as the probability of the data packet loss in a finite buffer, the mean queue length, the mean waiting time, the arrival rate impact on the performance measures, are investigated using appropriate analytical models. In the general case (say, non-Poisson data packet flow, non-exponential service time distribution), an exact analytical model turns out to be very complicated. So, simulation is recommended to achieve task-oriented investigation results.

Obviously, the proposed formal approach to the analysis of asymmetric systems is nothing but the starting point for those who are interested in the processes associated with asymmetric data packet transmission systems, i.e. for those specialists who wish to identify new research trends in the area of asymmetric transmission systems for better resource sharing and increasing transmission link performance measures.

Undoubtedly, accurate modelling of the data packet transmission processes in asymmetric systems is truly an important step in optimizing any data transmission network.

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